The Dilemma with AdS/QCD

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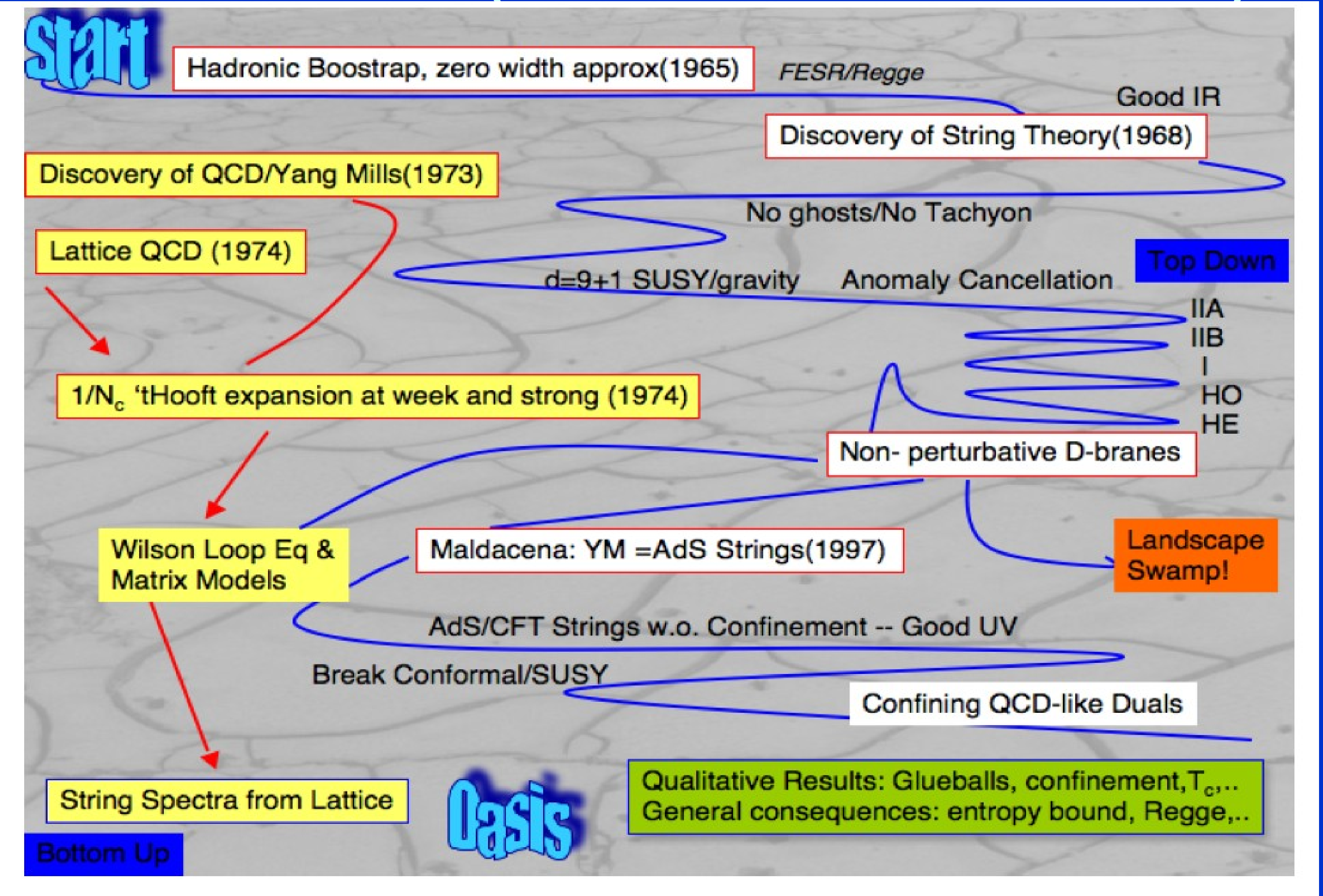
Outline

- Bottom-Up Approach
- Breaking Conformal Invariance in AdS/CFT
- Implementing asymptotic freedom and confinement
- Checking the equation of state of the plasma
- The dilemma

Short History of String Theory and QCD (after R.C. Brower)



Our approach

- Breakpoint: Malcacena‘s conjecture about the exact equivalence of 10-dimensional superstring theory AdS5xS5 and the conformal N=4 SUSY Yang Mills Theory
- But for QCD we do not need supersymmetry, at least not yet- and we do not want a conformal theory
- Therefore we discuss type 0 string theory in 5 dim AdS space without supersymmetry

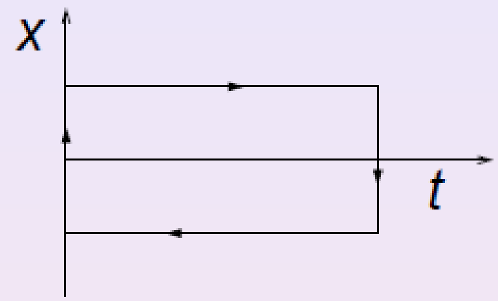
How can the fifth dimension enter

- Polyakov has proposed already long time ago, that the dilaton coupling maps out a fifth dimension related to energy resolution.
- The dilaton breaks the conformality and is related to the running coupling of QCD.
- In fact, averaging of gluon link variables in large Nc gives matrices which are a product HxSU(N)xU(1).
- The eigenvalues of these hermitian matrices H may be the origin of the fifth dimension (c.f Arodz and Pirner)

Scale Invariance and Confinement

Consider a rectangular Wilson loop:

$$W(\mathcal{C}) = \exp\left(ig\int_{\mathcal{C}} A_\mu dx^\mu\right)$$



It is related to the potential $V_{q\bar{q}}(R)$ acting between charges q and $\bar{q}$:

$$\langle W(\mathcal{C})\rangle \underset{t\to\infty}{\sim} \exp\left(-t\cdot V_{q\bar{q}}(R)\right)$$

Scale transformations: $t \to \lambda t$, $R \to \lambda R$,
The only scale invariant solution is the Coulomb Potential:

$$V_{q\bar{q}} \sim \frac{1}{R}$$

Running coupling and string tension break scale invariance:

$$V_{q\bar{q}}(R) = -\frac{4}{3}\frac{\alpha_s(R)}{R} + \sigma R\,.$$

Model of Confinement with running coupling

$$ds^2_{QCD} = h(z) \cdot ds^2 = h(z) \underbrace{\frac{\ell^2}{z^2}\left(-dt^2 + d\vec{x}^2 + dz^2\right)}_{AdS_5} .$$

- $h(z) = 1 \Longrightarrow$ Conformal
- $h(z) \neq 1 \Longrightarrow$ Non conformal

Breaking of scaling invariance in QCD is given by the running coupling:

$$\Delta \equiv \frac{\epsilon - 3p}{T^4} = \frac{\beta(\alpha_s)}{4\alpha_s^2}\langle F^2_{\mu\nu}\rangle .$$

where $\beta(\alpha_s) = \mu\frac{d\alpha_s}{d\mu}$ and $\alpha_s(E) \sim 1/\log(E/\Lambda)$.

