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*DPG Physics School on Heavy Particles at the LHC*

*Theory of*

*Top Quark Physics*

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## overview

- why top physics
- tops @ Tevatron, LHC and ILC
- what do we want to know

## $t\bar{t}$

- top production at (N)NLO
- resummation
- including the decay of top
- off-shell effects

## top mass

- renormalon issue with pole mass
- issue with  $m_t$  from invariant mass
- 'alternative'  $m_t$  determinations
- $m_t$  @ ILC

## single top

- recap (resummation, decay, off-shell effects)
- definition of process
- 4-flavour scheme vs 5-flavour scheme

forward-backward asymmetry  $A_{FB}$

- theory vs. experiment
- Tevatron vs. LHC
- BSM effects

testing the SM

- spin correlations
- anomalous couplings vs. effective theory
- Higgs and top

conclusions

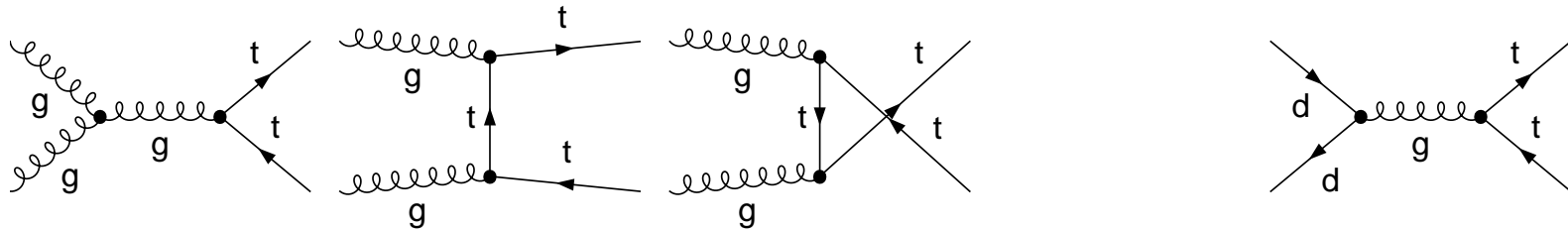
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Part I

Overview

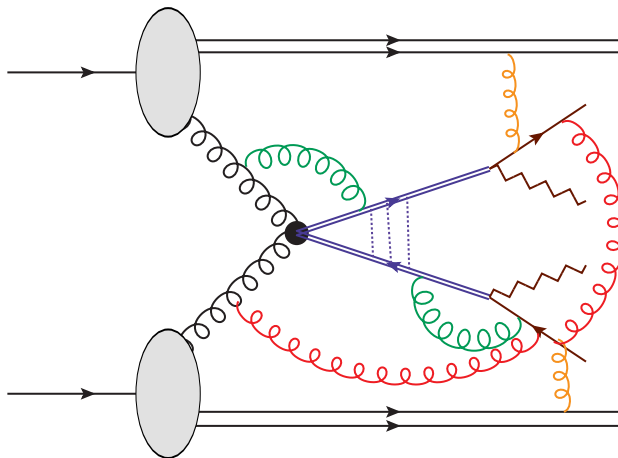
## why top physics?

- top is a “free” quark
  - typical hadronization time governed by  $\Lambda_{\text{QCD}}^{-1} \sim (250 \text{ MeV})^{-1}$
  - top lifetime  $(\Gamma_t)^{-1} \sim (1.4 \text{ GeV})^{-1}$
  - top quark does not (quite) form bound states and decays before hadronization does its dirty business
- top is relevant in many BSM scenarios
  - top has proper Yukawa coupling  $y_t = \sqrt{2}m_t/v \sim 1$
  - top plays important role in EW symmetry breaking
- **a lucky coincidence !!**
  - top observables can be computed (hadronization not a show stopper)
  - top observables can be measured (“easy” to produce)
  - top observables are relevant (window for BSM)
- the top is the only quark that behaves properly!
  - ⇒ It’s the white sheep in a herd of black sheep
- also input for other branches of particle physics



