Rethinking Saturation

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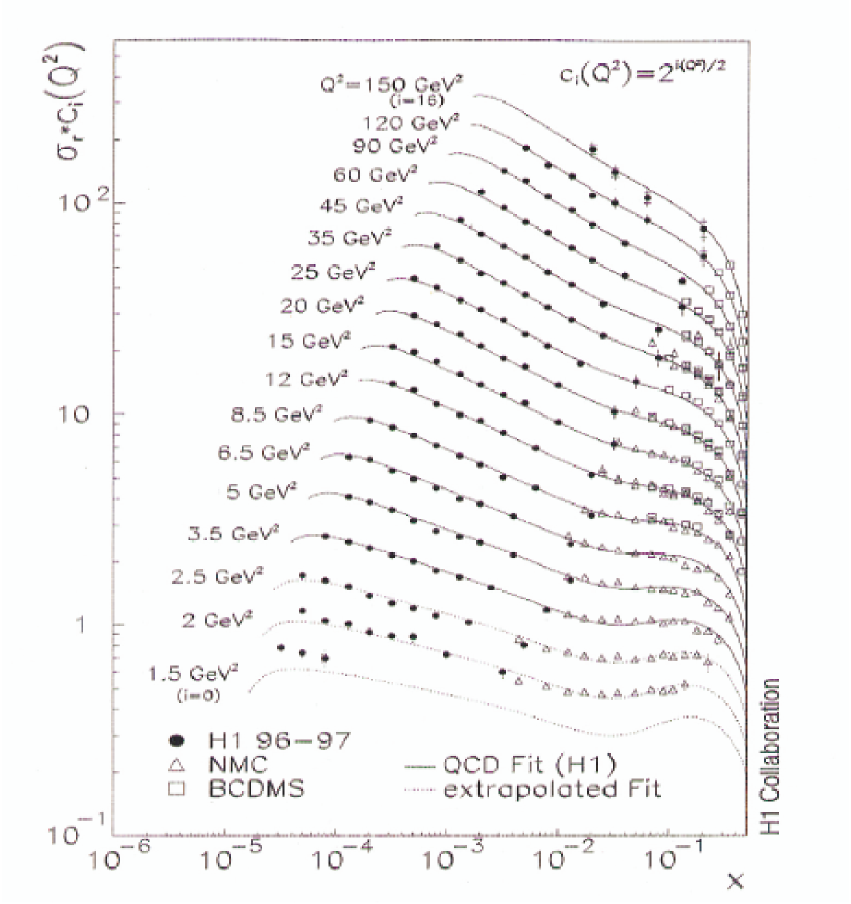
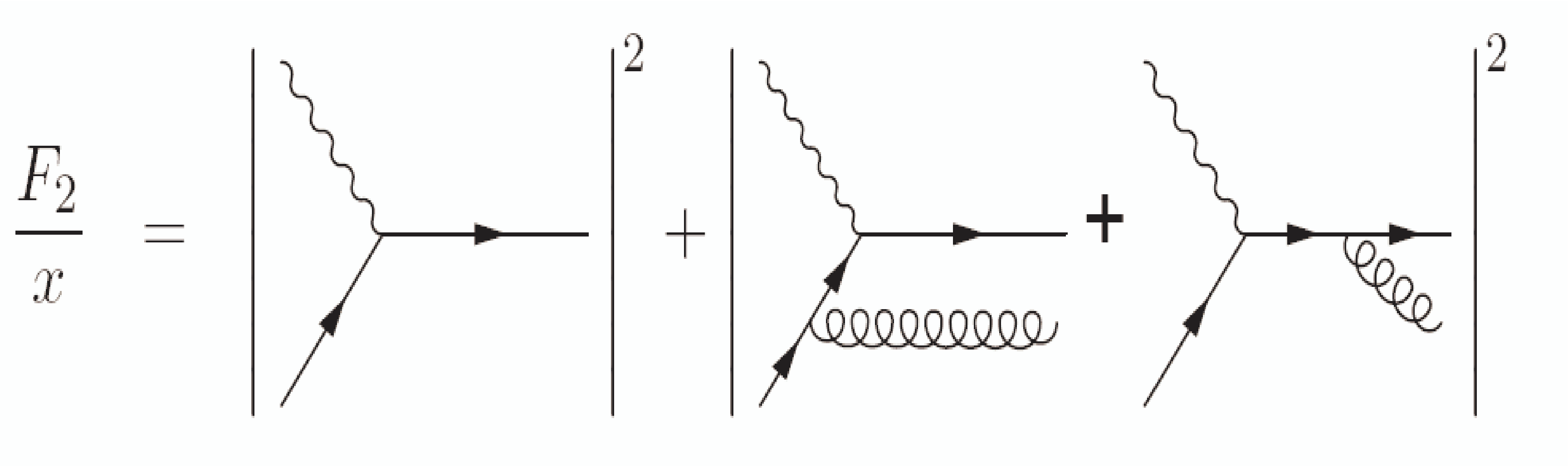
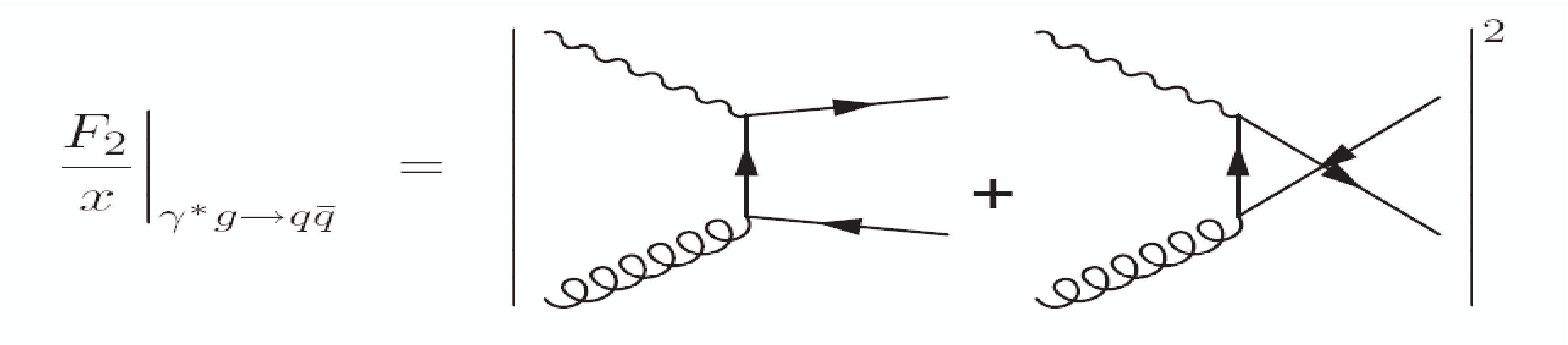
Discussion about saturation starts with HERA e-proton scattering



