Lattice Field Theory on Spin Foams

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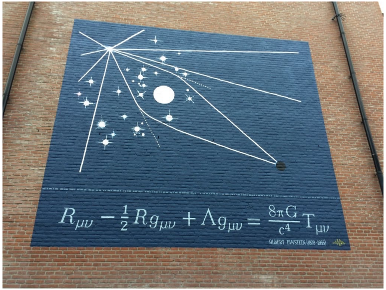
Gravity and quantum matter?

- **Matter indispensable** to describe our **universe**
  - Cosmology, early universe, black holes...
- **Quantum matter on dynamical space-time?**
  - Reconcile general relativity with quantum field theory
- Does quantum gravity affect the **matter sector**?
  - **Asymptotic safety**: restriction on matter sector from quantum gravity [Dona, Eichhorn, Percacci '14] and quantum gravity influences particle physics [Eichhorn, Held '18]

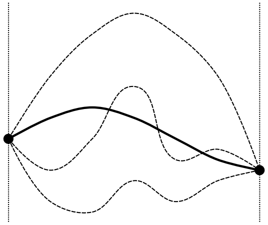


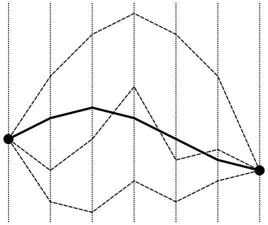
Incorporating matter is **vital** to approaches of **pure quantum gravity**.

Consistency check: recover quantum field theory on fixed (background) space-time

This talk: **free, massive scalar field** on a simplified spin foam

Spin foam gravity [Rovelli, Reisenberger, Barrett, Crane, Freidel, Livine, Krasnov, Perez, Speziale, Engle, Pereira, Kaminski...]



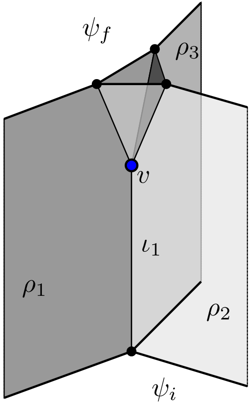


- **Path integral of geometries**
- Regulator: **Discretization** / 2-complex
- **Quantum geometric** building blocks
  - (Constrained) **topological quantum field theory**
  - Discrete area spectrum
- Physical content: **Transition amplitudes**
  - Assign an amplitude $\mathcal{A} \sim e^{iS_{\text{EH}}}$ to each geometry
  - **Single building block** $\sim$ **discrete gravity** [Conrady, Freidel '08, Barrett, Dowdall, Fairbairn, Gomes, Hellmann '09, Kaminski, Kisielowski, Sahlmann '17, Liu, Han '18, Simão, S.St. '21]
  - Quantum amplitudes (not Wick-rotated)

No reference to background geometry

Aim to implement **diffeomorphism symmetry**

Spin foam gravity - Basics

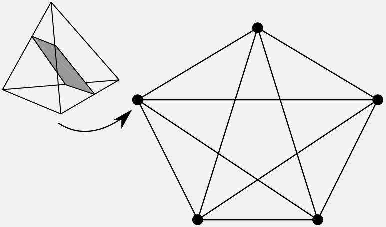


- Regulator: (dual) **2-complex** Δ^*
  - Vertices v, edges e, faces f
- Coloured with group theoretic data $\{\rho_f, \iota_e\}$
- Boundary graph $\sim$ **3D geometry**
  - **Polyhedra** $\sim$ intertwiner ι_e
  - **Area** of face $\sim$ representation ρ_f
- **Evolution**: bulk geometry
  - History interpolating between boundaries
- **Sum over all histories**
  - Sum over all ι and ρ
  - Assign amplitude to each history
- **Amplitude functionals**: $\mathcal{A}_b : \mathcal{H}_b \to \mathbb{C}$
  - From initial to final state: $\mathcal{H}_i \otimes \mathcal{H}_f^*$: $\langle \psi_f, \psi_i \rangle_{\mathcal{A}}$

