



# Random Geometry in Heidelberg

# 16-20 May, 2022



# Participants:

- Seth Asante
- Joseph Ben Geloun
- Dario Benedetti
- Maarten DeKieviet
- Nicolas Delporte
- Léonard Ferdinand
- L Glaser
- Răzvan Gurău<sup>\*</sup>
- Sabine Harribey\*

- Hannes Keppler
- Thomas Krajewski
- Davide Lettera
- Luca Lionni
- Matteo M. Maglio
- Thomas Muller
- Victor Nador
- Daniele Oriti
- Mohamed Ouerfelli
- Romain Pascalie

- Sylvie Paycha
- Carlos Pérez-Sánchez\*
- Vincent Rivasseau
- Saswato Sen
- José D. Simão
- Sebastian Steinhaus
- Adrian Tanasă
- Johannes Thürigen
- Reiko Toriumi
- Raimar Wulkenhaar

# Monday, 16/05

11:00-11:40	Welcome	
11:40-12:30	Nicolas Delporte OIST	Phase transitions in spherical models
12:30-14:30	Lunch break	
14:30-15:20	Dario Benedetti	Instability of complex CFTs with operators in
	CPHT, Ecole Polytechnique	the principal series
15:20-15:50	Coffee break	
15:50-16:40	<b>Mohamed Ouerfelli</b> Université Paris Saclay - CEA	Tensorial Principal Components Analysis

# Tuesday, 17/05

09:30-10:20	<b>Raimar Wulkenhaar</b> WWU Münster	Solution of the $\lambda\Phi^4$ -matrix model
10:20-10:50	Coffee break	
10:50-11:40	<b>Sebastian Steinhaus</b> FSU, Jena	Lattice Field Theory on Spin Foams
11:40-12:30	<b>Adrian Tanasa</b> LaBRI, Université Bordeaux	Combinatorial aspects of tensor models
12:30-14:30	Lunch break	
14:30-15:20	Carlos Perez Sanchez	Yang-Mills theory on a certain 'quantum
	Heidelberg University	space'
15:20-15:50	Coffee break	
15:50-16:40	<b>Johannes Thürigen</b> WWU Münster	Algebraic Structures in Renormalization of Tensorial Fields

# Wednesday, 18/05

09:30-10:20	<b>Davide Lettera</b> Heidelberg University	The $F$ -theorem in the melonic limit
10:20-10:50	Coffee break	
10:50-11:40	<b>Joseph Ben Geloun</b> LIPN, Paris	Dimensional flow from nonlocality: some results on a cyclic melonic Tensor Field Theory
11:40-12:30	Matteo Maglio Heidelberg University	Conformal Field Theory in Momentum Space and Anomaly Actions in Gravity
12:30-14:30	Lunch break	
14:30-15:20	Romain Pascalie ULB	JT gravity at finite cutoff
	Free afternoon	

# Thursday, 19/05

09:30-10:20	<b>Daniele Oriti</b> LMU, Munich	Landau-Ginzburg analysis of tensorial group field theories
10:20-10:50	Coffee break	
10:50-11:40	<b>Luca Lionni</b> Heidelberg University	A family of triangulated 3-spheres constructed from trees
11:40-12:30	<b>Reiko Toriumi</b> OIST	Trisections in colored tensor models
12:30-14:30	Lunch break	
14:30-15:20	<b>L Glaser</b> University of Vienna	What's that spectral triple?
15:20-15:50	Coffee break	
15:50-16:40	<b>Sylvie Paycha</b> University of Potsdam	Evaluating meromorphic functions at poles under the prisma of locality: an ubiquitous problem in mathematics and QFT
19:00	Conference dinner	

# Friday, 20/05

09:00-09:50	Seth Asante	Gravity dynamics from effective spin foam
	FSU Jena	models
09:50-10:20	Coffee break	
10:20-11:10	Victor Nador	Double scaling mechanism for multi-matrix
	LaBRI, Université Bordeaux	models
11:10-12:00	Thomas Krajewski	Loop vertex expansion for random matrices
	CPT, Aix-Marseille Université	with higher order interactions
		End

# **List of Abstracts**

# Monday 16/05

### Phase transitions in spherical models

#### Nicolas Delporte

Okinawa Institute of Science and Technology (OIST) Graduate University, Japan

I will contrast three spherical models (vector, matrix and tensor like), with their different phase transitions (still conjectural in the latter case) and distinct tools at our disposal to study them.