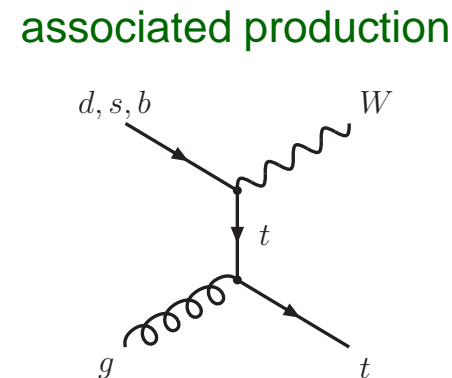
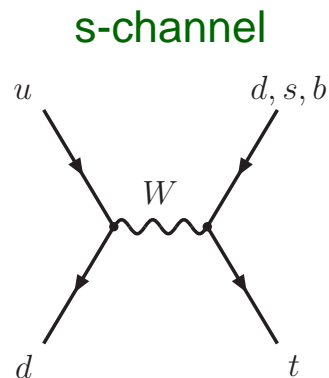
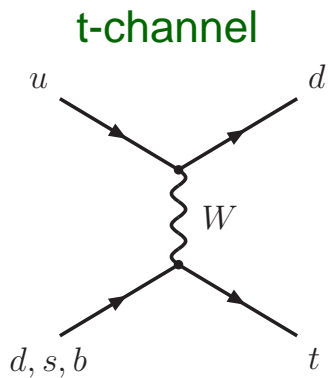
approximate (!)  
 expected / measured  
 SM cross sections in pb

	Tevatron	7 TeV LHC	14 TeV LHC
$t\bar{t}$	7	160	900
$q\bar{q}$	~ 90%	~ 20%	~ 10%
$gg$	~ 10%	~ 80%	~ 90%



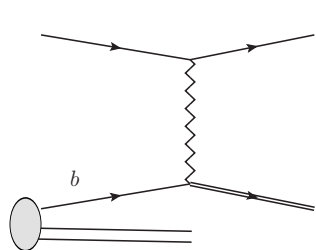
- cross sections are large
- tops are seen only through their decay products  $t \rightarrow Wb \rightarrow \{l\nu, q'\bar{q}\} b$
- information from top quark carried over to decay products
- the full process is still far from simple

- fully exclusive known at  $\sim$  one-loop
  - electroweak corrections known [Bernreuther et.al., Kuhn et.al.]
  - spin correlations included [Bernreuther et.al., Melnikov et.al.]
  - non-factorizable corrections computed [Denner et.al., Bevilacqua et.al.]
  - included in MC@NLO and POWHEG [Frixione, Nason, Webber . . . . .]
  - two-loop corrections on their way . . .
  
- inclusive cross section(s) known at  $\sim$  two-loop
  - two-loop nearly known [Czakon et.al, Moch et.al, . . .]
  - bound-state effects computed [Hagiwara et.al., Kiyo et.al.]
  - non-factorizable corrections computed [Beenakker et.al.]
  - resummation of logs under control [Ahrens et.al, Beneke et.al . . .]
  
- further processes known at one-loop:
  - $t\bar{t}H$  [Beenakker et.al] and  $t\bar{t}j$  [Dittmaier et.al.] ;  $\Rightarrow$  MC@NLO and POWHEG
  - $t\bar{t}bb$  [Bredenstein et.al; Bevilacqua et.al.] and  $t\bar{t}jj$  [Bevilacqua et.al.]
  - “background” processes  $V + jets$

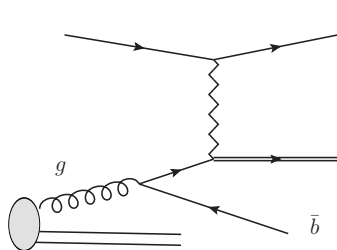


approximate (!)  
expected / measured  
SM cross sections in pb

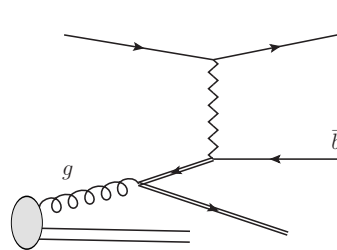
	Tevatron	7 TeV LHC	14 TeV LHC
$t (\bar{t})$ "t"-ch	1.2	40 (20)	150 (100)
$t (\bar{t})$ "s"-ch	0.55	2.5 (1.4)	7 (4)
$t W^-$	0.15	8	45



LO 5 Flavour



LO 4 Flavour



cross sections not much smaller than for  $t\bar{t}$

where does  $b$  come from?