Partition function and geometric interpretation

- **Amplitudes locally** assigned to building blocks

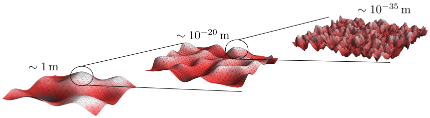
$$Z(\Delta^*) = \sum_{\rho_f, \iota_e} \prod_{f \in \Delta^*} \mathcal{A}_f(\rho_f) \prod_{e \in \Delta^*} \mathcal{A}_e(\iota_e) \prod_{v \in \Delta^*}$$



Quantum space-time as a **superposition of quantum geometric building blocks**

Matter in spin foams

- How to **incorporate matter** in spin foam quantum gravity?
  - Matter on top of spin foam [Oriti, Pfeiffer '03, Speziale '07, Bianchi, Han, Rovelli, Wieland, Magliaro, Perini '13]
  - Unification scenarios [Crane '00, Smolin '09]
  - Massless scalar field [Lewandowski, Sahlmann '15, Kisielowski, Lewandowski '18]
- **Computational challenges** for pure spin foams:
  - **Vertex amplitude:** numerical algorithm for EPRL/FK model [Dona, Fanizza, Sarno, Speziale '19, Dona, Gozzini, Sarno '20]
  - **Sum over configurations:** effective spin foam algorithm [Asanta, Dittrich, Haggard PRL '20, Asante, Dittrich, Padua-Argüelles '21]
  - **Observables**: MCMC on Lefshetz thimbles [Han, Huang, Liu, Qu, Wan '20]



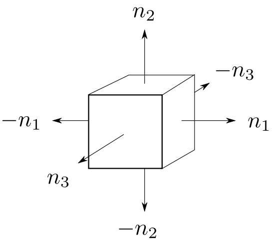
- What is **matter at the Planck scale** and how can we connect to **observable physics**?

First test: **Massive scalar field** on a **restricted 4D spin foam**

Outline

Towards a simplified model [Bahr, S.St. '15]

- **Strategy**: study a **subset** of the full spin foam path integral
- **Quantum cuboids**: 4D Riemannian EPRL model [Engle, Pereira, Rovelli, Livine '08] on hypercubic 2-complex [Lewandowski, Kaminski, Kisielowski '09']



- **Restrict shape** of intertwiner
  - **Coherent** SU(2)-intertwiner [Livine-Speziale '07]

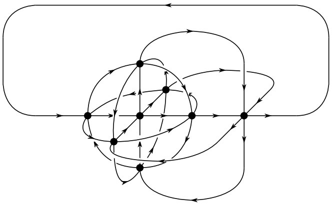
$$|\iota_{j_1,j_2,j_3}\rangle = \int_{\mathrm{SU}(2)} dg\; g \triangleright \bigotimes_{i=1}^{3} |j_i, e_i\rangle \otimes |j_i, -e_i\rangle$$

  - Peaked on the shape of a cuboid
  - e_i normal unit vectors in $\mathbb{R}^3$.

Drastic restrictions on spins and intertwiners.

Asymptotic expansion of full amplitude.

Semi-classical spin foam amplitudes [Bahr, S.St. '15]



- Vertex amplitude $\mathcal{A}_v$

$$\mathcal{A}_v \sim \int_{\mathrm{SU}(2)^8} \prod_a dg_a \, e^{\sum_{ab} 2j_{ab} \ln(\langle -\vec{n}_{ab} | g_a^{-1} g_b | \vec{n}_{ba} \rangle)}$$

- Face amplitude $\mathcal{A}_f \sim (2j+1)^{2\alpha}$

- **Stationary phase approximation**:

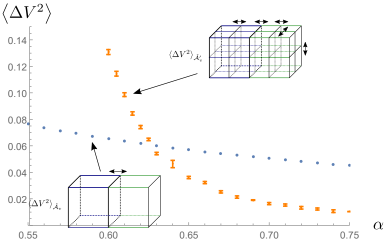
$$\hat{\mathcal{A}}_v(j_1, j_2, j_3, j_4, j_5, j_6) \sim j_i^{2\alpha} \left(\frac{1}{\sqrt{D}} + \frac{1}{\sqrt{D^*}} \right)^2$$

