Supersymmetry

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DPG School on Heavy Particles at the LHC
September 16-21, 2012

The purpose of these lectures is to

~~question~~

~~argue~~

~~annoy~~

~~make fun of~~

~~not reference~~

~~drink beer with~~

...

Introduction to Supersymmetry

The best single reference to supersymmetry at the weak scale is:

S.P. Martin, "A supersymmetry primer", hep-ph//9709356
(updated to v6; 09/2011)

For part of my lectures, I'll be heavily using a nice set of two lectures on supersymmetry given by Steve at PreSUSY 2010 in Bonn
http://zippy.physics.niu.edu/PreSUSY10.pdf

Aside from many comments, we'll deviate more substantially when we get to models, mediation, flavor, and of course, impact of LHC searches.

In addition to using notation consistent with the Primer, this will give you an idea of what has not changed, and perhaps more interestingly, what has changed in just two years of LHC running.

Here we go!

Introduction and Motivation

Good reasons to believe that the next discoveries beyond the presently known Standard Model will involve **supersymmetry (SUSY)**:

- A possible cold dark matter particle
- A light Higgs boson, in agreement with indirect constraints
- More generally, easy agreement with precision electroweak constraints
- Unification of gauge couplings
- Mathematical beauty

However, they are all insignificant compared to the one really good reason to suspect that supersymmetry is real:

- **The Hierarchy Problem**

An analogy: Coulomb self-energy correction to the electron's mass

(H. Murayama, `hep-ph/0002232`)

If the electron is really point-like, the classical electrostatic contribution to its energy is infinite.

Model the electron as a solid sphere of uniform charge density and radius R:

$$\Delta E_{\text{Coulomb}} = \frac{3e^2}{20\pi\epsilon_0 R}$$

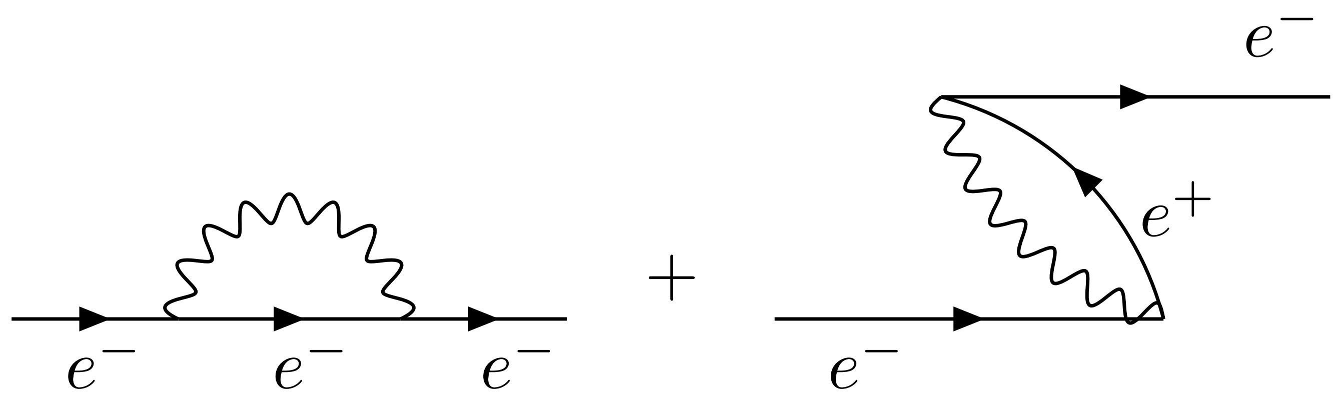
This implies a correction $\Delta m_e = \Delta E_{\text{Coulomb}}/c^2$ to the electron mass:

$$m_{e,\text{physical}} = m_{e,\text{bare}} + (1\,\text{MeV}/c^2)\left(\frac{0.86 \times 10^{-15}\,\text{meters}}{R}\right).$$

A divergence arises if we try to take $R \rightarrow 0$. Naively, we might expect $R \gtrsim 10^{-17}$ meters, to avoid having to tune the bare electron mass to better than 1%, for example:

$$0.511\,\text{MeV}/c^2 = -100.000\,\text{MeV}/c^2 + 100.511\,\text{MeV}/c^2.$$

However, there is another important quantum mechanical contribution:



The virtual positron effect cancels most of the Coulomb contribution, leaving:

$$m_{e,\text{physical}} = m_{e,\text{bare}} \left[1 + \frac{3\alpha}{4\pi} \ln \left(\frac{\hbar / m_e c}{R} \right) + \ldots \right]$$

with $\hbar/m_e c = 3.9 \times 10^{-13}$ meters. Even if R is as small as the Planck length 1.6×10^{-35} meters, where quantum gravity effects become dominant, this is only a 9% correction.

The existence of a "partner" particle for the electron, the positron, is responsible for eliminating the dangerously huge contribution to its mass.

The “reason” for the positron’s existence can be understood from a **symmetry**, namely the Poincaré invariance of quantum electrodynamics.

If we did not yet know about relativity or the positron, we would have three options:

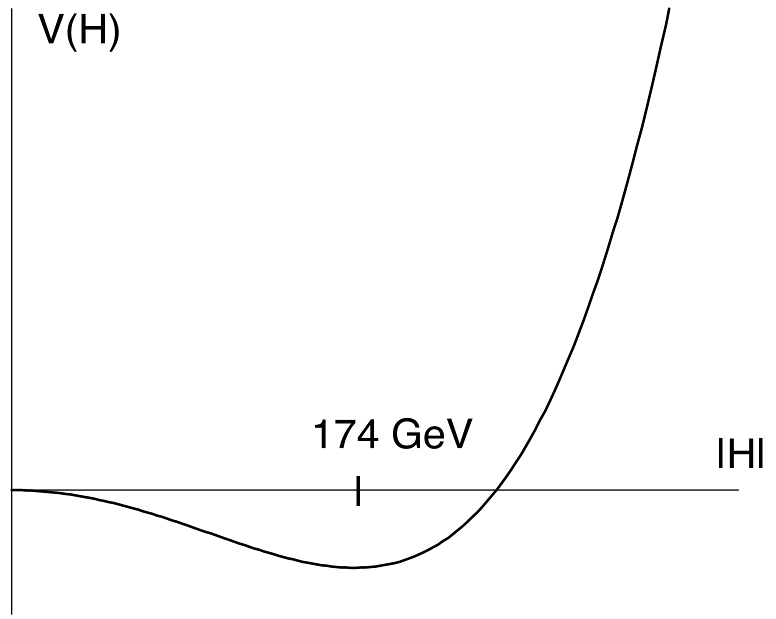
- Assume that the electron is not point-like, and has structure at a measurable size $R \gtrsim 10^{-17}$ meters.
- Assume that the electron is (nearly?) point-like, and there is a mysterious fine-tuning between the bare mass and the Coulomb correction to it.
- Predict that the electron’s symmetry “partner”, the positron, must exist.

Today we know that the last option is the correct explanation.