- x- dependence from HERA
- x- dependence means energy (s) dep. of the photon-proton cross section
- s=Q^2(1/x-1) dependence is much stronger than in hadronic collisions
- Can be obtained by gluon radiation processes

Gluon Radiation und Photon Gluon Fusion





DGLAP Evolution Equations

$$\frac{d}{d\ln Q^2}\begin{pmatrix} q_i(x,Q^2) \\ g(x,Q^2) \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi}\sum_j \int_x^1 \frac{dy}{y}$$

$$\times \begin{pmatrix} P_{q_i q_j}\left(\frac{x}{y}\right) & P_{q_i g}\left(\frac{x}{y}\right) \\ P_{g q_j}\left(\frac{x}{y}\right) & P_{gg}\left(\frac{x}{y}\right) \end{pmatrix} \begin{pmatrix} q_j(y,Q^2) \\ g(y,Q^2) \end{pmatrix}$$

The functions P (x/y) give the probability that a daughter parton with momentum x is produced by the splitting of the ancestor parton with momentum y.

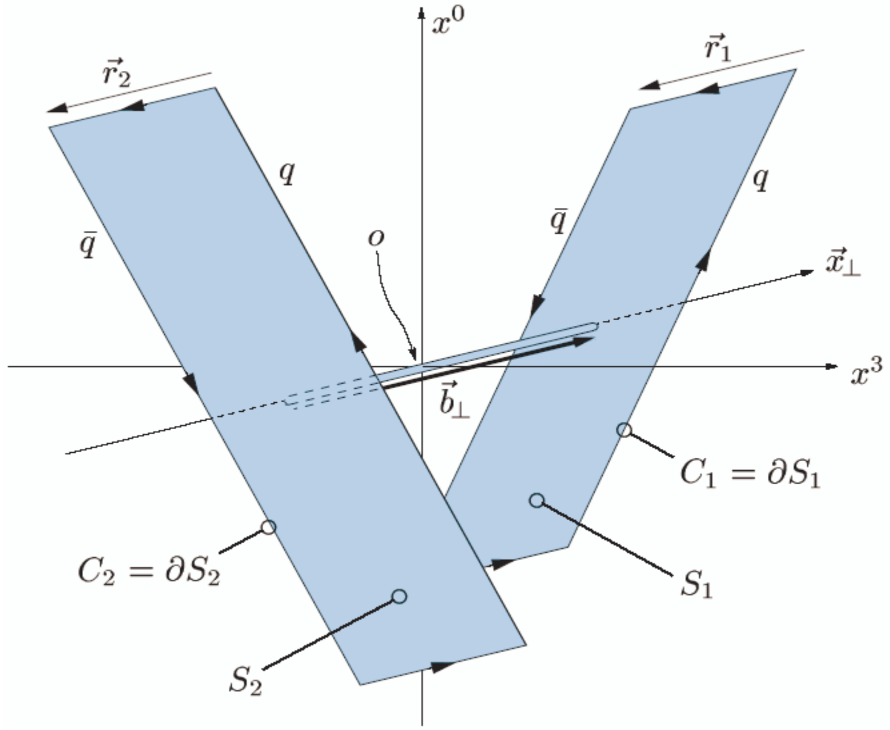
They are called splitting functions, they exist for all four combinations of quark q and gluon g processes. For P_qg a leading gluon splits into a leading q and an antiquark.

Challenge of Small x -Physics

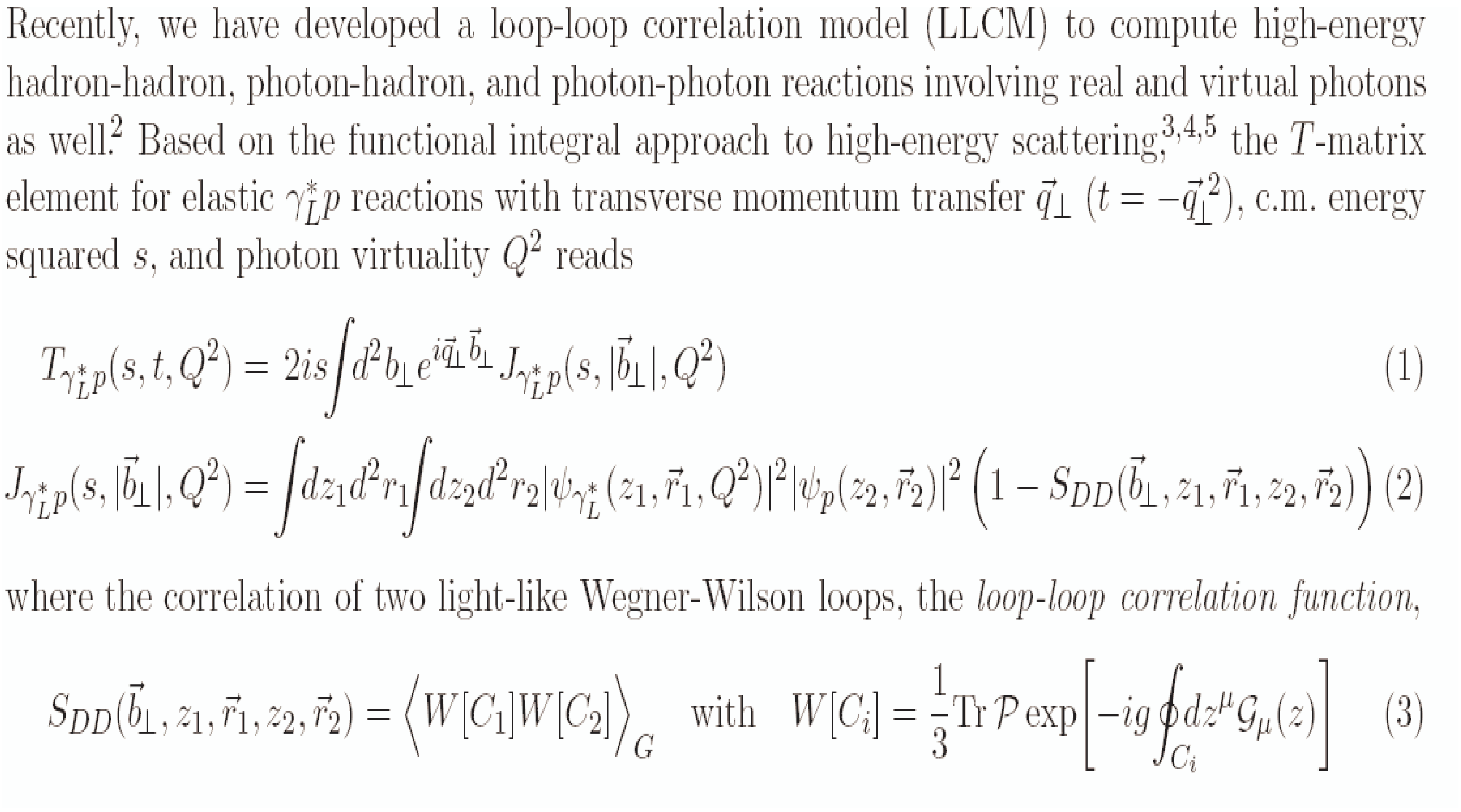
- By radiation the number of soft gluons in the proton is rapidly increasing
- Each gluon has a transverse size of about 1/Q^2
- This leads to overlapping gluons or a failure of the independent parton model
- At large gluon densities one may reach a regime of saturation
- This can be best studied in similar way to hadron hadron scattering where the gluon clouds of the individual constituents overlap also
- Build a scattering theory which preserves unitarity