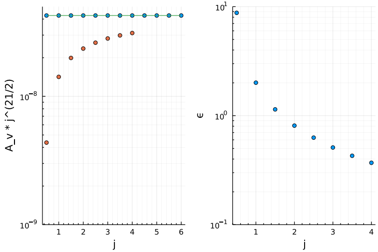
- Spins $\{j_i\}_{i=1,2,\ldots,6}$ $\rightarrow$ edge lengths $\{l_k\}_{k=1,2,3,4}$

Flat space-time: discrete gravity / Regge action vanishes.

Model: **superpostion of hypercuboidal, flat lattices**.

Cuboids as a test case





- **Renormalization group flow** by comparing **observables** on different foams [Bahr, S.St. PRL '16, PRD '17]

$$\langle \mathcal{O} \rangle_{\alpha}^{\text{fine}} \approx \langle \mathcal{O} \rangle_{\alpha'}^{\text{coarse}}$$

Indication for a **UV-attractive fixed point**

and restored **diffeomorphism symmetry**

- **Spectral dimension** [S.St., Thürigen '18]
  - Superposition of geometries $\rightarrow$ change of effective dimension measure
- **Quantum amplitudes** [Allen, Girelli, S.St. '22]
  - Amplitude non-oscillatory also in quantum regime

Outline

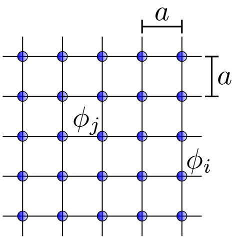
Interlude: scalar field on a regular lattice

- **Continuum** free scalar field with mass M_0:

$$Z = \int \mathcal{D}\phi \, e^{-\int d^4x \, \mathcal{L}(\phi, \partial_\mu \phi)}, \quad \mathcal{L}(\phi, \partial_\mu \phi) = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{M_0^2}{2}\phi^2$$

- Discretisation: **lattice constant** a

$$Z = \int \prod_i d\phi_i \, e^{-\frac{1}{2}\phi_i K_{ij} \phi_j}, \quad K_{ij} = -a^2 \sum_\mu (\delta_{i+e_\mu,j} + \delta_{i-e_\mu,j} - 2\delta_{ij}) + a^4 M_0^2 \delta_{ij}$$



- Theory depends on **relation** of M_0 and a
  - Lattice mass $M(a) = aM_0$
- Correlations: $\langle \phi_i \phi_j \rangle = K_{ij}^{-1} \sim \exp\{-d_{ij} M_0\}$

Straightforward continuum limit for $a \to 0$.

Correlation length (in units of a) diverges for $M(a) \to 0$.

Discrete exterior calculus

- **Generalize** lattice field theory to irregular lattice
- **Discrete exterior calculus** [Desbrun, Hirani, Leok, Marsden '05, Arnold, Falk, Winther '09, McDonal, Miller '10, Sorkin '75, Calcagni, Oriti, Thürigen '12]
  - Project p-forms onto p-dim objects $|\sigma_p\rangle$
  - ϕ p-form smeared on p-surface $\mathcal{S} = \sum_i |\sigma_p^i\rangle$:

$$\phi(\mathcal{S}) = \langle\phi|\mathcal{S}\rangle = \sum_i \langle\phi|\sigma_p^i\rangle = \sum_i V_{\sigma_p^i}\phi_{\sigma_p^i} = \sum_i \int_{\sigma_p^i}\phi = \int_{\mathcal{S}}\phi$$

- Dual $(d-p)$-cells defined on **dual complex** $\langle\star\sigma_p|$.
  - $\langle\star\sigma_p|\sigma_{p'}\rangle = \delta_{p,p'}$ and $\sum_\sigma |\sigma\rangle\langle\star\sigma| = \mathrm{id}$.
- Discrete exterior derivative:

$$d\phi(\sigma_{p+1}) = \int_{\sigma_{p+1}} d\phi = \int_{\partial\sigma_{p+1}}\phi = \phi(\partial\sigma_{p+1}) = \sum_{\sigma_p\subset\sigma_{p+1}} \mathrm{sgn}(\sigma_p,\sigma_{p+1})\phi(\sigma_p)$$

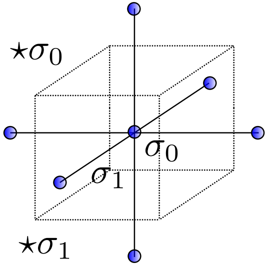
Derive scalar field action for **general lattices**.