The Hierarchy Problem

Potential for H, the complex scalar field that is the electrically neutral part of the Standard Model Higgs field, is:

$$V(H) = m_H^2|H|^2 + \frac{\lambda}{2}|H|^4$$



For electroweak symmetry breaking to give the experimental m_Z, we need:

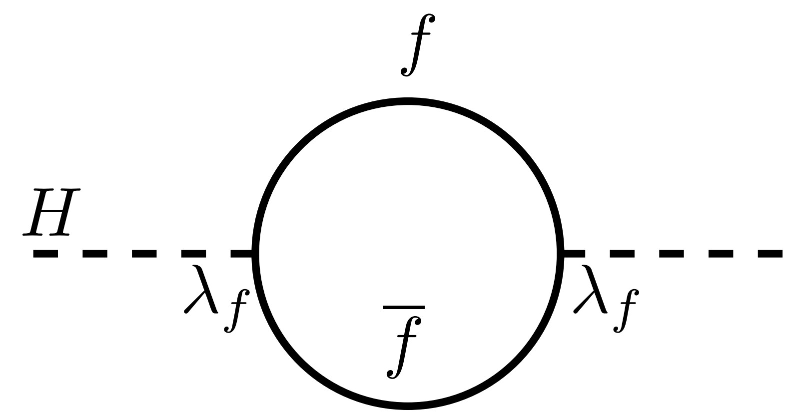
$$\langle H \rangle = \sqrt{-m_H^2/\lambda} \approx 174 \text{ GeV}$$

The requirement of unitarity in the scattering of Higgs bosons and longitudinal W bosons tells us that λ is not much larger than 1. Therefore,

$$-(\text{few hundred GeV})^2 \lesssim m_H^2 < 0.$$

However, this appears fine-tuned (in other words, incredibly and mysteriously lucky!) when we consider the likely size of quantum corrections to m_H^2.

Contributions to m_H^2 from a Dirac fermion loop:



The correction to the Higgs squared mass parameter from this loop diagram is:

$$\Delta m_H^2 = \frac{\lambda_f^2}{16\pi^2}\left[-2M_{\mathrm{UV}}^2 + 6m_f^2 \ln\left(M_{\mathrm{UV}}/m_f\right) + \ldots\right]$$

where λ_f is the coupling of the fermion to the Higgs field H.

M_{UV} should be interpreted as the ultraviolet cutoff scale(s) at which new physics enters to cut off the loop integrations.

So m_H^2 is sensitive to the **largest** mass scales in the theory.

The Hierarchy Problem

Why should:

$$\frac{|m_H^2|}{M_{\rm Planck}^2} \lesssim 10^{-32}$$

if individual radiative corrections Δm_H^2 are of order $M_{\rm Planck}^2$ or $M_{\rm string}^2$, multiplied by loop factors?

The problem is present even if String Theory is wrong and some other unspecified effects modify physics at $M_{\rm Planck}$, or any other very large mass scale, to make the loop integrals converge.

An incredible coincidence seems to be required to make the corrections to the Higgs squared mass cancel to give a much smaller number.

Supersymmetry Solution

The systematic cancellation of loop corrections to the Higgs mass squared requires the type of conspiracy that is better known to physicists as a **symmetry**.

Fermion loops and boson loops gave contributions with opposite signs:

$$\Delta m_H^2 = -\frac{\lambda_f^2}{16\pi^2}(2M_{\rm UV}^2) + \ldots \qquad \text{(Dirac fermion)}$$

$$\Delta m_H^2 = +\frac{\lambda_S}{16\pi^2}M_{\rm UV}^2 + \ldots \qquad \text{(complex scalar)}$$

The cancellation is not because "all fermion loops cancel all boson loops" (this is false!). Instead, the chirality of fermions has been promoted to scalars.

This is how supersymmetry makes the cancellation not only possible, but automatic.

This requires *two* complex scalars for every Dirac fermion, and $\lambda_S = \lambda_f^2$.

Supersymmetry

Supersymmetry

A SUSY transformation turns a boson state into a fermion state, and vice versa. So the operator Q that generates such transformations acts, schematically, like:

$$Q|\text{Boson}\rangle = |\text{Fermion}\rangle; \qquad\qquad Q|\text{Fermion}\rangle = |\text{Boson}\rangle$$

This means that Q must be an anticommuting spinor. This is an intrinsically complex object, so $Q^\dagger$ is also a distinct symmetry generator:

$$Q^\dagger|\text{Boson}\rangle = |\text{Fermion}\rangle; \qquad\qquad Q^\dagger|\text{Fermion}\rangle = |\text{Boson}\rangle$$

The possible forms for such theories are highly restricted by the Haag-Lopuszanski-Sohnius extension of the Coleman-Mandula Theorem. In a 4-dimensional theory with chiral fermions (like the Standard Model) and non-trivial scattering, then Q carries spin-$1/2$ with L helicity, and $Q^\dagger$ has spin-$1/2$ with R helicity, and they must satisfy...

The Supersymmetry Algebra

$$\begin{aligned}\{Q, Q^\dagger\} &= P^\mu \\ \{Q, Q\} &= \{Q^\dagger, Q^\dagger\} = 0 \\ [P^\mu, Q] &= [P^\mu, Q^\dagger] = 0 \\ [T^a, Q] &= [T^a, Q^\dagger] = 0\end{aligned}$$

Here $P^\mu = (H, \vec{\mathbf{P}})$ is the generator of spacetime translations, and T^a are the gauge generators. This is schematic, with spinor indices suppressed

The single-particle states of the theory fall into irreducible representations of this algebra, called **supermultiplets**. Fermion and boson members of a given supermultiplet are **superpartners** of each other. By definition, if $|\Omega\rangle$ and $|\Omega'\rangle$ are superpartners, then $|\Omega'\rangle$ is equal to some combination of $Q, Q^\dagger$ acting on $|\Omega\rangle$.

Therefore, since P^2 and T^a commute with $Q, Q^\dagger$, all members of a given supermultiplet must have the same (mass)^2 and gauge quantum numbers.

Types of supermultiplets

Chiral (or "Scalar" or "Matter" or "Wess-Zumino") supermultiplet:

1 two-component Weyl fermion, helicity $\pm\frac{1}{2}$. $(n_F = 2)$

2 real spin-0 scalars = 1 complex scalar. $(n_B = 2)$

The Standard Model quarks, leptons and Higgs bosons must fit into these.

Gauge (or "Vector") supermultiplet:

1 two-component Weyl fermion gaugino, helicity $\pm\frac{1}{2}$. $(n_F = 2)$

1 real spin-1 massless gauge vector boson. $(n_B = 2)$

The Standard Model $\gamma, Z, W^{\pm}, g$ must fit into these.

Gravitational supermultiplet:

1 two-component Weyl fermion gravitino, helicity $\pm\frac{3}{2}$. $(n_F = 2)$

1 real spin-2 massless graviton. $(n_B = 2)$

How do the Standard Model quarks and leptons fit in?

Each quark or charged lepton is 1 Dirac = 2 Weyl fermions

Electron: $\Psi_e = \begin{pmatrix} e_L \\ e_R \end{pmatrix}$ $\leftarrow$ two-component Weyl LH fermion; $\leftarrow$ two-component Weyl RH fermion

Each of e_L and e_R is part of a chiral supermultiplet, so each has a complex, spin-0 superpartner, called $\widetilde{e}_L$ and $\widetilde{e}_R$ respectively. They are called the "left-handed selectron" and "right-handed selectron", although they carry no spin.