Loop Loop Correlation Model

- The loop loop correlation model describes high energy scattering as the interaction of colour dipoles (q qbar in the photon) with (q diquark in the proton)



Loop Loop Correlation Model



Recently, we have developed a loop-loop correlation model (LLCM) to compute high-energy hadron-hadron, photon-hadron, and photon-photon reactions involving real and virtual photons as well.[2] Based on the functional integral approach to high-energy scattering,[3,4,5] the T-matrix element for elastic $\gamma_L^* p$ reactions with transverse momentum transfer $\vec{q}_\perp$ ($t = -\vec{q}_\perp^{\,2}$), c.m. energy squared s, and photon virtuality Q^2 reads

$$T_{\gamma_L^* p}(s, t, Q^2) = 2is \int d^2 b_\perp e^{i \vec{q}_\perp \vec{b}_\perp} J_{\gamma_L^* p}(s, |\vec{b}_\perp|, Q^2) \tag{1}$$

$$J_{\gamma_L^* p}(s, |\vec{b}_\perp|, Q^2) = \int dz_1 d^2 r_1 \int dz_2 d^2 r_2 |\psi_{\gamma_L^*}(z_1, \vec{r}_1, Q^2)|^2 |\psi_p(z_2, \vec{r}_2)|^2 \Big(1 - S_{DD}(\vec{b}_\perp, z_1, \vec{r}_1, z_2, \vec{r}_2)\Big) \tag{2}$$

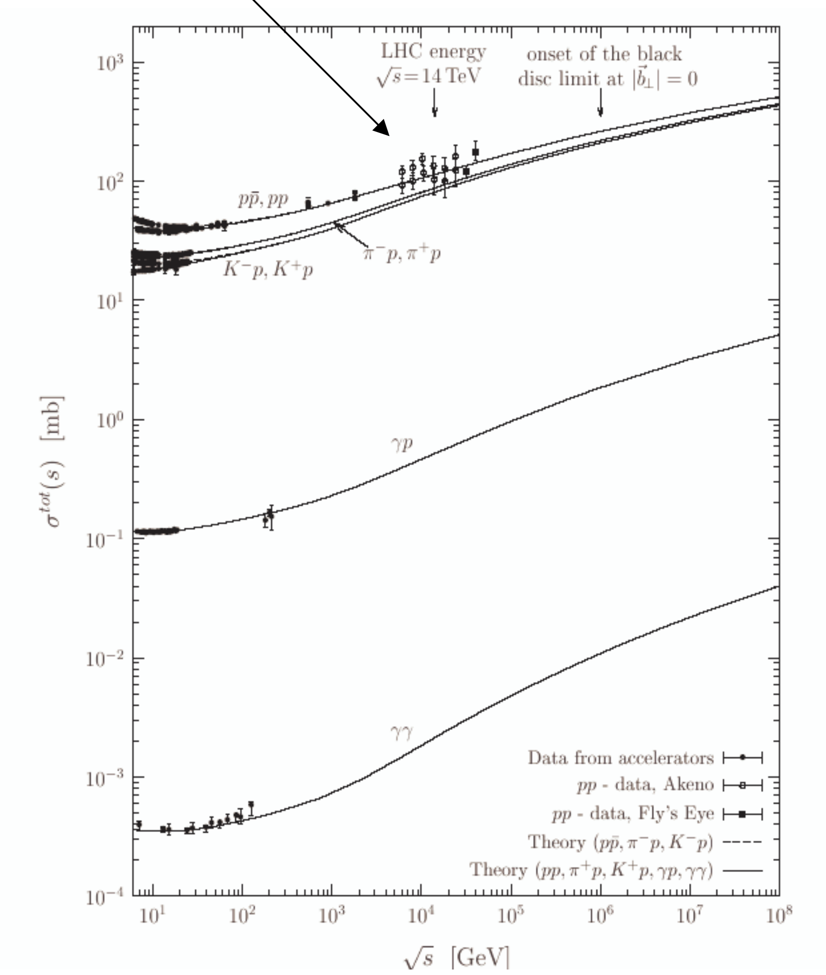
where the correlation of two light-like Wegner-Wilson loops, the *loop-loop correlation function*,

$$S_{DD}(\vec{b}_\perp, z_1, \vec{r}_1, z_2, \vec{r}_2) = \Big\langle W[C_1] W[C_2] \Big\rangle_G \quad \text{with} \quad W[C_i] = \frac{1}{3} \mathrm{Tr}\, \mathcal{P} \exp\left[-ig \oint_{C_i} dz^\mu \mathcal{G}_\mu(z)\right] \tag{3}$$