Scalar field on an irregular lattice

- Scalar field action in terms of **forms**:

$$S = \frac{1}{2}\int d\phi \wedge * d\phi + \frac{M_0^2}{2}\int \phi \wedge *\phi \;\rightarrow\; S = \frac{1}{2}\sum_{\sigma_1}\langle d\phi | d\phi\rangle + \frac{M_0^2}{2}\sum_{\sigma_0}\langle \phi|\phi\rangle \,.$$

  - $\langle d\phi|d\phi\rangle = \langle \star d \star d\phi|\phi\rangle$ (periodic boundary conditions)



Scalar field action: $S = \frac{1}{2}\phi(\sigma_0^i)\, K_{ij}\, \phi(\sigma_0^j)$ [Hamber, Williams '93]:

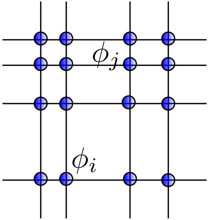
$$K_{ij} = \begin{cases} \sum_{\sigma_1 \supset \sigma_0^i} \frac{V_{\star\sigma_1}}{V_{\sigma_1}} + M_0^2\, V_{\star\sigma_0} & i = j \\ -\sum_{\sigma_1 \supset \sigma_0^i} \frac{V_{\star\sigma_1}}{V_{\sigma_1}} & i \neq j \end{cases}$$

- $V_{\star\sigma_0}$: 4-volume dual to vertex σ_0
- $V_{\star\sigma_1}$: 3-volume dual to edge σ_1.
- V_{σ_1}: length of edge σ_1.

Scalar field coupled to cuboid spin foams

- **Ansatz** for coupling scalar field and spin foams
  - **Define scalar LTF for each spin foam state**
  - Spin foam amplitude unchanged
  - Summing over spin foam states $\rightarrow$ **superposition of scalar LTFs**
- **Partition function** then reads:

$$Z = \int \prod_{\sigma_1} dl_{\sigma_1} \prod_{\sigma_0} d\phi(\sigma_0) \prod_{\sigma_4} \hat{A}_{\sigma_4}(\alpha, \{l_{\sigma_1}\}) \; \text{exp} \left(-\frac{1}{2} \phi(\sigma_0^i) \, K_{ij}(\{l_{\sigma_1}\}, M_0) \, \phi(\sigma_0^j) \right)$$



- **“Minimal” coupling**
  - Assumption scalar field diagonalized by spin foam states

Massive, free scalar field on a **superposition of hypercubodial lattices** weighted by **spin foam amplitudes**.

Outline

What can we learn from this model?

- How to **numerically** explore the dynamics?
  - Markov Chain Monte Carlo for lengths and fields
  - Spin foam amplitudes **non-oscillatory**
- What can we **measure**?
  - Geometric observables: $\langle V \rangle$ or $\langle l \rangle$
  - Matter observables: $\langle \phi_i \phi_j \rangle$
- **Physical meaning** of “observables”?
  - **Correlations** between ϕ at vertex i and j?
  - Not meaningful: labels $i, j \sim$ coordinates
- **Diffeomorphism invariance**?
  - Lattice dependence! Can we define a continuum / refinement limit?

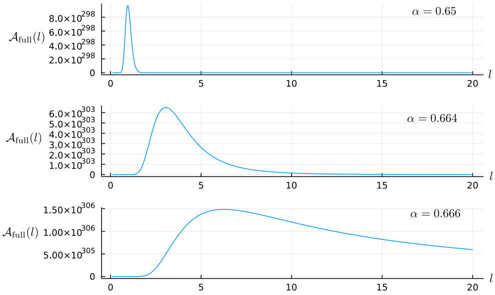
Conceptual question: What are good observables in quantum gravity?