$\Longrightarrow$Assume an ansatz for conformal invariance breaking similar to 1-loop running coupling (H.J.Pirner & B. Galow '09):

$$h(z) = \frac{\log(\epsilon)}{\log(\epsilon + (\Lambda z)^2)} , \qquad z \sim \frac{1}{E} .$$

Calculation in the string frame

(11)

The Nambu-Goto action S_{NG} is given by

$$S_{NG} = \frac{1}{2\pi l_s^2} \int d^2\xi \sqrt{\det h_{ab}}, \qquad (12)$$

where l_s is the string length and h_{ab} is the induced worldsheet metric: The indices a, b are reserved to the ξ_1, ξ_2-coordinates on the worldsheet, the greek indices μ, ν to the coordinates of the embedding five-dimensional space

$$h_{ab} = G_{\mu\nu} \frac{\partial X^\mu}{\partial \xi^a} \frac{\partial X^\nu}{\partial \xi^b}. \qquad (13)$$

Using the condition eq. (20) for the maximum and rescaling $z = \nu z_0$, we obtain the inter-quark distance R as a function of z_0:

$$R(z_0) = 2z_0 \int_0^1 d\nu \nu^2 \frac{h(z_0)}{h(\nu z_0)} \frac{1}{\sqrt{1 - \nu^4 \left(\frac{h(z_0)}{h(\nu z_0)}\right)^2}}. \qquad (22)$$

By similar transformations we can write the energy, which we get from the Nambu-Goto string action, as a function of z_0

$$V_{Q\bar{Q}}(z_0) = \frac{1}{\pi\epsilon} \frac{1}{z_0} \int_0^1 d\nu \frac{h(\nu z_0)}{\nu^2} \frac{1}{\sqrt{1 - \nu^4 \left(\frac{h(z_0)}{h(\nu z_0)}\right)^2}}. \qquad (23)$$

Good Heavy Quark Potential

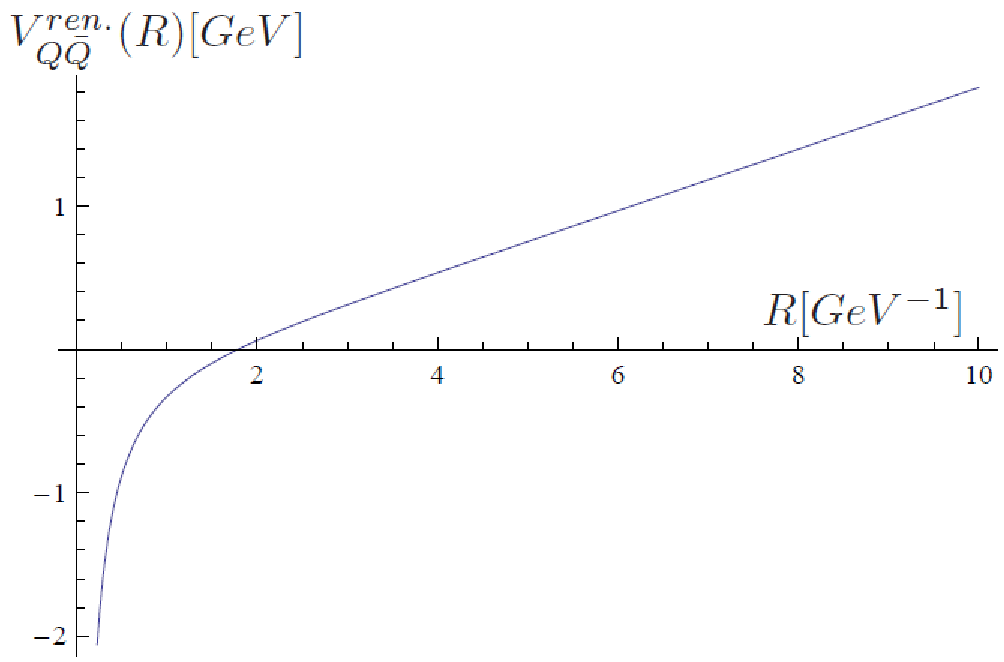


Figure 4: Numerically calculated heavy-quark potential for the modified metric, eq. (11), using $h(z)$ from eq. (7).

But for running coupling, the resulting beta-function is not good

Dual 5-dim Gravity model based on the running of α

5D Einstein-dilaton model (Gürsoy et al. '08):

$$S_{5D} = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left(R - \frac{4}{3}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right) - \frac{1}{8\pi G_5}\int_{\partial M} d^4x\sqrt{-h}\,K$$

One to one relation between β-function and dilaton potential $V(\phi)$:

$$V(\phi) = -\frac{12}{\ell^2}\left(1 - \left(\frac{\beta(\alpha)}{3\alpha}\right)^2\right)\exp\left[-\frac{8}{9}\int_0^\alpha \frac{\beta(a)}{a^2}da\right], \qquad \alpha = e^\phi .$$

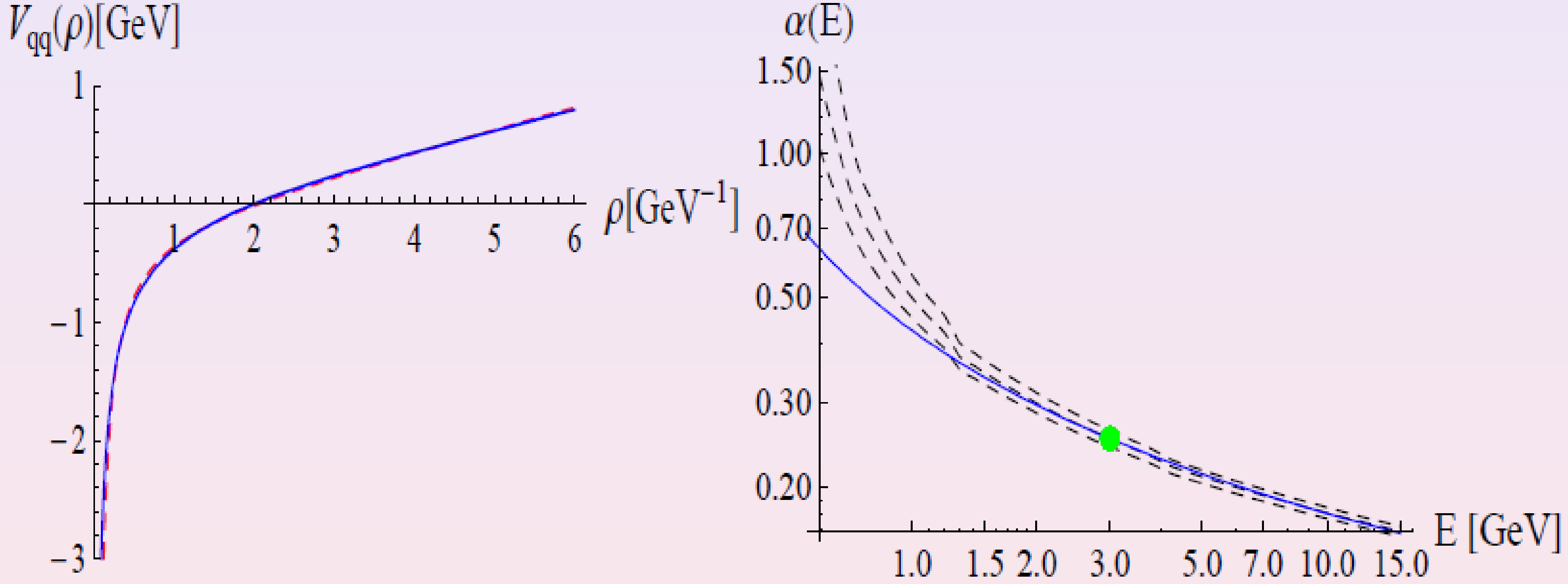
Ansatz (E.Megías et al., NPB834, 2010):

$$\beta(\alpha) = -b_2\alpha + \left[b_2\alpha + \left(\frac{b_2}{\bar\alpha} - \beta_0\right)\alpha^2 + \left(\frac{b_2}{2\bar\alpha^2} - \frac{\beta_0}{\bar\alpha} - \beta_1\right)\alpha^3\right]e^{-\alpha/\bar\alpha} .$$