The conjugate of a right-handed Weyl spinor is a left-handed Weyl spinor. Define two-component left-handed Weyl fields: $e \equiv e_L$ and $\bar{e} \equiv e_R^\dagger$. So, there are two left-handed chiral supermultiplets for the electron:

$$(e,\ \widetilde{e}_L) \quad \text{and} \quad (\bar{e},\ \widetilde{e}_R^*).$$

The other charged leptons and quarks are similar.

Chiral supermultiplets of the Minimal Supersymmetric Standard Model (MSSM):

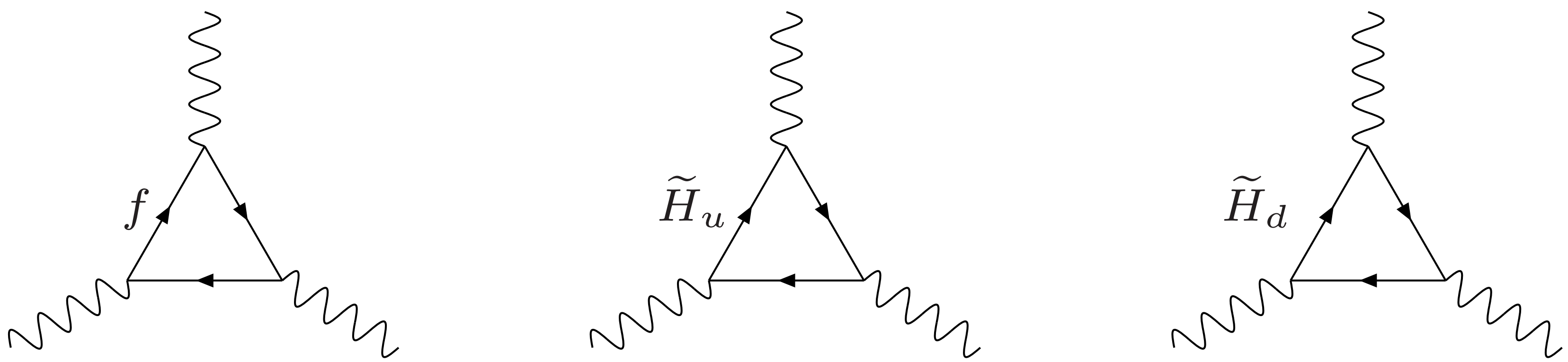
Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L \quad \widetilde{d}_L)$	$(u_L \quad d_L)$	$(\,\mathbf{3},\ \mathbf{2}\,,\ \frac{1}{6})$
($\times 3$ families)	$\bar{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\bar{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu} \quad \widetilde{e}_L)$	$(\nu \quad e_L)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$
($\times 3$ families)	$\bar{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\,\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H_u^+ \quad H_u^0)$	$(\widetilde{H}_u^+ \quad \widetilde{H}_u^0)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ +\frac{1}{2})$
	H_d	$(H_d^0 \quad H_d^-)$	$(\widetilde{H}_d^0 \quad \widetilde{H}_d^-)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$

The superpartners of the Standard Model particles are written with a $\widetilde{\ }$. The scalar names are obtained by putting an “s” in front, so they are generically called **squarks** and **sleptons**, short for “scalar quark” and “scalar lepton”.

The Standard Model Higgs boson requires two different chiral supermultiplets, H_u and H_d. The fermionic partners of the Higgs scalar fields are called **higgsinos**. There are two charged and two neutral Weyl fermion higgsino degrees of freedom.

Why do we need two Higgs supermultiplets? Two reasons:

1) Anomaly Cancellation



$$\sum_{\text{SM fermions}} Y_f^3 = 0 \qquad + \; 2\left(\frac{1}{2}\right)^3 \qquad + \; 2\left(-\frac{1}{2}\right)^3 \;=\; 0$$

This anomaly cancellation occurs if and only if **both** $\widetilde{H}_u$ and $\widetilde{H}_d$ higgsinos are present.

2) Quark and Lepton masses

Only the H_u Higgs scalar can give masses to charge $+2/3$ quarks (top).

Only the H_d Higgs scalar can give masses to charge $-1/3$ quarks (bottom) and the charged leptons. We will show this later.

The vector bosons of the Standard Model live in gauge supermultiplets:

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\ \mathbf{8},\ \mathbf{1}\ ,\ 0)$
winos, W bosons	$\widetilde{W}^{\pm}\quad \widetilde{W}^{0}$	$W^{\pm}\quad W^{0}$	$(\ \mathbf{1},\ \mathbf{3}\ ,\ 0)$
bino, B boson	$\widetilde{B}^{0}$	B^{0}	$(\ \mathbf{1},\ \mathbf{1}\ ,\ 0)$

The spin-$1/2$ **gauginos** transform as the adjoint representation of the gauge group. Each gaugino carries a $\widetilde{\ }$. The color-octet superpartner of the gluon is called the **gluino**. The $SU(2)_L$ gauginos are called **winos**, and the $U(1)_Y$ gaugino is called the **bino**.

However, the winos and the bino are not mass eigenstate particles; they mix with each other and with the higgsinos of the same charge.

Exact versus Broken Supersymmetry

Recall that if supersymmetry were an exact symmetry, then superpartners would have to be exactly degenerate with each other. For example,

$$m_{\tilde{e}_L} = m_{\tilde{e}_R} = m_e = 0.511 \text{ MeV}$$

$$m_{\tilde{u}_L} = m_{\tilde{u}_R} = m_u$$

$$m_{\tilde{g}} = m_{\text{gluon}} = 0 + \text{QCD confinement effects}$$

etc.

But new particles with these properties have been ruled out long ago, so:

Supersymmetry must be broken in the vacuum state chosen by Nature.

Supersymmetry is thought to be spontaneously broken and therefore hidden, the same way that the full electroweak symmetry $SU(2)_L \times U(1)_Y$ is hidden from very low-energy experiments.

“Soft” Supersymmetry Breaking

For a clue as to the nature of SUSY breaking, return to our motivation in the Hierarchy Problem. The Higgs mass parameter gets corrections from each chiral supermultiplet:

$$\Delta m_H^2 = \frac{1}{16\pi^2}(\lambda_S - \lambda_F^2)M_{\rm UV}^2 + \ldots$$

The corresponding formula for Higgsinos has no term proportional to $M_{\rm UV}^2$; fermion masses always diverge at worst like $\ln(M_{\rm UV})$. Therefore, if supersymmetry were exact and unbroken, it must be that:

$$\lambda_S = \lambda_F^2,$$

in other words, the dimensionless (scalar)^4 couplings are the squares of the (scalar)-(fermion)-(antifermion) couplings.

If we want SUSY to be a solution to the hierarchy problem, we must demand that this is still true even after SUSY is broken:

The breaking of supersymmetry must be “soft”. This means that it does not change the dimensionless terms in the Lagrangian.

Hierarchy Problem versus Soft Supersymmetry Breaking

The effective Lagrangian of the MSSM is therefore:

$$\mathcal{L} = \mathcal{L}_{\rm SUSY} + \mathcal{L}_{\rm soft}$$

- $\mathcal{L}_{\rm SUSY}$ contains all of the gauge, Yukawa, and dimensionless scalar couplings, and preserves exact supersymmetry
- $\mathcal{L}_{\rm soft}$ violates supersymmetry, and contains only mass terms and couplings with *positive* mass dimension.