### Instability of complex CFTs with operators in the principal series

#### Dario Benedetti

#### CPHT, CNRS - Ecole Polytechnique, France

I will outline a proof of instability of d-dimensional conformal field theories having in the operatorproduct expansion of two fundamental fields a primary operator of scaling dimension h = d/2 + i r. From an AdS/CFT point of view, this corresponds to a well-known tachyonic instability, associated to the violation of the Breitenlohner-Freedman bound in (d+1)-dimensional anti-de Sitter space; I will show how to derive it directly on the CFT side, by applying the harmonic analysis for the Euclidean conformal group and the two-particle irreducible formalism. I will conclude with some explicit examples, such as melonic tensor models and the biscalar fishnet model.

#### **Tensorial Principal Components Analysis**

#### Mohamed Ouerfelli

#### Université Paris Saclay - CEA, France

Tensor Principal Components Analysis (PCA) is a model that have been extensively studied for its multiple application and theoretical interests in machine learning. We introduce a novel theoretical framework based on Random Tensor Theory that provides new theoretical results and an experimental improvement with respect to state of the art on synthetic and real data . Furthermore, we present a new gradient-descent based algorithm that provides new insights on the conjectured statistical-computational gap in Tensor PCA (the gap between the information theoretically optimal performance and the performance achieved by computationally efficient algorithms).

# Tuesday 17/05

# Solution of the $\lambda \Phi^4$ -matrix model

#### Raimar Wulkenhaar

#### Mathematisches Institut, WWU Münster, Germany

The  $\lambda \Phi^4$ -model is one of the simplest examples for quantum field theory. It is nevertheless full of mathematical challenges. We report on a programme which intended to circumvent triviality by approaching  $\lambda \Phi_4^4$  from noncommutative geometry. Early results about vanishing of the  $\beta$ -function have been extended into a complete description in terms of a system of Dyson-Schwinger equations. The explicit solution of the planar sector is accomplished, even in spectral dimension 4, where triviality is avoided. The solution for higher topologies, in case of an approximation by large but finite matrices, makes contact with topological recursion and the moduli space of stable complex curves.

### **Lattice Field Theory on Spin Foams**

#### Sebastian Steinhaus

#### FSU, Jena, Germany

Any approach to pure quantum gravity must eventually face the question of coupling quantum matter to the theory. In this talk, I present a model, in which I couple massive, free scalar lattice field theory to a simplified 4D spin foam model, called quantum cuboids, which can be understood as a superposition of flat irregular lattices. To this end, I briefly introduce spin foam models, generalize scalar lattice field theory to irregular discretizations via discrete exterior calculus and explain the idea of how to couple both theories. I explore the coupled system using Monte Carlo techniques and present results for geometrical and matter observables. In particular, there exists an extended regime in parameter space of the model, where the spin foam is sharply peaked around a regular lattice. This lattice spacing is a function e.g. of the mass of the scalar field. Furthermore, I measure the 2-point correlation function of the scalar field and its correlation length, which matches well the 2-point correlation function of a scalar field of the same mass defined on the average spin foam. Thus, effectively the coupled system can be described as a scalar lattice field theory defined on regular lattice.

### **Combinatorial aspects of tensor models**

#### Adrian Tanasa

#### LaBRI, Université Bordeaux, France

Tensor models are known to have a rich combinatorial structure. In this talk I will present how combinatorial tools can be used to implement the double scaling limit for two such tensor models, namely the multi-orientable and the  $O(N)^3$ -invariant models.

### Yang-Mills theory on a certain 'quantum space'

#### **Carlos Perez Sanchez**

#### Heidelberg University, Germany

The problem of quantization of noncommutative spaces (spectral triples) is addressed first in a finitedimensional, Euclidean setting (thus referred to as *random noncommutative geometry*, or *Dirac ensambles*). To address the problem of quantization of a space with gauge fields, first the structure that in the smooth (i.e. commutative) spacetime case is well-known to yield the Yang-Mills(-Higgs) theory, namely *almost-commutative manifolds*, has to be replaced by its 'fuzzy' counterpart already the classical level. We provide new the structure that allows this, whose quantisation turns out to be lead to a multimatrix ensemble.

### **Algebraic Structures in Renormalization of Tensorial Fields**

#### Johannes Thürigen

#### WWU Münster, Germany

It has been shown some time ago that the Connes-Kreimer Hopf algebra encoding the combinatorics of perturbative renormalization applies also to some theories of tensorial fields. I will explain how this generalizes to combinatorially non-local interactions in general, how it can be used to compute amplitudes, and discuss an aspect where the algebraic structure might differ from usual local field theories.