- $\alpha << \bar\alpha \Longrightarrow$ Ultraviolet: $\beta(\alpha) \approx -\beta_0\alpha^2 - \beta_1\alpha^3$
- $\alpha >> \bar\alpha \Longrightarrow$ Infrared: $\beta(\alpha) \approx -b_2\alpha$

Potential and Running α

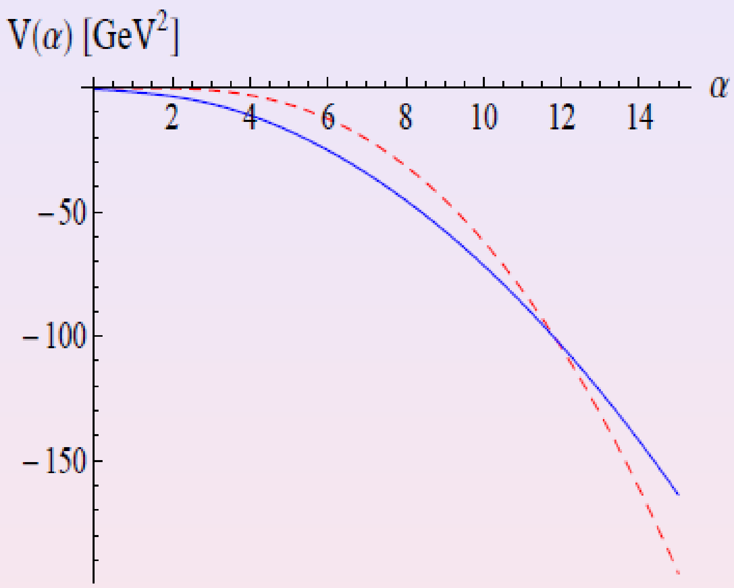
$$W(\mathcal{C}) \approx \exp\left(-S_{\rm NG}\right) \propto \exp\left(-t \cdot V_{q\bar{q}}(R)\right)$$

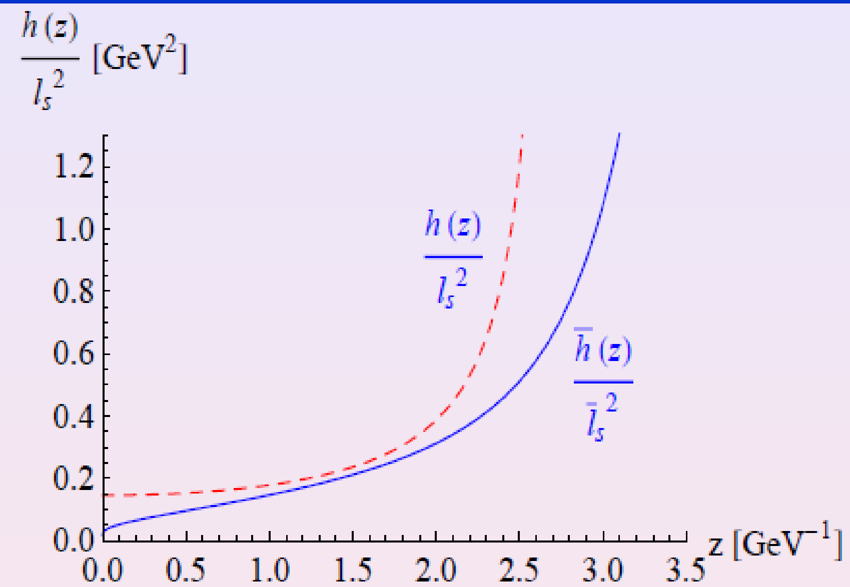


- $\sigma = (0.425\,\mathrm{GeV})^2 \Longrightarrow \frac{b_2}{\bar{\alpha}} = 3.51\mathrm{GeV} \cdot \bar{l}_s$
- Fit of running coupling $\Longrightarrow \frac{b_2}{\bar{\alpha}} = 5.09$

$$\bar{l}_s = 1.45\,\mathrm{GeV}^{-1}$$

Dilaton potential and warp h(z)





$V(\alpha) \sim -(b_2^2 - 9)\alpha^{\frac{8}{9}b_2}\,, \quad \alpha \to \infty;$ $\quad \alpha(z) \sim \dfrac{1}{(z_{IR} - z)^{\frac{b_2}{\frac{4}{9}b_2^2 - 1}}}\,, \quad z \to z_{IR}$

(Confinement) $1.5 < b_2 < 2.37$ (IR singularity repulsive to physical modes)

Black hole thermodynamics in 4 dimensions

$$Z = \mathrm{Tr}\left(e^{-\beta H}\right), \qquad \beta = \frac{1}{T}$$

Periodicity in euclidean time ($\tau = it$): $\Phi(\tau + \beta) = \Phi(\tau)$

*Regularity: Expansion around the horizon $f(r_h) = 0$; $r = r_h(1+\rho^2)$:

$$ds^2_{\mathrm{BH}} = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_2^2 \underset{\rho\to 0}{\sim} 4r_h^2\left(d\rho^2 + \rho^2 \underbrace{\left(\frac{d\tau}{2r_h}\right)^2}_{d\theta^2} + \frac{1}{4}d\Omega_2^2\right)$$

$\Longrightarrow$ Periodicity: $\frac{\tau}{2r_h} \to \frac{\tau}{2r_h} + 2\pi \Longrightarrow \tau \to \tau + 4\pi r_h =: \tau + \beta$

$$T = \frac{1}{8\pi M G_4}$$

Thermodynamics interpretation of black holes:

$$dM = TdS \Longrightarrow S = \int \frac{dM}{T} = 4\pi G_4 M^2$$

$$\mathcal{A} = 4\pi r_h^2 = 16\pi (G_4 M)^2 \Longrightarrow S_{\text{Black Hole}}(T) = \frac{\mathcal{A}(r_{\text{horizon}})}{4G_D} \quad \text{Bek-Hawking}$$

Thermodynamics in 5-dim AdS

$$S_{5D} = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left(R - \frac{4}{3}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right) - \frac{1}{8\pi G_5}\int_{\partial M} d^4x\sqrt{-h}\,K$$

Finite temperature solutions (E. Kiritsis et al. JHEP (2009) 033):

- Thermal gas solution (confined phase):

$$ds^2_{\mathrm{th}} = b_0^2(z)(-dt^2 + d\vec{x}^2 + dz^2)\,, \qquad t \sim t + i\beta$$

- Black hole solution (deconfined phase):

$$ds^2_{\mathrm{BH}} = b^2(z)\left[-f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)}\right]$$

In the UV ($z \simeq 0$): flat metric $b(z) \simeq \ell/z$ and $f(0) = 1$.
There exists an horizon $f(z_h) = 0$.
Regularity at the horizon $\Longrightarrow T = \frac{|\dot{f}(z_h)|}{4\pi}$.