Emergent / effective behavior of the coupled system?

Analytical study: regular lattices

- Restrict edge lengths to **regular lattice**
- Integrate out scalar fields: **probability distribution for lengths**

$$\frac{1}{Z}\frac{l^{N^4(24\alpha-14)}}{\sqrt{l^{2N^4}\left(\sum_{i=1}^{N^4} a_i l^{2i} M^{2i}\right)}}$$



- **Three regimes**, depending on α:
  - $\alpha < 0.625$: smallest lengths dominate
  - $\alpha > 0.666$: largest lengths dominate
  - $0.625 < \alpha < 0.666$: **peak at finite lengths**
- Plots for $M_0 = 10$, various α, lattice size 3

Peak position and width depends on mass M_0 and α.

Numerical setup

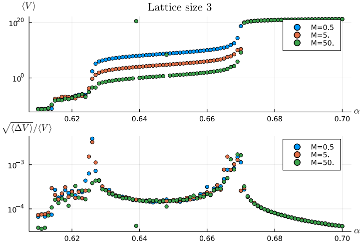
- $N \times N \times N \times N$ lattice with periodic boundary conditions
- Monte Carlo: **importance sampling** for probability distribution

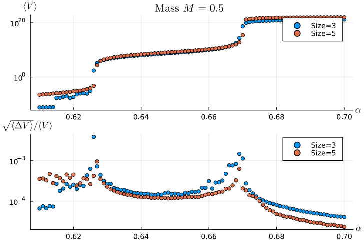
$$\frac{1}{Z} \prod_{\sigma_4} \hat{A}_{\sigma_4}(\alpha, \{l_{\sigma_1}\}) \text{ exp} \left(-\frac{1}{2} \phi(\sigma_0^i) \, K_{ij}(\{l_{\sigma_1}\}, M_0) \, \phi(\sigma_0^j) \right)$$

- **Metropolis algorithm**
  - Randomly choose to sample four lengths or one scalar field
  - Lengths: pick one lengths per dimension and rescale them individually
- **Simulation parameters**:
  - Two parameters: α and M_0
  - Upper and lower cut-off on edge lengths
  - Sample size: 5000 (lattice size 3,4) and 2000 (lattice size 5)

System difficult to sample: **long auto-correlation time**.

Geometric observables



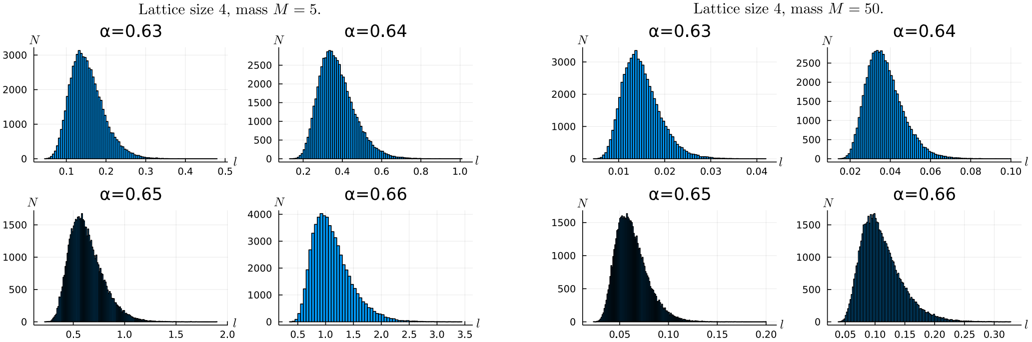


- Total 4-volume $\langle V\rangle$ and (normalized) variance $\frac{\sqrt{\langle\Delta V\rangle}}{\langle V\rangle}$
- Recognize **three distinct regimes**
  - “Plateau”: Finite volume for intermediate α
  - Small / large α: cut-off dependent
- **Normalized variance has two peaks**
  - Localized at end of plateau
  - Peaks do not appear to grow with lattice size
- Scalar field **mass** has **strong effect** on total volume
- Lattice size small impact
  - Different volume, slightly shifted plateau.