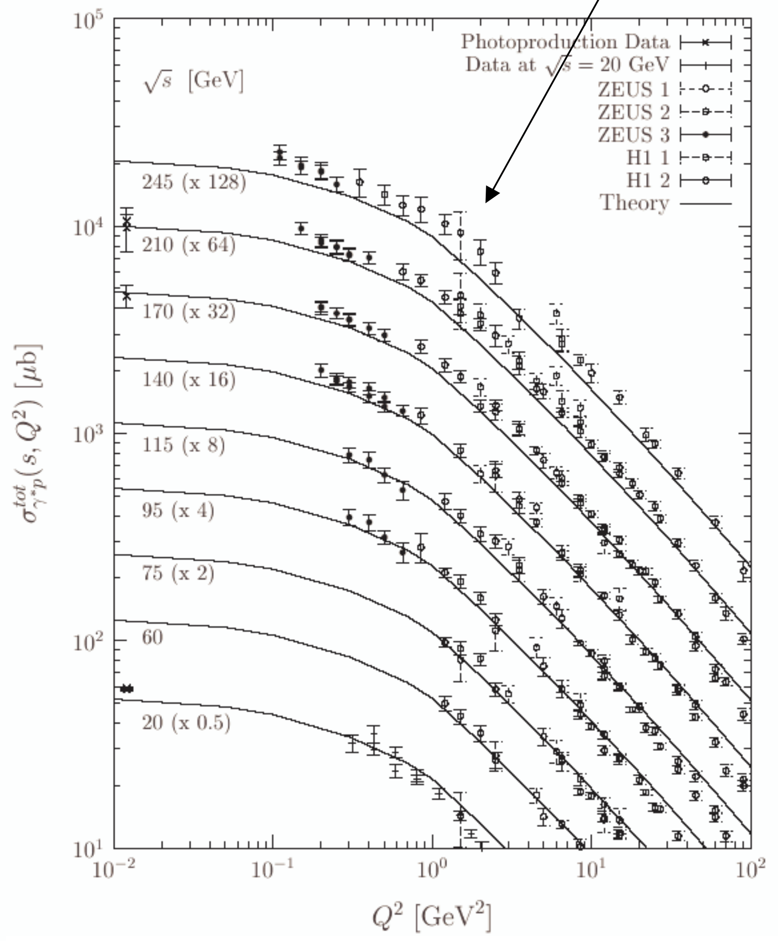
Cross Sections

A. Shoshi, F. Steffen, H.J. Pirner, Nucl. Phys. A 709,131,2002

LHC



HERA



Golec Biernat Wuesthoff Dipole Model of Saturation

- Describes the structure function with a dipole proton cross section
- The integral is weighted with the transverse or longitudinal q-anti quark density in the photon

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha}(\sigma_{\gamma p}^{T,tot} + \sigma_{\gamma p}^{L,tot}),$$

$$\sigma_{\gamma p}^{T/L,tot} = \int d^2x_\perp \int_0^1 dz \rho_\gamma^{T/L}(x_\perp, z)\sigma_{dip}(x_\perp)$$

The GBW dipol cross section saturates

$$\sigma_{GBW}(x_\perp, R_0) = \sigma_0(1 - e^{-\frac{x_\perp^2}{4R_0^2}}),$$

$$R_0 = \frac{1}{1GeV}(\frac{x_0}{3 * 10^{-4}})^{0.145} \ .$$

- For small dipol sizes the cross section is quadratic in the transverse size x-of the dipole
- For large sizes it saturates at a Radius R_0 which shrinks at higher energy or smaller x_0

Controversy

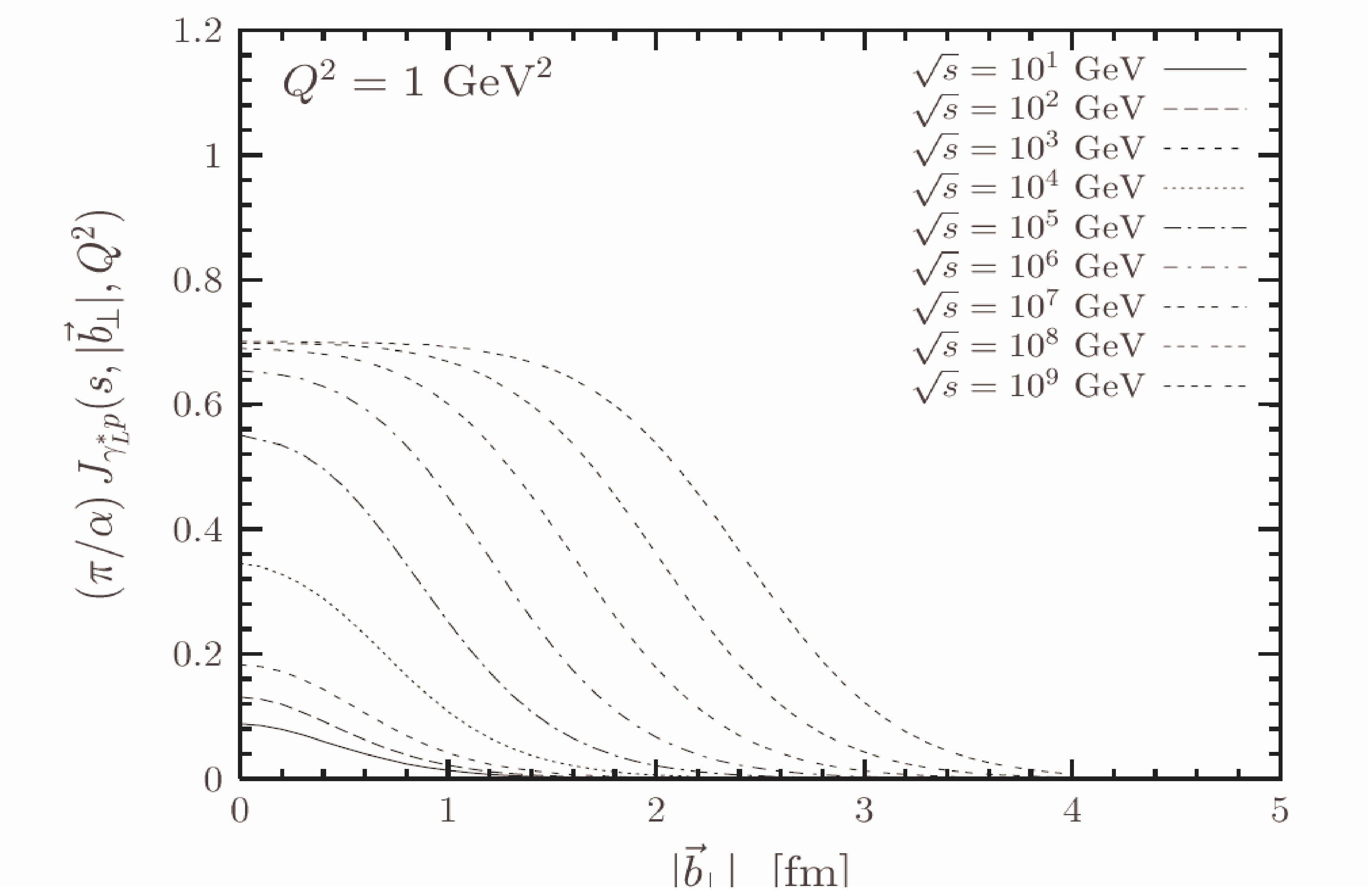
- We claim that this successful model has nothing to do with gluon saturation in the proton as advocated by Mueller, Mc Lerran et al.
- In order to look for gluon saturation one must look at the profile function of the gluon density
- This can be done best by analysing the longitudinal structure function in impact parameter space b
- Shoshi, Steffen and HJP have done this for different energies s
- Indeed one sees indications of saturation, but at extremely large cm energies of sqrt(s)=10^5 GeV