If $m_{\rm soft}$ is the largest mass scale in $\mathcal{L}_{\rm soft}$, then by dimensional analysis,

$$\Delta m_H^2 = m_{\rm soft}^2 \left[\frac{\lambda}{16\pi^2} \ln(M_{\rm UV}/m_{\rm soft}) + \ldots \right],$$

where λ stands for dimensionless couplings. This is because Δm_H^2 must vanish in the limit $m_{\rm soft} \rightarrow 0$, in which SUSY is restored. Therefore, we expect that $m_{\rm soft}$ should not be much larger than roughly 1000 GeV.

This is already in some tension with LHC searches; more on this later.

Constructing a Supersymmetric Lagrangian

The simplest SUSY model: a free chiral supermultiplet

The minimum particle content for a SUSY theory is a complex scalar ϕ and its superpartner fermion ψ. We must at least have kinetic terms for each, so:

$$S = \int d^4x \left(\mathcal{L}_{\rm scalar} + \mathcal{L}_{\rm fermion}\right)$$

$$\mathcal{L}_{\rm scalar} = -\partial^\mu \phi^* \partial_\mu \phi \qquad\qquad \mathcal{L}_{\rm fermion} = -i\psi^\dagger \overline{\sigma}^\mu \partial_\mu \psi$$

A SUSY transformation should turn ϕ into ψ, so try:

$$\delta\phi = \epsilon\psi; \qquad\qquad \delta\phi^* = \epsilon^\dagger \psi^\dagger$$

where $\epsilon =$ infinitesimal, anticommuting, constant spinor, with dimension [mass]$^{-1/2}$, that parameterizes the SUSY transformation. Then we find:

$$\delta\mathcal{L}_{\rm scalar} = -\epsilon\partial^\mu\psi\partial_\mu\phi^* - \epsilon^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi.$$

We would like for this to be canceled by an appropriate SUSY transformation of the fermion field. . .

To have any chance, $\delta\psi$ should be linear in $\epsilon^\dagger$ and in ϕ, and must contain one spacetime derivative. There is only one possibility, up to a multiplicative constant:

$$\delta\psi_\alpha = i(\sigma^\mu \epsilon^\dagger)_\alpha \partial_\mu \phi; \qquad\qquad \delta\psi^\dagger_{\dot\alpha} = -i(\epsilon\sigma^\mu)_{\dot\alpha}\partial_\mu\phi^*$$

With this guess, one finds:

$$\delta\mathcal{L}_{\text{fermion}} \;=\; -\delta\mathcal{L}_{\text{scalar}} + \big(\text{total derivative}\big)$$

so the action S is indeed invariant under the SUSY transformation, justifying the guess of the multiplicative factor. This is called the free Wess-Zumino model.

Furthermore, if we take the commutator of two SUSY transformations:

$$\delta_{\epsilon_2}(\delta_{\epsilon_1}\phi) - \delta_{\epsilon_1}(\delta_{\epsilon_2}\phi) \;=\; i(\epsilon_1\sigma^\mu\epsilon_2 - \epsilon_2\sigma^\mu\epsilon_1)\partial_\mu\phi$$

Since ∂_μ corresponds to the spacetime 4-momentum P_μ, this has exactly the form demanded by the SUSY algebra discussed earlier.

The fact that two SUSY transformations give back another symmetry (namely a spacetime translation) means that the SUSY algebra "closes".

If we do the same check for the fermion ψ:

$$\begin{aligned}\delta_{\epsilon_2}(\delta_{\epsilon_1}\psi_\alpha) - \delta_{\epsilon_1}(\delta_{\epsilon_2}\psi_\alpha) \quad &= \quad i(\epsilon_1\sigma^\mu\epsilon_2 - \epsilon_2\sigma^\mu\epsilon_1)\partial_\mu\psi_\alpha \\ &\quad -i\epsilon_{1\alpha}(\epsilon_2^\dagger\overline{\sigma}^\mu\partial_\mu\psi) + i\epsilon_{2\alpha}(\epsilon_1^\dagger\overline{\sigma}^\mu\partial_\mu\psi)\end{aligned}$$

The first line is expected, but the second line only vanishes on-shell (when the classical equation of motion $\overline{\sigma}^\mu\partial_\mu\psi = 0$ is satisfied). This seems like a problem, since we want SUSY to be a valid symmetry of the quantum theory (off-shell)!

To show that there is no problem, we introduce another bosonic spin-0 field, F, called an auxiliary field. Its Lagrangian density is:

$$\mathcal{L}_{\text{aux}} = F^*F$$

Note that F has no kinetic term, and has dimensions [mass]^2, unlike an ordinary scalar field. It has the not-very-exciting equations of motion $F = F^* = 0$.

The auxiliary field F does not affect the dynamics, classically or in the quantum theory. But it does appear in modified SUSY transformation laws:

$$\begin{aligned} \delta\phi &= \epsilon\psi \\ \delta\psi_\alpha &= i(\sigma^\mu\epsilon^\dagger)_\alpha\partial_\mu\phi + \epsilon_\alpha F \\ \delta F &= i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\psi \end{aligned}$$

Now the total Lagrangian

$$\mathcal{L} = -\partial^\mu\phi^*\partial_\mu\phi - i\psi^\dagger\overline{\sigma}^\mu\partial_\mu\psi + F^*F$$

is still invariant, and also one can now check:

$$\delta_{\epsilon_2}(\delta_{\epsilon_1}X) - \delta_{\epsilon_1}(\delta_{\epsilon_2}X) \;=\; i(\epsilon_1\sigma^\mu\epsilon_2 - \epsilon_2\sigma^\mu\epsilon_1)\partial_\mu X$$

for each of $X = \phi, \phi^*, \psi, \psi^\dagger, F, F^*$, without using equations of motion.

So in the “modified” theory, SUSY does close off-shell as well as on-shell.

The auxiliary field F is really just a book-keeping device to make this simple. We can see why it is needed by considering the number of degrees of freedom on-shell (classically) and off-shell (quantum mechanically):

	ϕ	ψ	F
on-shell ($n_B = n_F = 2$)	2	2	0
off-shell ($n_B = n_F = 4$)	2	4	2

(Going on-shell eliminates half of the propagating degrees of freedom of the fermion, because the Lagrangian density is linear in time derivatives, so that the fermionic canonical momenta are not independent phase-space variables.)

The auxiliary field also plays an important role when we add interactions to the theory, and in gaining a simple understanding of SUSY breaking.

Supersymmetric Masses and Interactions

Masses and Interactions for Chiral Supermultiplets

The Lagrangian describing a collection of free, massless, chiral supermultiplets is

$$\mathcal{L} = -\partial^\mu \phi^{*i} \partial_\mu \phi_i - i\psi^{\dagger i} \overline{\sigma}^\mu \partial_\mu \psi_i + F^{*i} F_i.$$

Question: How do we make mass terms and interactions for these fields, while still preserving supersymmetry invariance?

Answer: choose a **superpotential**,

$$W = L^i \Phi_i + \frac{1}{2} M^{ij} \Phi_i \Phi_j + \frac{1}{6} y^{ijk} \Phi_i \Phi_j \Phi_k$$

in terms of chiral superfields, where

$$\Phi = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$$

W cannot depend on Φ_i^* , only the Φ_i. It must be an analytic function of the superfields treated as complex variables.