# Wednesday 18/05

## The F-theorem in the melonic limit

#### Davide Lettera

#### Heidelberg University, Germany

In this talk I will present a non-trivial test of the F-theorem, which states that in three dimensions the sphere free energy of a field theory must decrease between ultraviolet and infrared fixed points of the renormalization group flow. We consider the long-range bosonic  $O(N)^3$  model on a spherical background, at next-to-next-to-leading order of the 1/N expansion. The model displays four large-N fixed points and we test and confirm the F-theorem holds in this case. This is non-trivial as one of the couplings is imaginary, and therefore the model is non-unitary at finite N. Despite this, several tests indicating that the large-N CFTs are in fact unitary have been performed: for instance all the OPE coefficients computed so far in the large-N limit are real, and the spectrum of bilinear operators is real and above unitarity bounds. Our result, namely that the F theorem holds at large N, can be viewed as further indication that such theories are unitary. As an added bonus, we show how conformal partial waves expansions in conformal field theory can be used to resum infinite classes of vacuum diagrams. Non-perturbatively, the jump in the value of the free energy has the interpretation of the inclusion at the ultraviolet fixed point of an extra non-normalizable contribution in the conformal partial wave expansion.

# Dimensional flow from nonlocality: some results on a cyclic melonic Tensor Field Theory

#### Joseph Ben Geloun

#### Laboratoire d'Informatique de Paris Nord, France

Tensor Field Theory (TFT), one of the simplest field theoretic counterparts of tensor models, enjoys two remarkable features: a large N expansion and nonlocality. Mixing these ingredients (and surely a bit more) in a balanced way is of course what makes nontrivial the renormalization group analysis of TFT. It is known that, e.g., the Functional Renormalization Group analysis of TFT with configuration space a compact group has one peculiarity: its sets of beta-functions becomes non autonomous. In the simplest instances, the large N limit (equivalently, the large radius size of the group) helps in setting up a notion dimension of the coupling constants. This makes the beta system autonomous and therefore computable. I will explain how not taking the large N limit and letting the RG flow fully non autonomous leads to a genuine definition of dimensional flow. Some preliminary results will be given for a particular TFT endowed with cyclic melonic interactions at arbitrary but finite valence.

## Conformal Field Theory in Momentum Space and Anomaly Actions in Gravity

#### Matteo Maglio

#### Heidelberg University, Germany

In this talk, we will present an overview of the essential features of the conformal anomaly. On general geometrical backgrounds, we will see how an amplitude that is both gauge and conformal invariant will contain anomalous trace contributions in the physical limit d = 4. We will show the procedure to obtain the anomaly effective action and illustrate the derivation of the conformal Ward identities directly from its path integral definition and its Weyl symmetry. Then we will present the method to write the general structure of correlation functions in momentum space by solving the conformal constraints. Considering correlation functions involving stress-energy tensors, we will show the appearance of (non-local) exchange of massless poles in specific form factors due to the conformal anomaly. The latter phenomenon is a signature that has been investigated both in free field theory and non-perturbatively by solving the conformal constraints.

### JT gravity at finite cutoff

#### **Romain Pascalie**

#### ULB, Brussels, Belgium

After reviewing Jackiw-Teitelboim (JT) gravity in the Schwarzian limit, I will present how to describe the theory at finite cutoff. The main step is to consider more general geometries, namely immersions, rather than a specific class of embeddings as in the Schwarzian limit. I will then show how this impacts the expansion of the action in powers of the cutoff and discuss briefly the path integral. Based on work in progress with Frank Ferrari and Nicolas Delporte.

# Thursday 19/05

# Landau-Ginzburg analysis of tensorial group field theories

#### Daniele Oriti

#### Arnold Sommerfeld Center for Theoretical Physics, LMU, Munich, Germany

In the tensorial group field theory approach to quantum gravity, the universe is described in terms of discrete building blocks, whose collective dynamics is then expected to give rise to a continuum spacetime (with usual local field theory as an effective description), possibly via critical behaviour and phase transitions. In parallel with detailed renormalization group analyses, mean-field methods can play a crucial role in understanding this collective dynamics, especially for the more involved quantum geometric models. In this talk, I outline the quantum gravity motivation and context for such analyses and then report on recent results obtained by adapting the Landau-Ginzburg approach to tensorial group field theories. These results concern the existence of phase transitions, the critical dimension for Gaussian fixed points and the validity of mean-field theory, for models with both compact and non-compact group domains, mixed local/non-local directions and some quantum geometric constraints.