Calculation of nonintegrated Gluon structure function

$$xG(x,Q^2,|\vec{b}_\perp|) \approx 1.305\,\frac{Q^2}{\pi^2\alpha_s}\,\frac{\pi}{\alpha}\,J_{\gamma_L^* p}(0.417x,|\vec{b}_\perp|,Q^2)$$

- Leading order photon quark scattering does not contribute to the longitudinal structure function (helicity conservation in the Breit frame requires +1/2-1=-1/2)
- Longitudinal structure function comes from photon-gluon fusion (pQCD)
- The profile function of longitudinal photon-proton scattering is related to the nonintegrated gluon structure function xG(x,Q^2,b)

Gluon Saturation –G(x,Q^2,b)



Saturation as a function of cm energy

- For the smallest Q^2 saturation sets in at cm energies of 50TeV
- For larger virtualties of the photon Q^2 one sees that saturation occurs later
- So Q^2^=1 GeV^2 is a favourable case

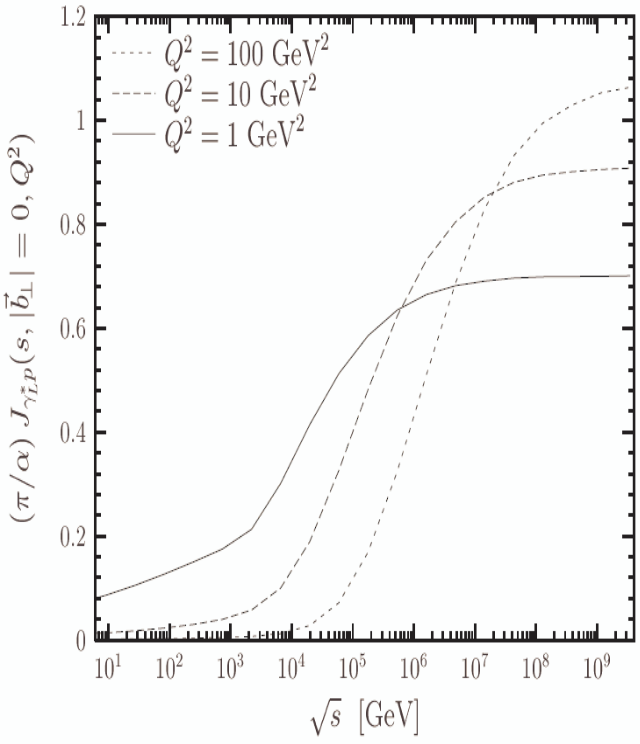


Figure 4: The profile function for a longitudinal photon scattering off a proton $J_{\gamma^*_L p}(s, |\vec{b}_\perp|, Q^2)$ divided by α/π is shown versus the c.m. energy $\sqrt{s}$ at zero impact parameter ($|\vec{b}_\perp| = 0$) for photon virtualities $Q^2 = 1$, 10, and $100\,\mathrm{GeV}^2$.

New Structure Functions for the Gluon

- G(x,Q^2,b) or the Fourier transform G(x,x,Q^2,pt^2) is called a nonforward S.F. It gives the gluon distribution function in impact parameter space b
- Differentiate from this the unintegrated gluon function F(x,kt^2) which gives the transverse momentum kt^2 which the gluon carries
- Note the two are not trivially related as every nuclear physicist knows:The impact parameter

  distribution of a lead nucleus is governed by the size of the nucleus 6 fm, whereas the momentum distribution is governed by the fermi momentum kf=270 MeV

Calcultion of the unintegrated gluon density (1)

$$\hat{\sigma}_{d-p}(s,r) = \frac{4\pi}{3}\int \frac{d^2k_t}{k_t^2}[1-e^{i\vec{r}\cdot\vec{k_t}}]\alpha_s f_g(x,k_t^2).$$

$$\begin{aligned}\alpha_s f_g(x,k_t^2) &= \frac{3}{16\pi^3}k_t\frac{d}{dk_t}k_t\frac{d}{dk_t}\int\frac{d^2r}{r^2}J_0(k_t r)\hat{\sigma}_{d-p}(x,r)\\ &= \frac{3}{16\pi^3}\int\frac{d^2r}{r^2}[-k_t r J_1(k_t r) - \frac{k_t^2 r^2}{2}(J_0(k_t r)-J_2(k_t r))]\times\hat{\sigma}_{d-p}(s,r),\end{aligned}$$

Calculation of the unintegrated Gluon density (2)

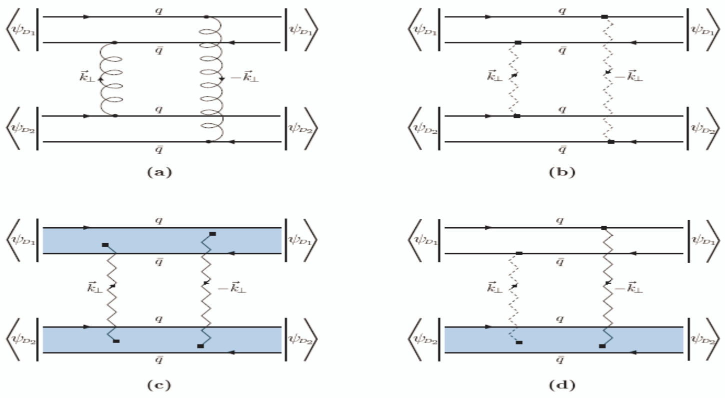


Figure 2: Perturbative and non-perturbative contributions to dipole-dipole scattering: (a) perturbative quark-quark interaction and non-perturbative (b) quark-quark, (c) string-string, and (d) quark-string interactions. The term quark is used genuinely for quarks

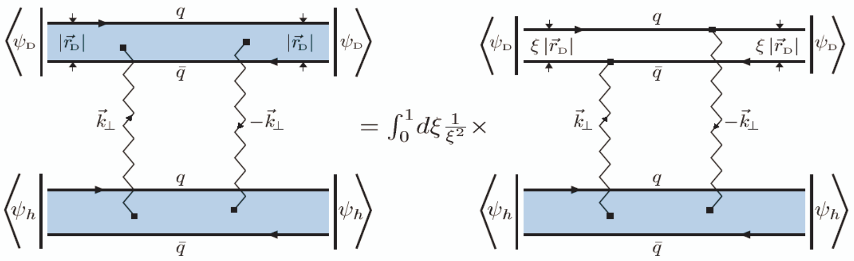


Figure 5: The string of length $|\vec{r}_D|$ is made up of stringless dipoles of size $\xi|\vec{r}_D|$ with $0 \leq \xi \leq 1$ and dipole number density $n(\xi) = 1/\xi^2$. The string-hadron scattering process reduces to an incoherent superposition of stringless dipole-hadron scattering processes.