Solve 5-dim Einstein Equations

Einstein equations $\frac{\delta}{\delta g_{\mu\nu}}$:

$$\underbrace{\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right)}_{E_{\mu\nu}} - \underbrace{\left(\frac{4}{3}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2} g_{\mu\nu}\left(\frac{4}{3}(\partial\phi)^2 + V(\phi)\right)\right)}_{T_{\mu\nu}} = 0$$

$$(a) \qquad \frac{\ddot{f}}{\dot{f}} + 3\frac{\dot{b}}{b} = 0\,, \Longrightarrow f(z) = 1 - \frac{\int_0^z \frac{du}{b(u)^3}}{\int_0^{z_h} \frac{u}{b(u)^3}}$$

$$(b) \qquad 6\frac{\dot{b}^2}{b^2} - 3\frac{\ddot{b}}{b} = \frac{4}{3}\dot{\phi}^2\,,$$

$$(c) \qquad 6\frac{\dot{b}^2}{b^2} + 3\frac{\ddot{b}}{b} + 3\frac{\dot{b}}{b}\frac{\dot{f}}{f} = \frac{b^2}{f} V(\phi)$$

Conformal solution:

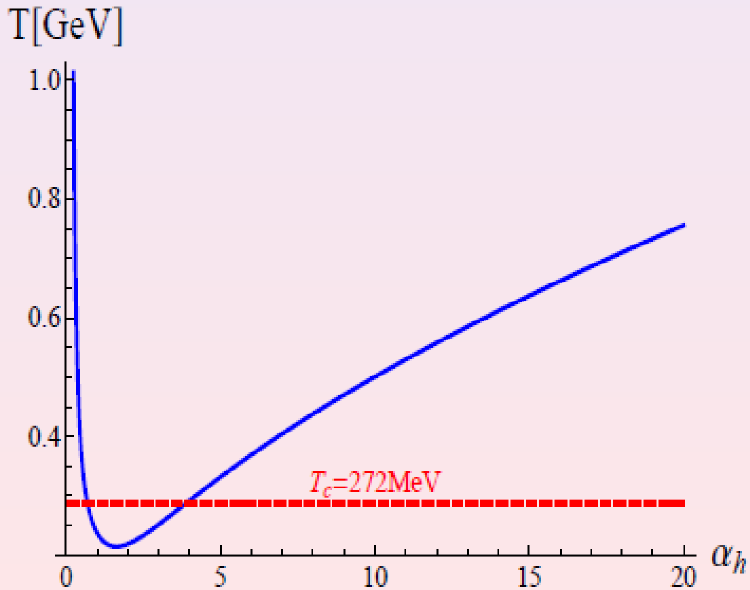
$$V(\phi) = -\frac{12}{\ell^2}\,, \quad \dot{\phi} = 0 \Longrightarrow b(z) = \frac{\ell}{z}\,, \quad f(z) = 1 - \left(\frac{z}{z_h}\right)^4\,, \quad T = \frac{1}{\pi z_h}$$

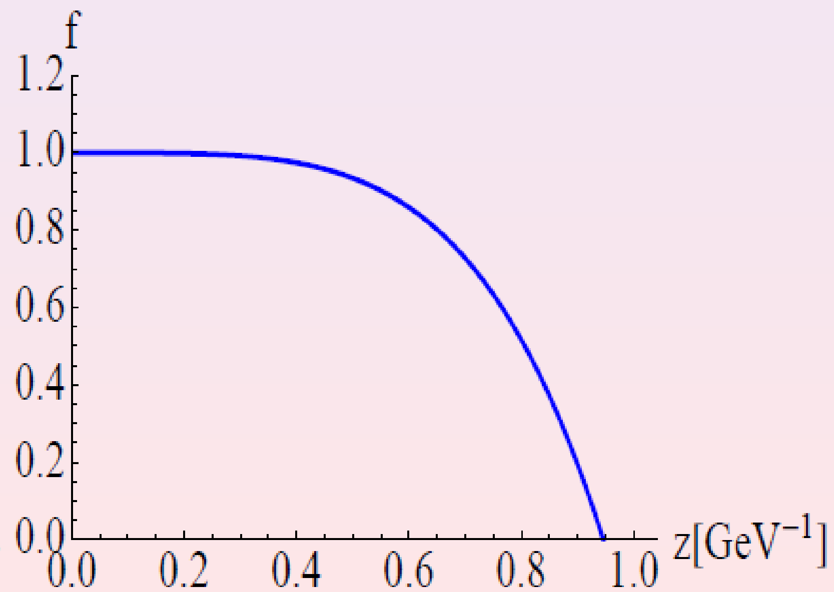
Running α and temperature

Input:

$$\beta(\alpha) = -b_2\alpha + \left[b_2\alpha + \left(\frac{b_2}{\bar\alpha} - \beta_0\right)\alpha^2 + \left(\frac{b_2}{2\bar\alpha^2} - \frac{\beta_0}{\bar\alpha} - \beta_1\right)\alpha^3\right] e^{-\alpha/\bar\alpha}$$

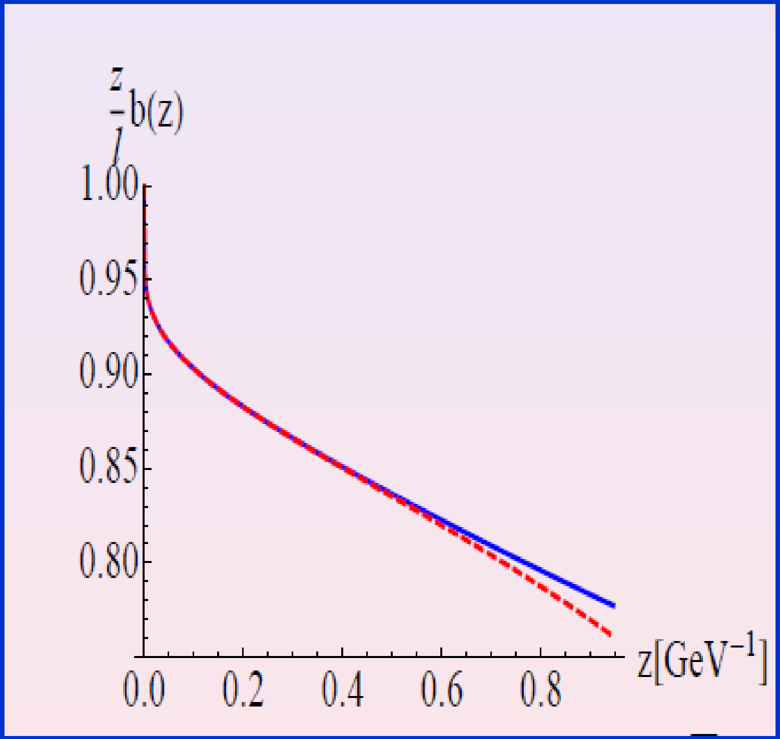
$$\Longrightarrow V(\alpha) = -\frac{12}{\ell^2}\left(1 - \left(\frac{\beta(\alpha)}{3\alpha}\right)^2\right)\exp\left[-\frac{8}{9}\int_0^\alpha \frac{\beta(a)}{a^2}da\right]$$, E.Megías NPB'10