The superpotential W contains masses M^{ij} and couplings y^{ijk}, which must be symmetric under interchange of i, j, k.

Supersymmetry is very restrictive; you cannot just do anything you want!

Superspace Interlude

Steve Martin's "Primer" (v1-v5) emphasized that supersymmetric Lagrangians can be obtained entirely in terms of components fields with a "scalar superpotential"

$$W = \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k$$

Most SUSY theorists, however, work with "superfields" in "superspace" where interactions are manifestly supersymmetric. The superspace superpotential is:

$$W = L^i\Phi_i + \frac{1}{2}M^{ij}\Phi_i\Phi_j + \frac{1}{6}y^{ijk}\Phi_i\Phi_j\Phi_k$$

where

$$\Phi = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$$

There is an elegant formalism to derive all of the results using superfields.
(and has many advantages, e.g. to understand what SUSY breaking terms are "soft", etc.).

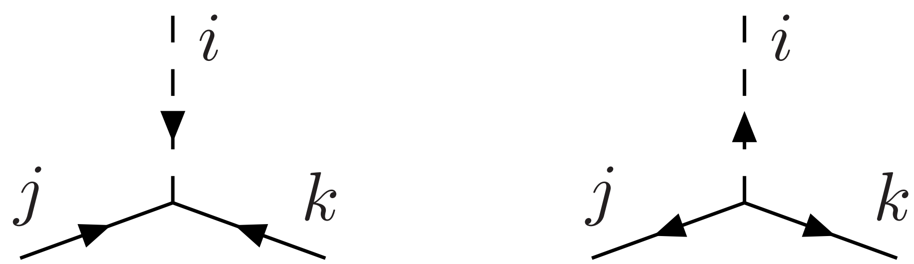
[The "Primer" v6 (2011/09) has a new chapter (Ch 4) devoted to superfields in superspace.]

The superpotential $W = L^i\Phi_i + \frac{1}{2}M^{ij}\Phi_i\Phi_j + \frac{1}{6}y^{ijk}\Phi_i\Phi_j\Phi_k$ **determines all non-gauge masses and interactions.**

Both scalars and fermions have squared mass matrix $M_{ik}M^{kj}$.

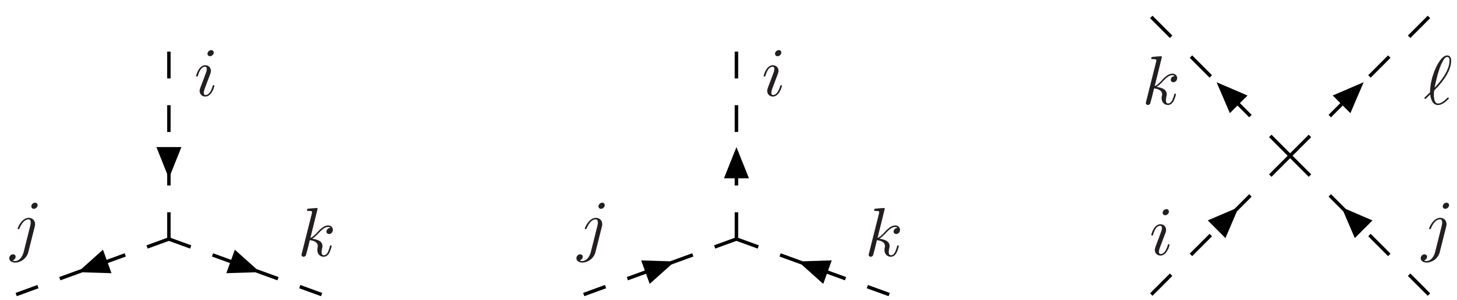
The interaction Feynman rules for the chiral supermultiplets are:

Yukawa interactions:



$-iy^{ijk}$ $\quad$ $-iy_{ijk}$

Scalar interactions:



$-iM^{in}y_{njk}$ $\quad$ $-iM_{in}y^{njk}$ $\quad$ $-iy^{ijn}y_{k\ell n}$

Supersymmetric Gauge Theories

A gauge or vector supermultiplet contains physical fields:

- a gauge boson A^a_μ
- a gaugino λ^a_α.

The index a runs over the gauge group generators $[1, 2, \ldots, 8$ for $SU(3)_C$; $1, 2, 3$ for $SU(2)_L$; $\;1$ for $U(1)_Y]$.

Suppose the gauge coupling constant is g and the structure constants of the group are f^{abc}. The Lagrangian for the gauge supermultiplet is:

$$\mathcal{L} = -\tfrac{1}{4} F^{\mu\nu}_a F^a_{\mu\nu} - i\lambda^{\dagger a}\overline{\sigma}^\mu \nabla_\mu \lambda^a + \tfrac{1}{2} D^a D^a$$

where D^a is a real spin-0 auxiliary field with no kinetic term, and

$$\nabla_\mu \lambda^a \equiv \partial_\mu \lambda^a - g f^{abc} A^b_\mu \lambda^c$$

The auxiliary field D^a is again needed so that the SUSY algebra closes on-shell. Counting fermion and boson degrees of freedom on-shell and off-shell:

	A_μ	λ	D
on-shell $(n_B = n_F = 2)$	2	2	0
off-shell $(n_B = n_F = 4)$	3	4	1

To make a gauge-invariant supersymmetric Lagrangian involving both gauge and chiral supermultiplets, one must turn the ordinary derivatives into covariant ones:

$$\partial_\mu \phi_i \quad \rightarrow \quad \nabla_\mu \phi_i = \partial_\mu \phi_i + igA_\mu^a (T^a \phi)_i$$
$$\partial_\mu \psi_i \quad \rightarrow \quad \nabla_\mu \psi_i = \partial_\mu \psi_i + igA_\mu^a (T^a \psi)_i$$

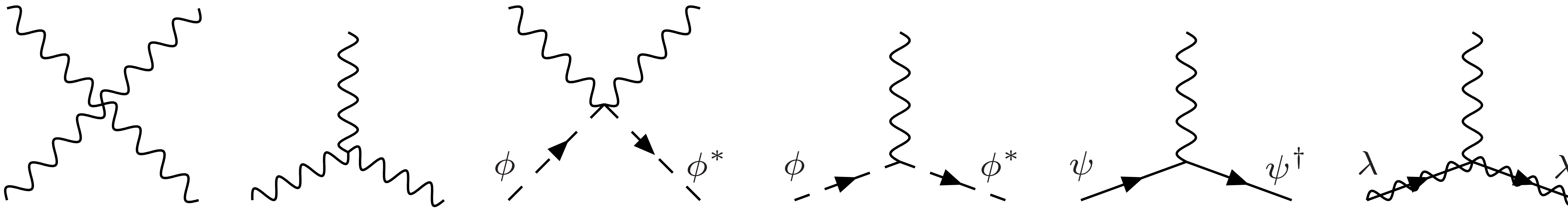
One must also add three new terms to the Lagrangian:

$$\mathcal{L} \quad = \quad \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{chiral}} - \sqrt{2} g (\phi^* T^a \psi) \lambda^a - \sqrt{2} g \lambda^{\dagger a} (\psi^\dagger T^a \phi)$$
$$+ g(\phi^* T^a \phi) D^a.$$