- The dipole dipole scattering cross section has four contributions: quarks interaction with perturbative gluon exchange (spiral line) and strings (blue shaded areas) interacting with one another (wiggly line)
- The string string interaction (wiggly lines)from the nonabelian field strength correlators can be represented as a superposition of perturbative dipole string interactions which can then be related to the gluon density

Unintegrated Gluon Distribution F(x,kt)

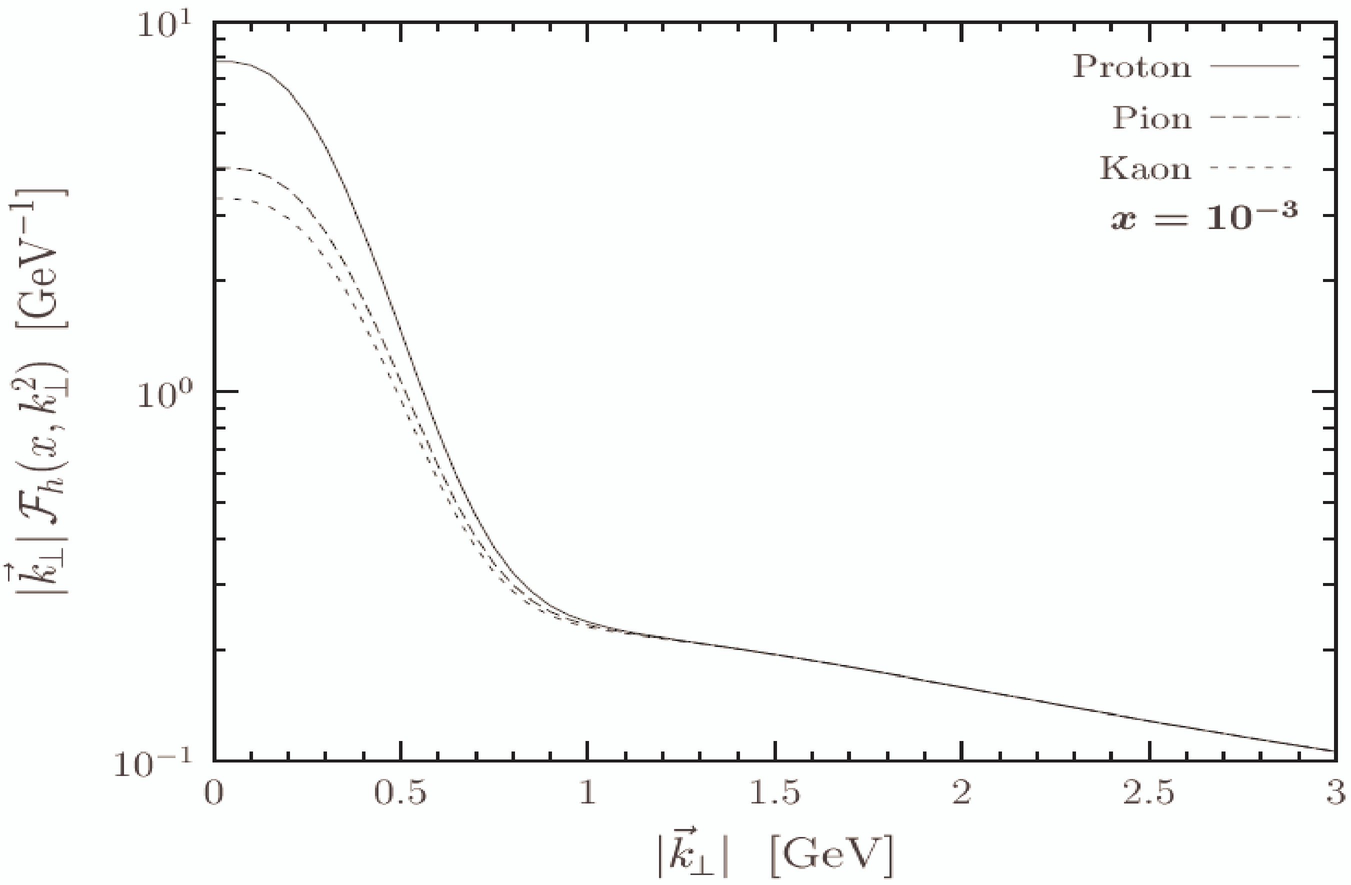
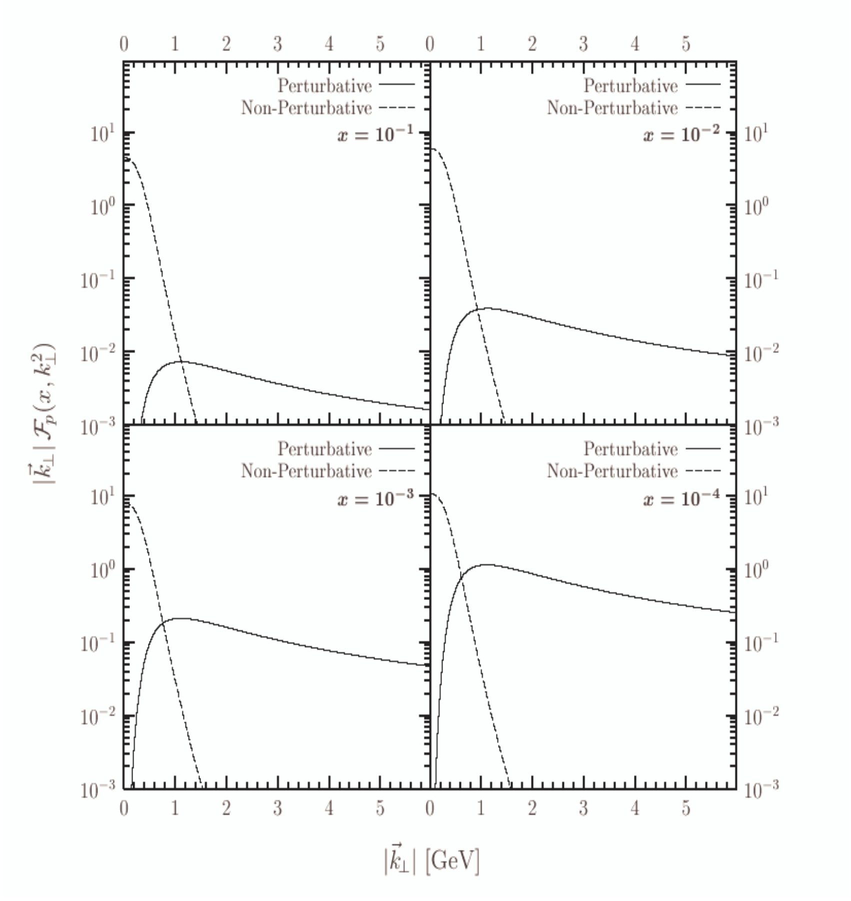