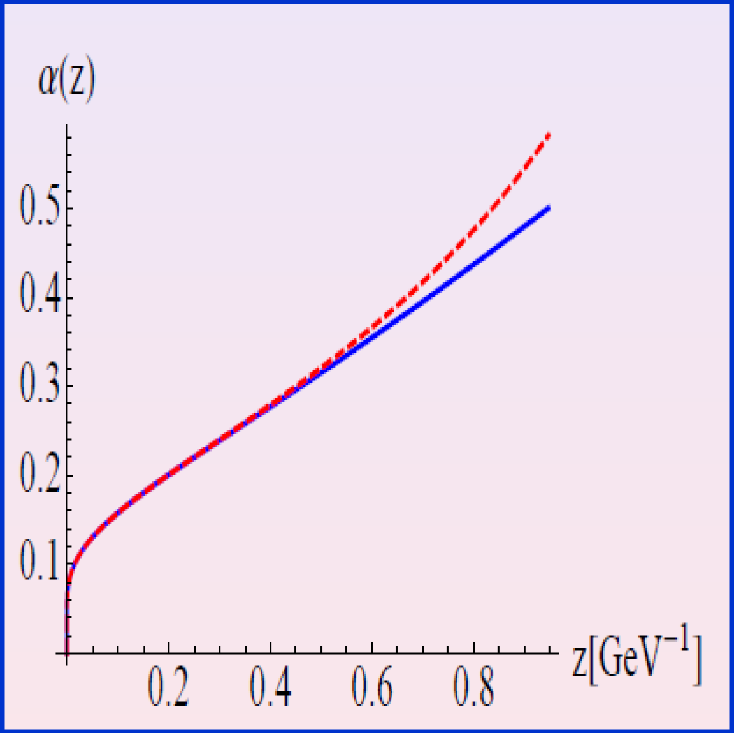




$T = 368\,\text{MeV}$

Small finite T - effects





The finite temperature metric deviates slightly in the IR

The coupling sees the finite Temperature in the IR

Two ways to calculate free energy

Free Energy from:
Bekenstein-Hawking entropy $\underset{\text{Classically}}{\Longleftrightarrow}$ Free Energy from:
Gibbons-Hawking action

$$S = \frac{\mathcal{A}}{4G_5} \qquad\qquad \beta\mathcal{F} = \mathcal{S}_{\text{reg}} \quad \Longrightarrow$$

$$\Longrightarrow \mathcal{F} = \frac{V_3}{16\pi G_5}\left(15G - \frac{C_f}{4}\right)$$

$$G = \frac{\pi G_5}{15}\frac{\beta(\alpha)}{\alpha^2}\langle \mathrm{Tr} F_{\mu\nu}^2 \rangle\,, \qquad C_f = 4\pi T b^3(z_h) \sim T \cdot s$$

$$b_T(z) = b_0(z)\left[1 + \frac{G}{\ell^3}z^4\left(1 + \frac{19}{12}\beta_0\alpha_0(z) + c_2^b\alpha_0^2(z) + \cdots\right) + \cdots\right]\,,\ z \to 0$$

Method I (Beckenstein)

$$S(T) = \frac{\mathcal{A}(z_h)}{4G_5} = \frac{V_3 b^3(z_h)}{4G_5}\,, \qquad z_h \simeq \frac{1}{\pi T}$$

High temperature limit: $s(T) \underset{T\to\infty}{\sim} \frac{\pi^3 \ell^3}{4G_5} T^3 = \frac{32}{45}\pi^2 T^3 =: s_{\mathrm{ideal}}(T)$

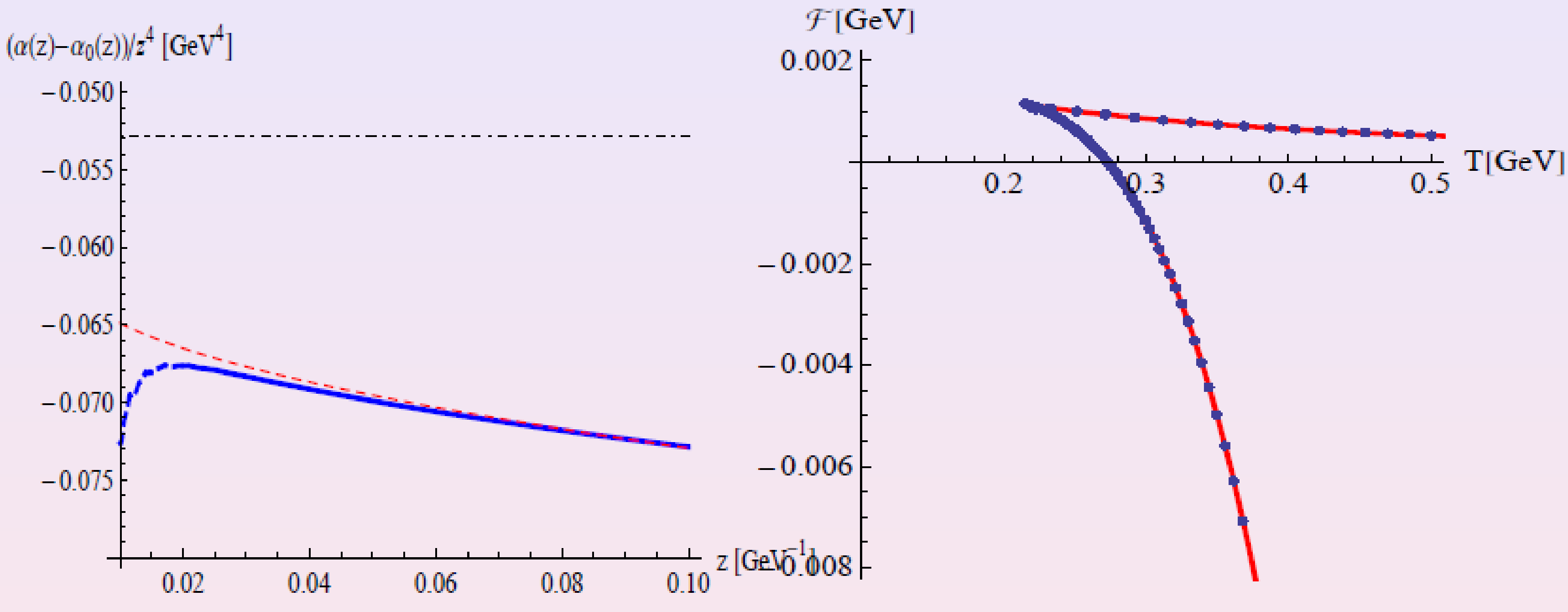
One can compute all the thermodynamics quantities:

$$s(T) = \frac{d}{dT} p(T)\,, \qquad \Delta(T) \equiv \frac{\epsilon - 3p}{T^4} = \frac{s}{T^3} - \frac{4p}{T^4}\,.$$

In the free energy $\Longrightarrow$ contributions from big and small black holes:

$$p(T) = p(T_0) + \int_{T_0}^{T} d\tilde{T} s(\tilde{T}) = \int_{+\infty}^{\alpha_h} d\tilde{\alpha}_h \left(\frac{dT}{d\tilde{\alpha}_h}\right) s(\tilde{\alpha}_h)\,.$$

Method II (Page-Hawking)

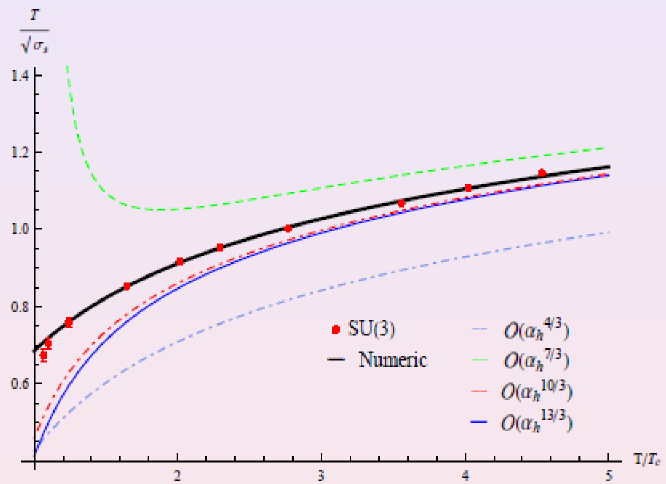


$$\frac{\alpha_T(z)-\alpha_0(z)}{z^4} = -\frac{45}{8}\frac{G}{\ell^3\beta_0}\left(1+\left(\frac{11}{6}\beta_0-\frac{\beta_1}{\beta_0}\right)\alpha_0(z)+c_2^{\alpha}\alpha_0^2(z)+\cdots\right)$$

$\Longrightarrow$ Corrections in α_0 are very important for agreement.