### A family of triangulated 3-spheres constructed from trees

#### Luca Lionni

#### Heidelberg University, Germany

If the zoo of Brownian trees and surfaces is well populated, no example is known in higher topological dimension. The goal would be for instance to find a 3d generalization of the Brownian 2-sphere, that is, to exhibit some sequences of random triangulations or more general random graphs that would converge to a Brownian 3-sphere through a continuum scaling limit. All efforts in that direction have produced previously known or likely trivial continuum limits. These conclusions are however mostly based on simulations and in most cases, such as uniform triangulations of the 3-sphere, exact results are out of reach. This can be explained by our lack of bijective encodings by simpler combinatorial objects, whose asymptotic properties are known, such as trees. I will describe a subset of triangulations of the 3-sphere encoded bijectively by three trees, introduced with Timothy Budd to address these issues, and for which early simulations show promising results.

### **Trisections in colored tensor models**

#### Reiko Toriumi

Okinawa Institute of Science and Technology (OIST) Graduate University, Japan

We give a procedure to construct trisections for closed manifolds generated by colored tensor models without restrictions on the number of simplices in the triangulation, therefore generalizing previous works in the context of crystallizations and PL-manifolds. We give a description of how trisection diagrams can arise from colored tensor model graphs for closed 4-manifolds.

# What's that spectral triple?

#### L Glaser

#### University of Vienna, Austria

Finite spectral triples can attache a geometric interpretation to a random multi matrix model. Computer simulations of this give an interesting phase diagram, with possible geometries arising at the phase transition. Having created these geometries, one starts to wonder: Can I make a picture of that geometry I created? One answer to that question are visualisation algorithms. After talking about finite spectra triples I will show how this visualisation works on a deformation of the fuzzy sphere, a fuzzy ellipsoid if you will.

# Evaluating meromorphic functions at poles under the prisma of locality: an ubiquitous problem in mathematics and QFT

#### Sylvie Paycha

#### University of Potsdam, Germany

Both in mathematics and QFT, one comes across the problem of evaluating divergent sums and integrals. Divergent sums may arise from "counting" discrete points on a cone, a problem relevant in toric geometry, while divergent integrals can arise from Feynman amplitudes which are building blocks in perturbative quantum field theory. To make sense of such divergent quantities, one can combine a "minimal subtraction scheme" (in one variable), which consists in "extracting divergences" (minimally) while retaining a (maximal) "finite part", with an "algebraic Birkhoff factorisation" which involves a coproduct.

I shall explain how by means of a multiparameter regularisation à la Speer, a mere minimal subtraction procedure for meromomorphic germs in several variables does the job. This uses an algebraic concept of locality reminiscent of locality in QFT. I shall then present our main result which is the classification of the resulting evaluators by means of what we call a Galois group.

This is joint work with Li Guo and Bin Zhang.

# Friday 20/05

### Gravity dynamics from effective spin foam models

#### Seth Asante

#### FSU, Jena, Germany

Recently, 'effective spin foam models' have been introduced as models which capture universal features of quantum gravity approaches based on discrete area variables. These models allow for a fast computation and hence overcome the computational challenges of spin foam models. They have therefore been used to perform semi classical analysis/study of spin foam models with configurations that are able to test quantum equations of motion. This study reveals a very rich semi classical regime as an interplay between parameters of the model and in addition it addresses the 'flatness problem' appearing in spin foam models. Also, the first steps towards studying the continuum limit of effective spin foam models at the linearized level show that there are more degrees of freedom from area variables compared to that from length (metric) variables. However, gravitons appear from the area variables at leading order in lattice constant. These results are very promising as a hope for emergence of general relativity from spin foam models. I will discuss these developments in the talk.

### Double scaling mechanism for multi-matrix models

#### Victor Nador

#### LaBRI, Université Bordeaux, France

Multi-matrix models are close cousins of tensor models. As such, many techniques developed for tensor models can be generalized to multi-matrix models. After presenting both models, I will explain how the double scaling mechanism can be implemented in the quartic  $U(N)^2 \times O(D)$  and in the quartic bipartite  $U(N) \times O(D)$  multi-matrix models.

#### Loop vertex expansion for random matrices with higher order interactions

#### Thomas Krajewski

#### CPT, Aix-Marseille Université, France

The loop vertex expansion is an alternative to the standard Feynman graph expansion which trades the latter for a convergent expansion over trees. In this talk, we present the general framework and apply it to some random matrix models. As a byproduct, we establish analyticity in the coupling in a domain independent of the size of the matrix, as well as Borel summability. This is based on work in collaboration with V. Rivasseau et V. Sazonov, see https://arxiv.org/abs/1910.13261.