Figure 8: The unintegrated gluon distribution of the proton, pion, and kaon $\mathcal{F}_h(x, k_\perp^2)$ times the transverse momentum $|\vec{k}_\perp|$ as a function of $|\vec{k}_\perp|$ at Bjorken-variable $x = 10^{-3}$.

Energy Dependence of the Unintegrated Gluon Distribution



- The small kt part of the distribution weakly increases with energy (1/x)
- The large kt part of the distribution strongly increases with energy (1/x)

For nucleus use the Glauber dipole nucleus cross section

$$\hat{\sigma}_{d-A}(x,r) = \int d^2b\,\hat{\sigma}_{d-A}(b,x,r),$$

$$\hat{\sigma}_{d-A}(b,x,r) = 2(1-e^{-\frac{1}{2}T(b)\hat{\sigma}_{d-p}(s,r)}).$$

- In this simplified formula only the dipole proton cross section prop. to r^2 is used together with the profile function of the nucleus T(b)
- Because of the larger exponent the cross section leads to more multiple scattering ,i.e. to earlier saturation

$$\sigma_{GBW}(x_\perp,R_0) = \sigma_0(1-e^{-\frac{x_\perp^2}{4R_0^2}}),$$

$$R_0 = \frac{1}{1GeV}(\frac{x_0}{3*10^{-4}})^{0.145}\ .$$

one finds supersaturation

Nonintegrated Gluon structure function´for Gold R_A=6.37 fm

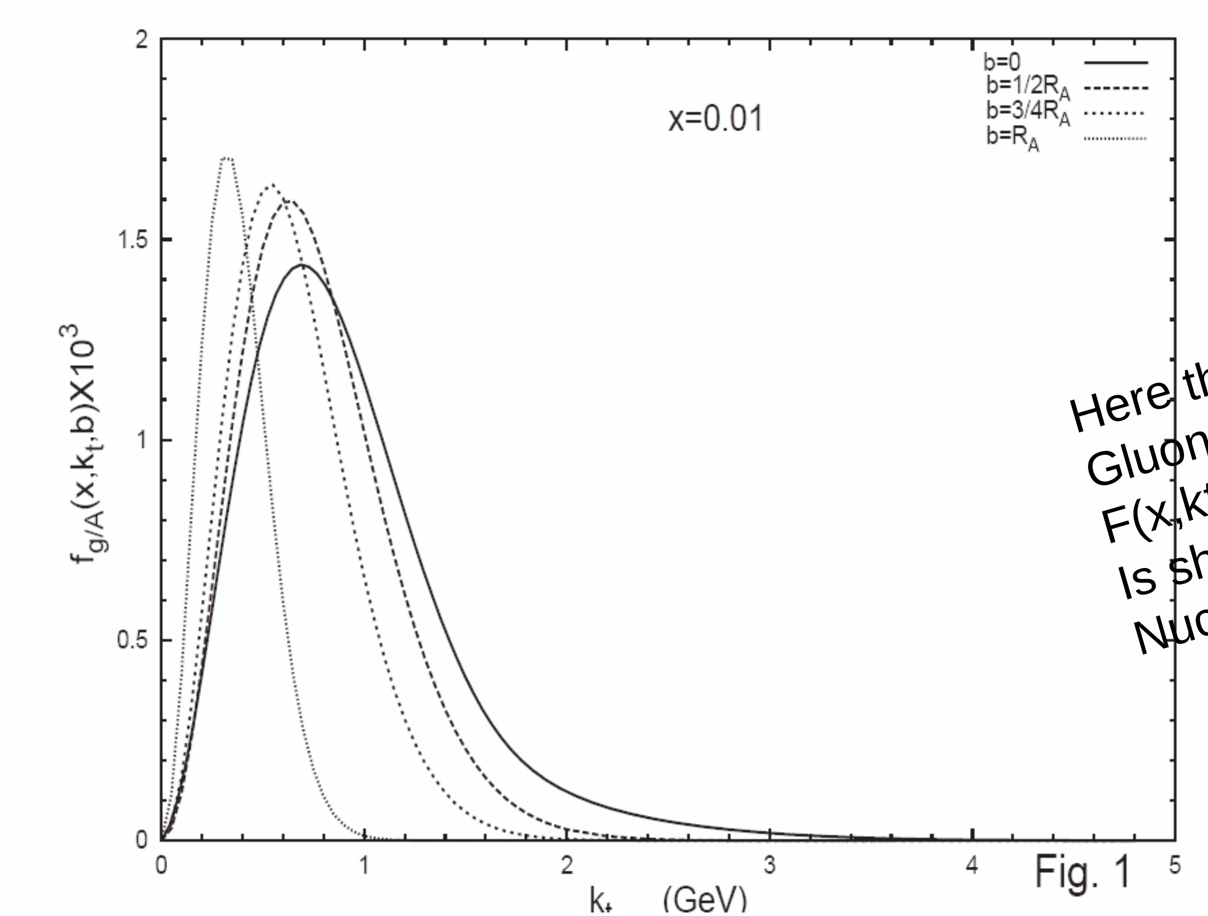


Fig. 1

Here the nonintegrated
Gluon Structure function
F(x,kt,b)
Is shown for different
Nuclear impact factors

Discussion

- Saturation with the scale Q_s = 1 GeV at RHIC is related to the k_t distribution of gluon, but has nothing to do with gluon saturation in impact parameter space b.
- The calculation of the k_t distribution of gluons in the nucleus is too simplified in the Glauber framework
- Especially it does not take into account confinement physics which starts to become important at this scale