Spatial string tension at finite T:

Spatial string tension: Lattice data: G. Boyd et al. NPB469 (1996)



$$l_s = 1.94\,\mathrm{GeV}^{-1}\,,$$
$$\chi^2/\mathrm{d.o.f.} = 0.87\,.$$

$$\sigma_s(T) = \frac{1}{2\pi l_s^2}\alpha_h^{\frac{4}{3}} b^2(z_h) = \frac{\ell^2}{2 l_s^2}\pi T^2 \alpha_h^{\frac{4}{3}}\left[1 - \frac{8}{9}\beta_0\alpha_h + \frac{2}{81}(25\beta_0^2 - 2\beta_1)\alpha_h^2 + \mathcal{O}(\alpha_h^3)\right]$$

This is in contradiction with pQCD: $\sigma_{\rm pQCD} \sim T^2\alpha_s^2(T)$.
See also Kajantie et al. PRD80 (2009).

Vacuum

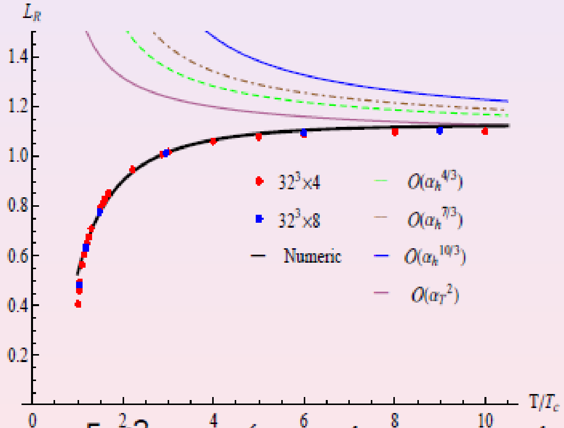
Let us focus first on the UV regime. Besides the numerical computation of the $Q\bar{Q}$-potential, we can perform an analytical study in the short distances regime by expanding Eqs. (45) and (48) in powers of α_0. The details of the computation are provided in Appendix B. The result is

$$V_{Q\bar{Q}}(\rho) = -\frac{2\ell^2}{\pi l_s^2}\frac{\alpha_0^{4/3}(\rho)}{\rho}\left(0.359 + 0.533 b_0\alpha_0(\rho) + (1.347 b_0^2 + 0.692 b_1)\alpha_0^2(\rho) + \mathcal{O}(\alpha_0^3)\right)\ . \quad (49)$$

Polyakov Loop

$$L(T) := \langle P \rangle = \int DX e^{-S_w} \underset{\text{semiclassically}}{\Longrightarrow} \langle P \rangle = \sum_i w_i e^{-S_i} \simeq e^{-S_0}$$

$$S_0^{\text{reg}} = \frac{1}{2\pi l_s^2 T} \left[\int_0^{z_h} dz\ \alpha^{4/3}(z) b^2(z) - \int_0^{z_c} dz\ \alpha_0^{4/3}(z) b_0^2(z) \right]$$



Lattice Gluodynamics SU(3): S.Gupta et al., PRD77(2008).

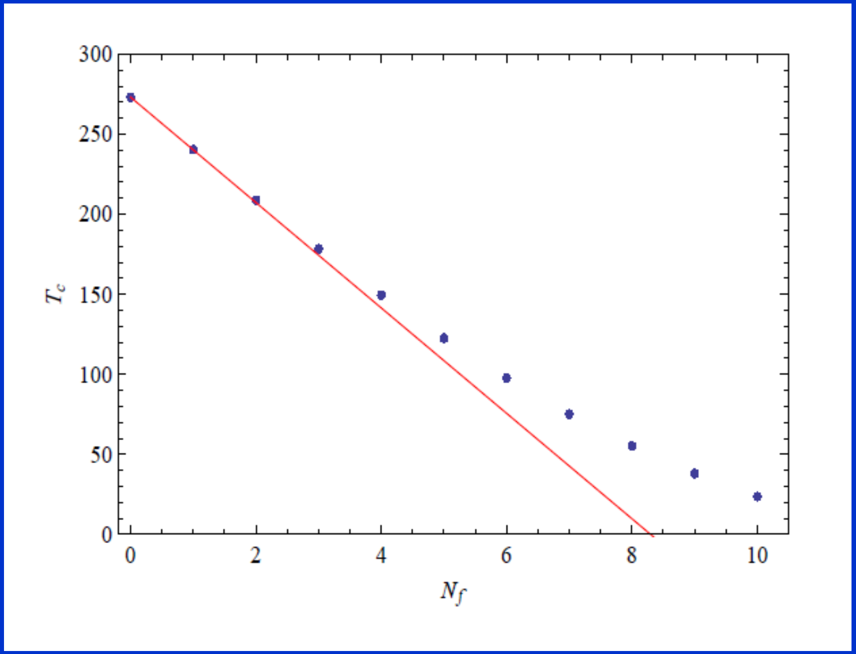
$l_s = 2.36\,\mathrm{GeV}^{-1}$,
$z_c = 0.43\,\mathrm{GeV}^{-1}$

$$L(T) = \exp\left[\frac{\ell^2}{2 l_s^2} \alpha_h^{\frac{4}{3}} \left(1 + \frac{4}{9}\beta_0 \alpha_h + \frac{1}{81}\left(161\beta_0^2 + 72\beta_1\right) \alpha_h^2 + \mathcal{O}(\alpha_h^3) \right) \right],$$

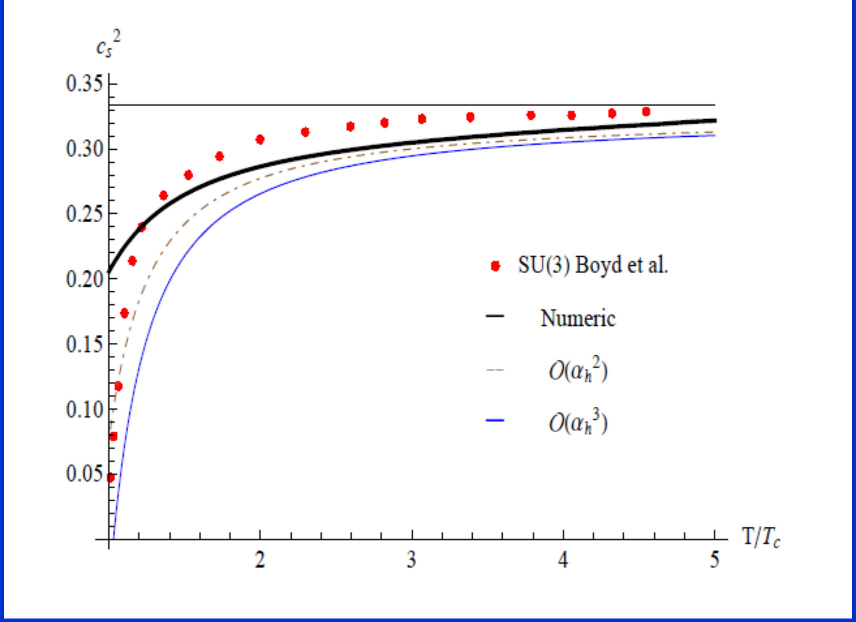
In contradiction with PT: $L_{\rm PT}(T) = \exp\left(\frac{4}{3}\sqrt{\pi}\alpha^{\frac{3}{2}} + \mathcal{O}(\alpha^2)\right)$.