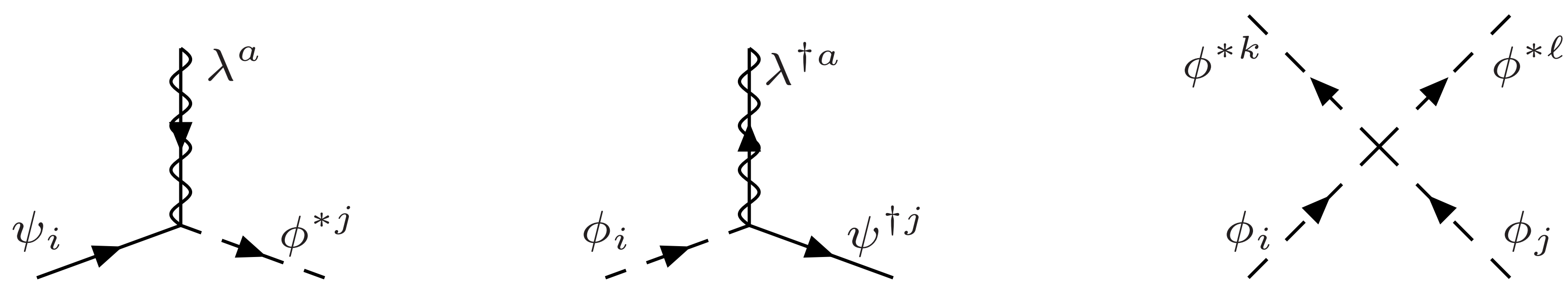
You can check (after some algebra) that this full Lagrangian is now invariant under both SUSY transformations and gauge transformations.

Supersymmetric gauge interactions

The following interactions are dictated by ordinary gauge invariance alone:



SUSY also predicts interactions that have gauge coupling strength, but are not gauge interactions in the usual sense:



$-i\sqrt{2}g_a(T^a)_i{}^j$ $\quad$ $-i\sqrt{2}g_a(T^a)_i{}^j$ $\quad$ $-ig_a^2(T_i^{ak}T_j^{a\ell}+T_i^{a\ell}T_j^{ak})$

These interactions are entirely determined by supersymmetry and the gauge group. Experimental measurements of the magnitudes of these couplings will provide an important test that we really have SUSY.

Soft SUSY-breaking Lagrangians

It has been shown that the quadratic sensitivity to $M_{\rm UV}$ is still absent in SUSY theories with these SUSY-breaking terms added in:

$$\begin{aligned}\mathcal{L}_{\rm soft} &= -\tfrac{1}{2}\left(M_a\,\lambda^a\lambda^a + \text{c.c.}\right) - (m^2)^i_j\phi^{*j}\phi_i \\ &\quad -\left(\tfrac{1}{2}b^{ij}\phi_i\phi_j + \tfrac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \text{c.c.}\right),\end{aligned}$$

They consist of:

- gaugino masses M_a,
- scalar (mass)^2 terms $(m^2)^j_i$ and b^{ij},
- (scalar)^3 couplings a^{ijk}

Building Supersymmetric Models

How to build a SUSY Model:

- Choose a gauge symmetry group.
  (In the MSSM, this is already done: $SU(3)_C \times SU(2)_L \times U(1)_Y$.)

- Choose a superpotential W; must be invariant under the gauge symmetry.
  (In the MSSM, this is almost already done: Yukawa couplings are dictated by the observed fermion masses.)

- Choose a soft SUSY-breaking Lagrangian, or else choose a method for spontaneous SUSY breakdown.
  (This is where almost all of the unknowns and arbitrariness in the MSSM are.)

Let us do this for the MSSM now, and then explore the consequences.

The Superpotential for the Minimal SUSY Standard Model:

$$W_{\rm MSSM} = \overline{u}\mathbf{y_u} Q H_u - \overline{d}\mathbf{y_d} Q H_d - \overline{e}\mathbf{y_e} L H_d + \mu H_u H_d \, .$$

The objects H_u, H_d, Q, L, $\bar{u}$, $\bar{d}$, $\bar{e}$ appearing here are the **superfields.**

Recall that $\bar{u}$, $\bar{d}$, $\bar{e}$ are the conjugates of the right-handed parts of the quark and lepton fields.

The dimensionless Yukawa couplings $\mathbf{y_u}$, $\mathbf{y_d}$ and $\mathbf{y_e}$ are 3×3 matrices in family space. Up to a normalization, and higher-order quantum corrections, they are the same as in the Standard Model.

We need both H_u and H_d, because terms like $u\mathbf{y_u}QH_d^*$ and $\bar{d}\mathbf{y_d}QH_u^*$ are not analytic, and so not allowed in the superpotential.

Yukawa Couplings

In the approximation that only the t, b, τ Yukawa couplings are included:

$$\mathbf{y_u} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}; \qquad \mathbf{y_d} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}; \qquad \mathbf{y_e} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

the superpotential becomes

$$\begin{aligned} W_{\rm MSSM} \;\approx\; & y_t(\bar{t}tH_u^0 - \bar{t}bH_u^+) - y_b(\bar{b}tH_d^- - \bar{b}bH_d^0) \\ & -y_\tau(\bar{\tau}\nu_\tau H_d^- - \bar{\tau}\tau H_d^0) + \mu(H_u^+ H_d^- - H_u^0 H_d^0) \end{aligned}$$

Here $\quad Q_3 = (t\, b);\;\; L_3 = (\nu_\tau\, \tau);$

$H_u = (H_u^+\, H_u^0);\;\; H_d = (H_d^0\, H_d^-)\quad \bar{u}_3 = \bar{t};\;\; \bar{d}_3 = \bar{b};\;\; \bar{e}_3 = \bar{\tau}.$

The minus signs are arranged so that if the neutral Higgs scalars get positive VEVs $\langle H_u^0 \rangle = v_u$ and $\langle H_d^0 \rangle = v_d$, and the Yukawa couplings are defined positive, then the fermion masses are also positive:

$$m_t = y_t v_u; \qquad m_b = y_b v_d; \qquad m_\tau = y_\tau v_d.$$

Baryon and Lepton Number Violating Terms

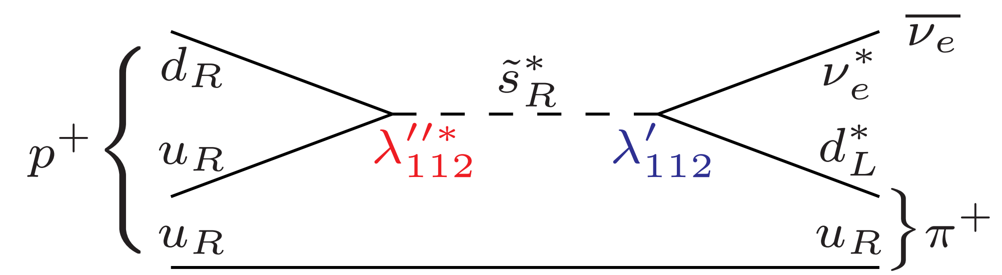
Actually, the most general possible superpotential would also include:

$$W_{\Delta L=1} = \frac{1}{2}\lambda_{ijk} L_i L_j \bar{e}_k + \lambda'_{ijk} L_i Q_j \bar{d}_k + \mu'_i L_i H_u$$

$$W_{\Delta B=1} = \frac{1}{2}\lambda''_{ijk} \bar{u}_i \bar{d}_j \bar{d}_k$$

These violate lepton number ($\Delta L = 1$) or baryon number ($\Delta B = 1$).