What does a **typical lattice** look like?

Lengths histograms

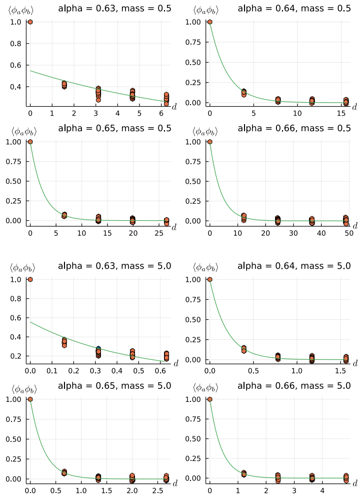
- Histogram for all lengths combined:



Distribution relatively sharply peaked; **regular lattice on average**

Mass linearly influences the average lengths.

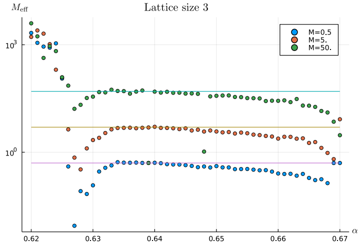
Correlations and correlation lengths

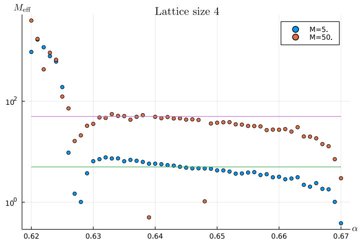


- We measure: **correlations** $\phi_i \phi_j$ and **distance** d_{ij}
  - Plot correlation over distance (forget labels)
  - Path distance d_{ij}
- **Correlations + distance** diffeo-invariant
- We can only measure at **probable** distances
  - Algorithm does not allow us to set distance manually.
- **Mass influences mostly spin foam**
  - $M \to 10M: \; l \to l/10$
  - Correlations hardly change
- **Lattice appears highly regular!**

What is the **correlation length** $\frac{1}{M_{\text{eff}}}$?

Effective mass





- **Good agreement** of M_0 and M_{eff} in plateau region
  - M_{eff} decreases as α increases
- **Probably no physical effect**
  - Large α implies large edge lengths
  - Fields are hard to sample then, probably overestimate correlations
- Scalar field apparently **not sensitive** to spin foam fluctuations.
  - Note: correlation length is non-local, coarse observable.

Effectively scalar field theory on a regular lattice.

Lattice spacing **dynamical**, function of M_0 and α.

Summary

- **Massive scalar field** coupled to a **superposition of hypercuboid lattices**
- 4D **cuboid spin foam model**
  - Superposition of flat irregular lattices
  - Not realistic, but remnant of **diffeomorphism symmetry**
- Matter action derived via **discrete exterior calculus**
  - Applicable to more general lattices than hypercuboids
  - Not unique, depend on choice of dual lattice
- Results: **on average regular lattice + scalar field**
  - Lattice spacing depends strongly on mass
  - Correlation lengths: measure correlations and distance in spin foams
  - Effectively recover scalar field theory on a regular lattice

Tentative evidence that we might **recover QFT on fixed space-time** from spin foams.

Outlook

- **Many assumptions / simplifications**
  - More general than cuboids: effective spin foams on triangulation [Asante, Dittrich, Haggard '20]
  - Matter: Yang-Mills, interacting scalar field theory
- **Conceptual questions**:
  - How to measure correlations for arbitrary distance? [Ambjorn, Goerlich, Jurkiewicz, Loll '12]
  - Lorentzian: mismatch between Wick-rotated matter action and spin foam.
  - Which numerical tools to use beyond Monte Carlo?
  - Role of gravitational constant / length scales?
- **Renormalization / coarse graining** of matter-spin foam system
  - Ambiguities, **phase diagram**, fixed points, continuum limit... [S.St. '15]
  - Restoration of **diffeomorphism symmetry**?
- **Restrictions on matter sector** from spin foams?

Thank you for your attention!