Additional results:

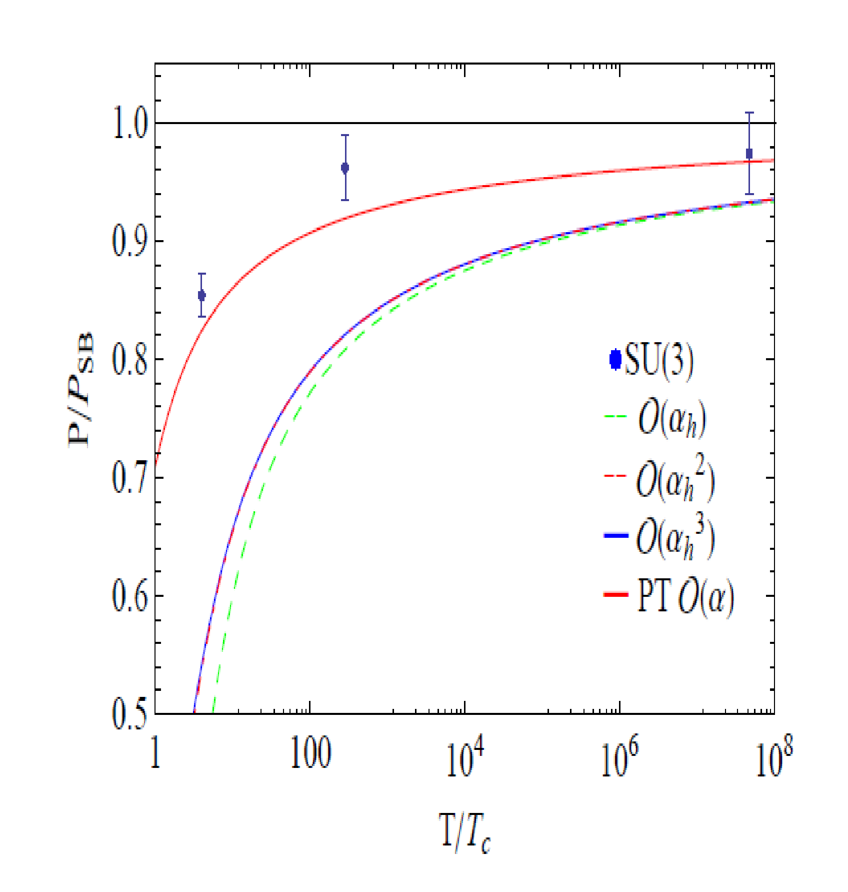
Dependence of critical temperature on the number of flavours



Speed of sound



Pressure is underestimated, if G5 is fitted to Stefan Boltzmann limit



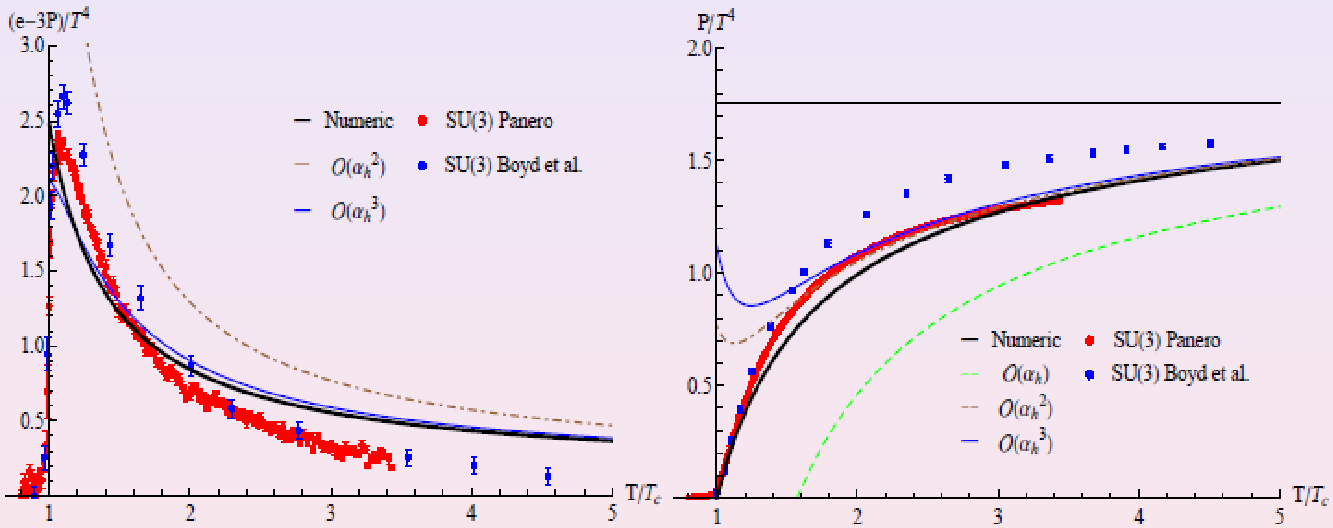
$$\frac{p_{QCD}(T)}{T^4} = \frac{8\pi^2}{45}\left[1 - \frac{15}{4\pi}\alpha + 30\left(\frac{\alpha}{\pi}\right)^{3/2} + \dots\right].$$

$$= \frac{\pi^3 \ell^3}{16 G_5}\left[1 - 2.33\alpha_h + 1.86\alpha_h^2\right.$$

In Ads

Fit G5 to latent heat at critical temperature

Input $\beta(\alpha)$.



Not Normalized To Stefan Boltzman pressure

$$N_c = 3\,, \qquad N_f = 0\,, \qquad b_2 = 2.3\,, \qquad \bar{\alpha} = 0.45$$

$$\frac{p(T)}{T^4} = \frac{\pi^3 \ell^3}{16 G_5}\left(1 - \frac{4}{3}\beta_0 \alpha_h + \frac{2}{9}(4\beta_0^2 - 3\beta_1)\alpha_h^2 + \dots\right), \quad \alpha_h \simeq \frac{1}{\beta_0 \log(\pi T/\Lambda)}$$

The dilemma

One can fit the pressure by choosing a beta function which goes very quickly from perturbative behaviour to nonperturbative behaviour, Scheme dependence allows for a large coefficient in fourth order, But then one does not know how to relate the coupling to the coupling known experimentally in the MS or Mom scheme.