If both types of couplings were present, and of order 1, then the proton would decay in a tiny fraction of a second through diagrams like this:



Many other proton decay modes, and other experimental limits on B and L violation, give strong constraints on these terms in the superpotential.

One cannot simply require B and L conservation, since they are already known to be violated by non-perturbative electroweak effects. Instead, in the MSSM, one postulates a new discrete symmetry called **Matter Parity**, also known as **R-parity**.

(on superfields) (on fields)

R-parity

Matter parity is a multiplicatively conserved quantum number defined as:

$$P_M = (-1)^{3(B-L)}$$

for each particle in the theory. All quark and lepton supermultiplets carry $P_M = -1$, and the Higgs and gauge supermultiplets carry $P_M = +1$. This eliminates all of the dangerous $\Delta L = 1$ and $\Delta B = 1$ terms from the superpotential, saving the proton.

R-parity is defined for each particle with spin S by:

$$P_R = (-1)^{3(B-L)+2S}$$

This is **exactly equivalent** to matter parity, because the product of $(-1)^{2S}$ is always $+1$ for any interaction vertex that conserves angular momentum.

R-parity

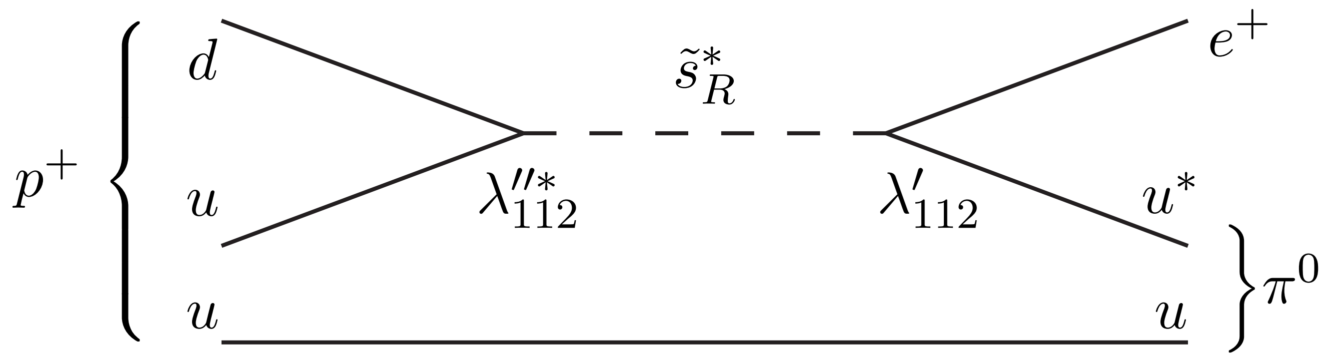
$$P_R = (-1)^{3(\mathrm{B}-\mathrm{L})+2s}$$

SM	spin	(super)partner	spin
gluon, g	1	gluino: $\tilde{g}$	1/2
$W^\pm, Z$	1	gaugino: $\tilde{W}^\pm, \tilde{Z}$	1/2
quark: q	1/2	squark: $\tilde{q}$	0
...		...	

All superpartners are odd under R-parity.

R-parity

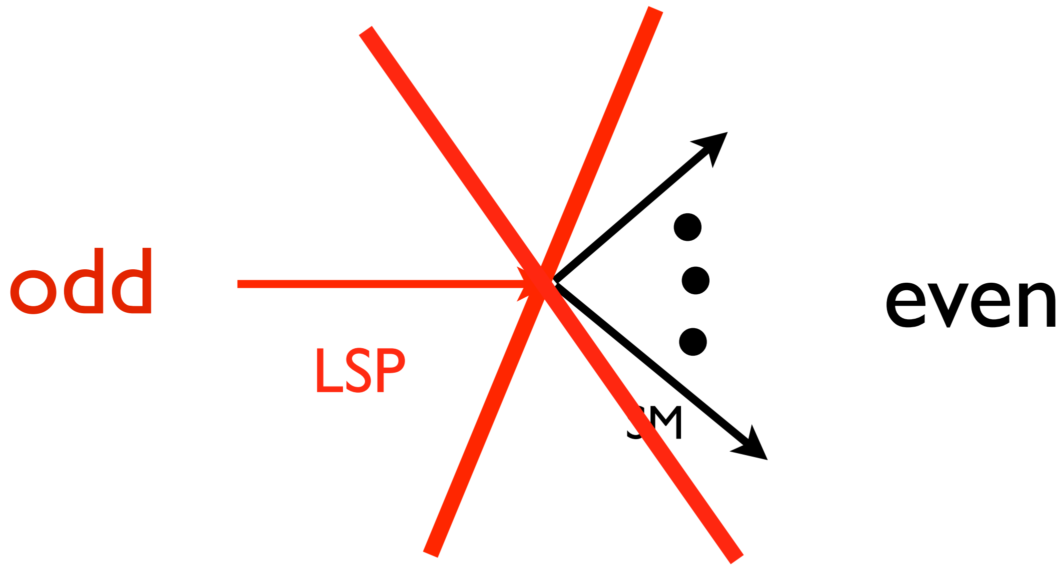
Proton decay through R-violating squark exchange:



forbidden!

R-parity

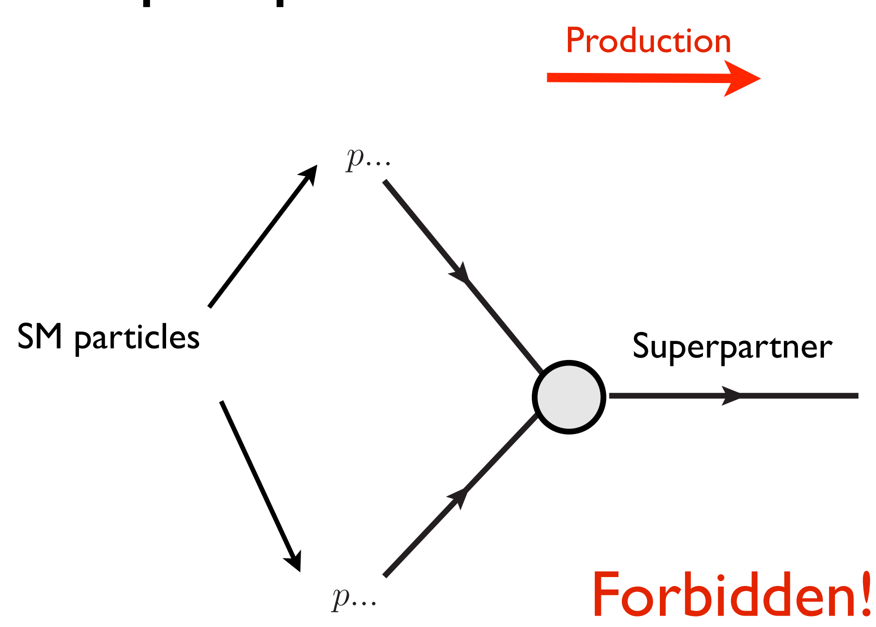
Lightest R-odd supersymmetric particle "LSP" is stable:



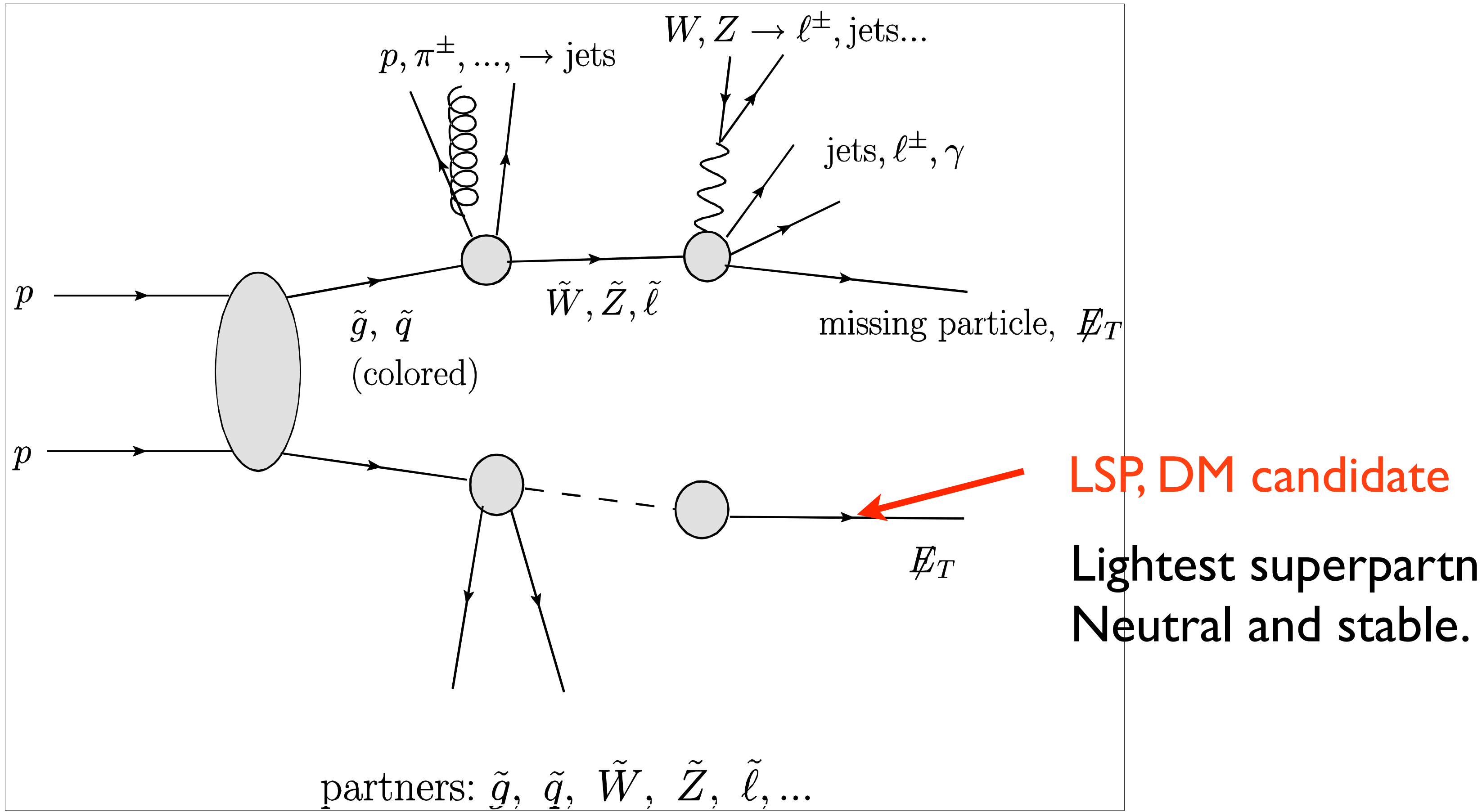
If neutral, the LSP is a natural candidate for WIMP dark matter.

R-parity

Superpartners must be pair-produced at colliders!



leaving two LSPs at the end of superpartner decay chains:



Lightest superpartner (LSP)
Neutral and stable.

The LSP could be Dark Matter

Recent results in experimental cosmology suggest the existence of cold dark matter with a density:

$$\Omega_{\text{CDM}} h^2 = 0.11 \pm 0.02 \qquad \text{(WMAP)}$$

where h = Hubble constant in units of 100 km/(sec Mpc).

A stable particle which freezes out of thermal equilibrium will have $\Omega h^2 = 0.11$ today if its thermal-averaged annihilation cross-section is, roughly:

$$\langle \sigma v \rangle = 1 \text{ pb}$$

One example of a "true" WIMP candidate, i.e., has full-strength SU(2) weak interactions, is the neutral "Wino", with a thermally-averaged annihilation rate

$$\langle \sigma_{eff} v \rangle = \frac{3g^4}{16\pi M_2^2}$$

This annihilation cross section gives a thermal abundance of

$$\Omega_{\tilde{W}} h^2 = 0.13 \left(\frac{M_2}{2.5\,\text{TeV}} \right)^2$$

Generically, the "WIMP miracle" yields a ≈ TeV-mass WIMP. (more on this later)

The Soft SUSY-breaking Lagrangian for the MSSM

$$\begin{aligned}\mathcal{L}_{\rm soft}^{\rm MSSM} \;=\;& -\tfrac{1}{2}\Big(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B}\Big) + \text{c.c.}\\ & -\Big(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d\Big) + \text{c.c.}\\ & -\widetilde{Q}^\dagger\,\mathbf{m^2_{\tilde{Q}}}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m^2_{\tilde{L}}}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m^2_{\tilde{\bar{u}}}}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m^2_{\tilde{\bar{d}}}}\,\widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\,\mathbf{m^2_{\tilde{\bar{e}}}}\,\widetilde{\overline{e}}^\dagger\\ & - m^2_{H_u}H_u^*H_u - m^2_{H_d}H_d^*H_d - (bH_uH_d + \text{c.c.})\,.\end{aligned}$$

The first line gives masses to the MSSM gauginos (gluino $\widetilde{g}$, winos $\widetilde{W}$, bino $\widetilde{B}$).

The second line consists of (scalar)^3 interactions.

The third line is (mass)^2 terms for the squarks and sleptons.

The last line is Higgs (mass)^2 terms.

If SUSY is to solve the Hierarchy Problem, we expect:

$$M_1,\ M_2,\ M_3,\ \mathbf{a_u},\ \mathbf{a_d},\ \mathbf{a_e}\ \sim\ m_{\rm soft};$$

$$\mathbf{m^2_{\tilde{Q}}},\ \mathbf{m^2_{\tilde{L}}},\ \mathbf{m^2_{\tilde{\bar{u}}}},\ \mathbf{m^2_{\tilde{\bar{d}}}},\ \mathbf{m^2_{\tilde{\bar{e}}}},\ m^2_{H_u},\ m^2_{H_d},\ b\ \sim\ m^2_{\rm soft}$$

where $m_{\rm soft}\ \lesssim\ 1$ TeV.

↑ The exact number is sparticle-dependent

(and been, perhaps not surprisingly, an upwardly moving target)

The soft SUSY-breaking Lagrangian of the MSSM contains 105 new parameters not found in the Standard Model.

Most of what we do not already know about SUSY is expressed by the question: "How is supersymmetry broken?"

Many proposals have been made.