Outlook: New Structure Functions

- Standard PQCD is based on the collinear parton scattering approach-i.e. all transverse momentum is obtained from gluon radiation (only good when coupling is weak)
- Kt-factorization starts with unintegrated structure functions and converges faster
- Assumption of color glass condensate is used in a regime where Q^2 is still too close to the confinement region
- The impact parameter structure of G(x,Q^2,b) does not indicate saturation,

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Profile function for pp-scattering

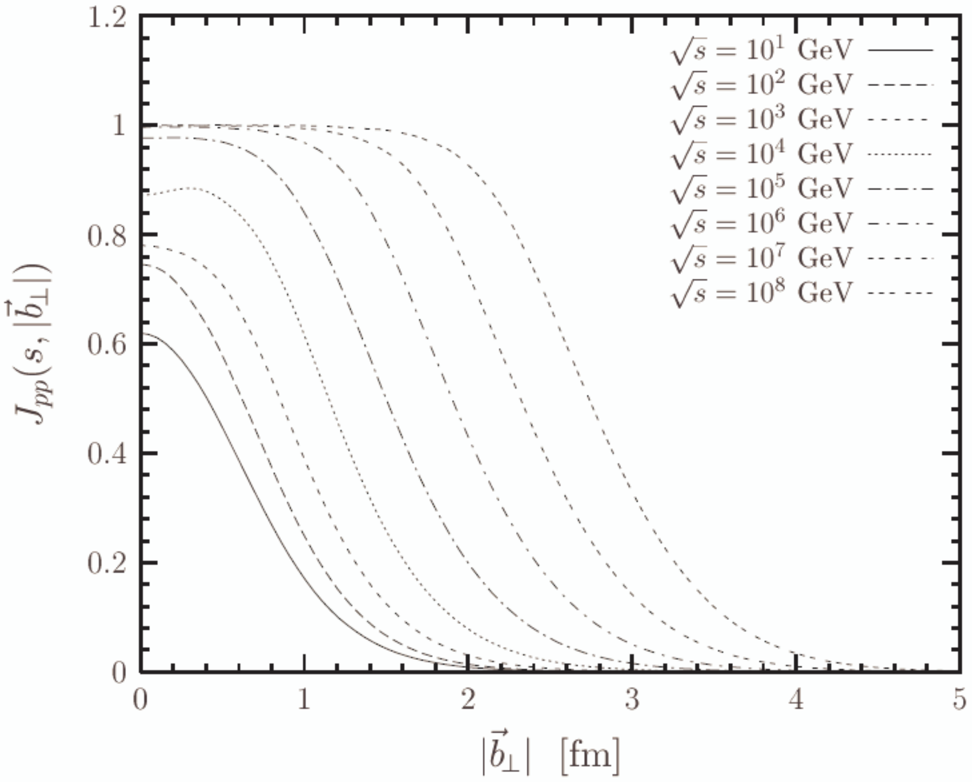

Figure 2: The profile function for proton-proton scattering $J_{pp}(s, |\vec{b}_\perp|)$ is shown versus the impact parameter $|\vec{b}_\perp|$ for c.m. energies from $\sqrt{s} = 10\,\text{GeV}$ to $\sqrt{s} = 10^8\,\text{GeV}$. The unitarity limit (3.4) corresponds to $J_{pp}(s, |\vec{b}_\perp|) = 2$ and the black disc limit (3.6) to $J_{pp}(s, |\vec{b}_\perp|) = 1$.

Saturation of the profile function and the Gluon distribution function G(x,Q^2,b=0)

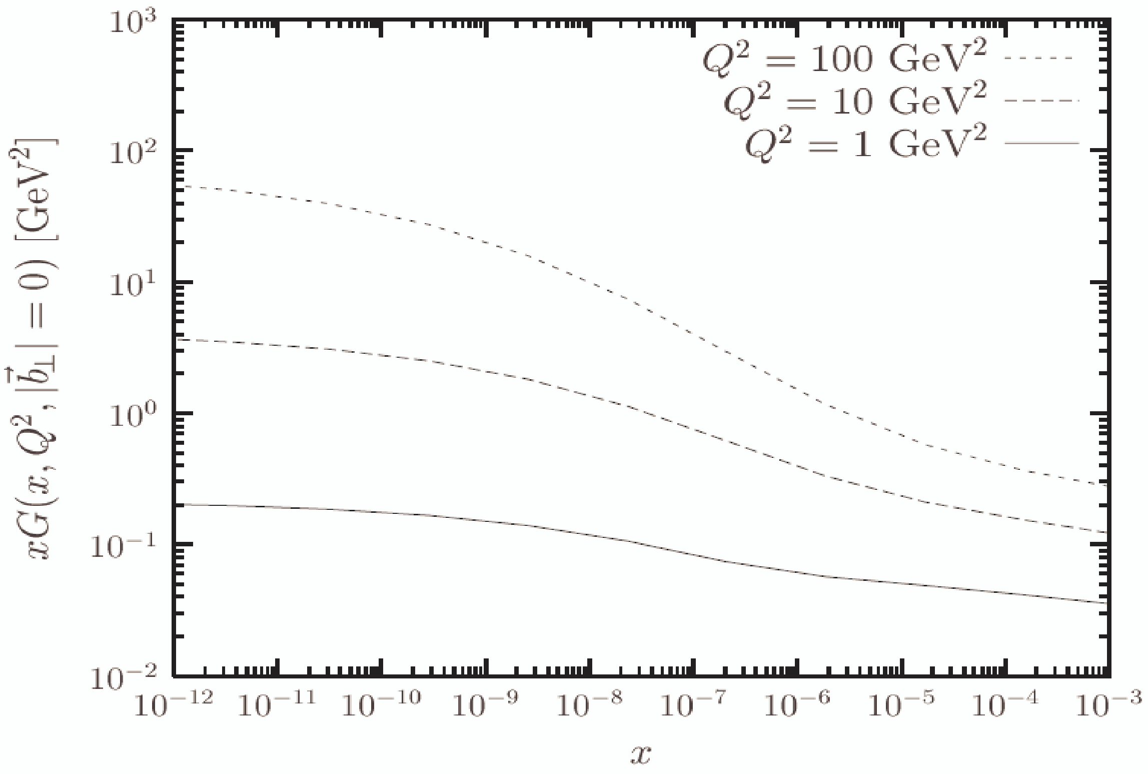

Figure 5: The gluon distribution of the proton at zero impact parameter $xG(x, Q^2, |\vec{b}_\perp| = 0)$ is shown as a function of x for $Q^2 = 1$, 10, and $100\,\mathrm{GeV}^2$. The results are obtained within the approximation (4.6).