Choose other extrapolation of beta-function

We assume the following toy β-function to demonstrate our case:

$$\beta(\alpha) = \begin{cases} \beta_{\text{pert}}(\alpha) - \hat{\beta}_2 \alpha^4 & \text{if } \alpha \leq \hat{\alpha} \\ \beta_{\text{pert}}(\hat{\alpha}) - \hat{\beta}_2 \hat{\alpha}^4 - 3(\alpha - \hat{\alpha}) & \text{if } \alpha > \hat{\alpha} \end{cases}, \tag{2.1}$$

where

$$\hat{\beta}_2 = \frac{3 - 2\beta_0 \hat{\alpha} - 3\beta_1 \hat{\alpha}^2}{4\hat{\alpha}^3}, \tag{2.2}$$

is chosen such that $\partial_\alpha \beta(\alpha)$ is continuous. There is only one parameter $\hat{\alpha}$ which controls the transition point.

Quick transition from pt to non pt helps:

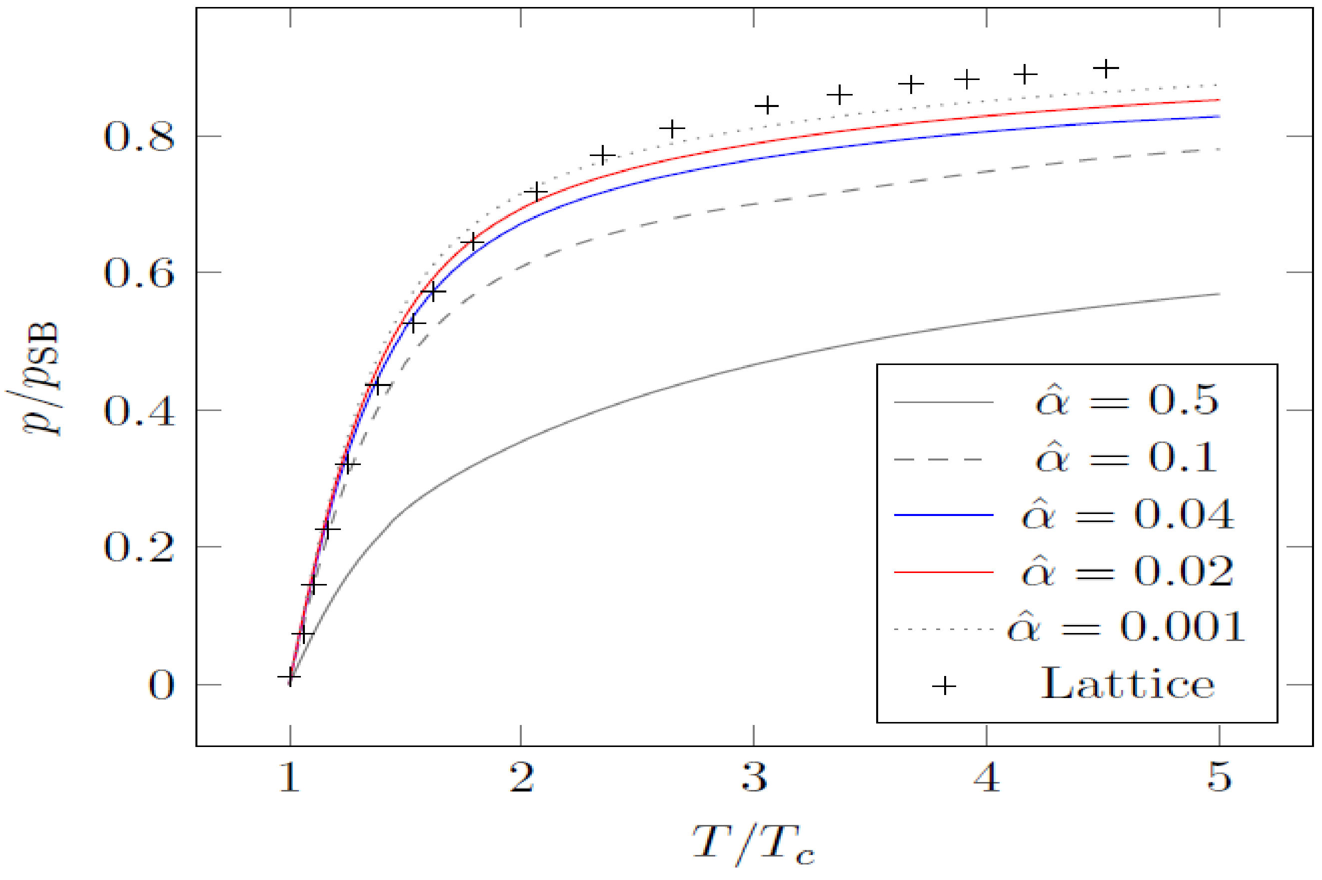


FIG. 1. Pressure normalized to the Stefan-Boltzmann pressure as a function of T/T_c compared with lattice data for $N_c = 3$ taken from ref. [5].

No connection to α in MS-scheme

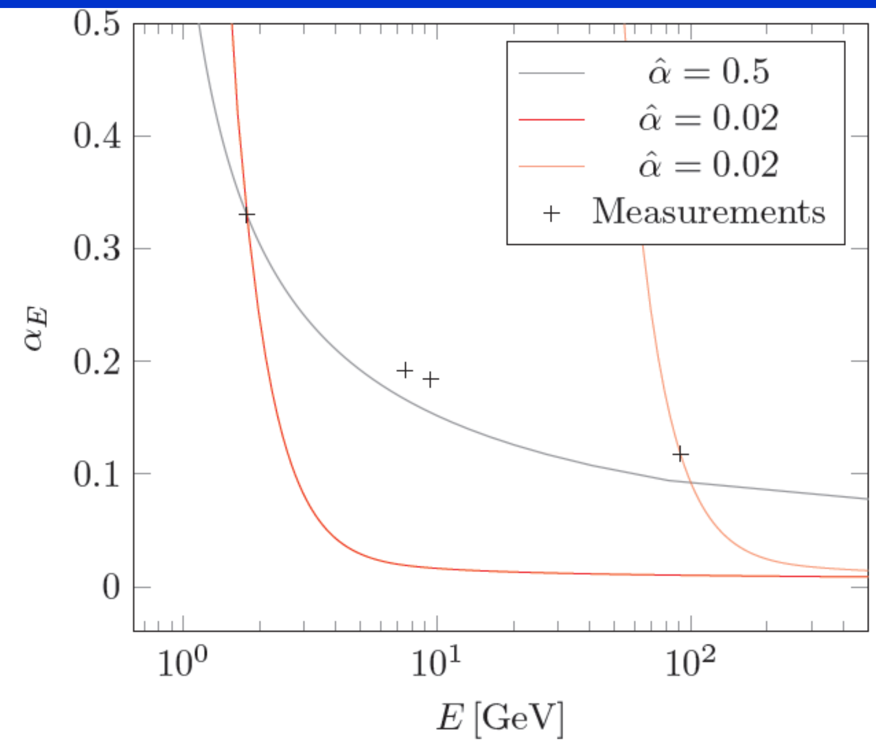


FIG. 2. The running coupling α as a function energy $E = \Lambda_E b$. Data points are take from ref. [16]. The curve corresponding to $\hat{\alpha} = 0.02$ is shown for two different choices of Λ_E.

Conclusions:

- 5-d gravity action with dilaton potential can well describe heavy quark potential and running coupling and glueballs
- With the same dilaton potential difficult to get correct thermodynamics both at critical temperature and at asymptotic temperatures
- Only a very swift transition from perturbative two loop running to a npt running can fit the pressure (Unknown α in MS scheme)