precise definition of process not obvious beyond LO

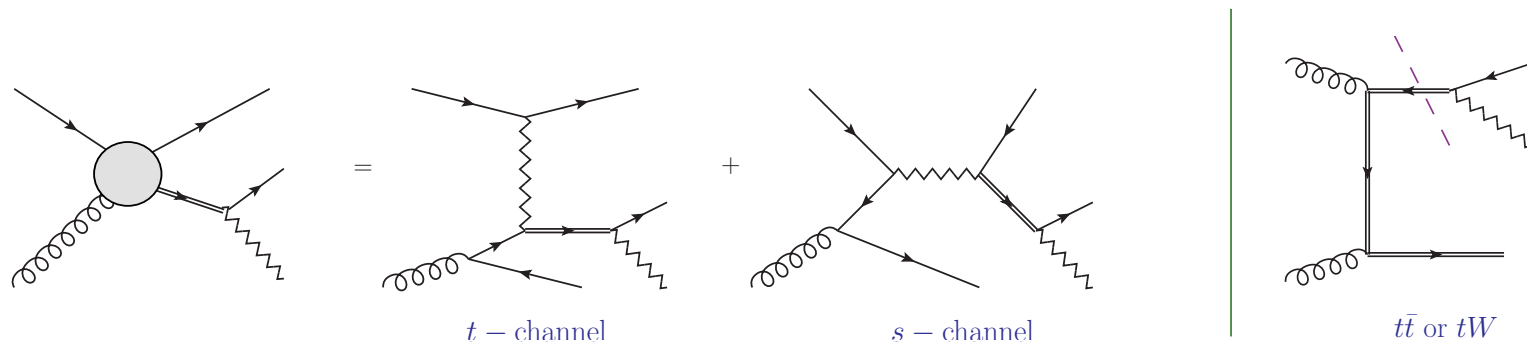


- NLO QCD corrections, production and hadronic decay for  $t$ -,  $s$ -channel and  $Wt$  known [Harris et.al; Campbell et.al; Cao et.al . . .]
- all channels included in MC@NLO and POWHEG [Frixione, Laenen, Motylinski, Alioli, Nason, Re, Webber, White . . . . .]
- EW corrections known [Beccaria et.al; Macorini et.al]
- non-factorizable corrections known [Falgari et.al.]
- resummation of inclusive cross section [Kidonakis, Wang et.al.]
- **Note:** issues with definition of cross section:

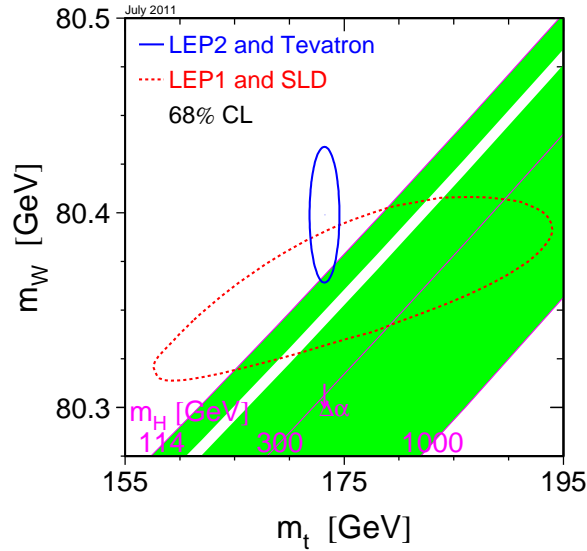
$s$  and  $t$  channel mix (beyond LO)

→ more appropriate to talk about  $(tJ)$ ,  $(tb)$  and  $(tW)$  cross sections

disentangling  $Wt$  vs  $t\bar{t}$  non-trivial [Frixione et.al.]



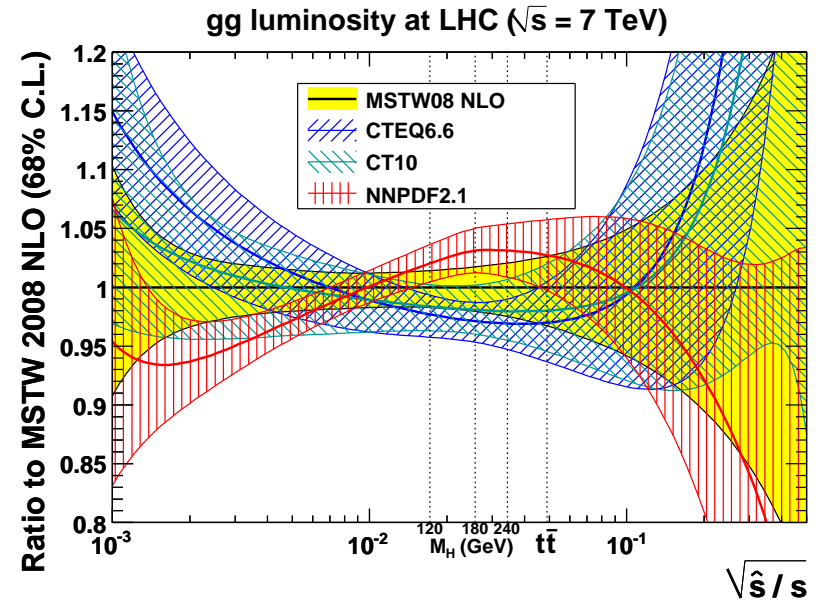
ew precision



LEP EWWG

$m_t$ , but also  $V_{tb}$

pdf



plot from G.Watt (HepForge)

$\sigma_{tt}$ , but also single top  $\sigma_t/\sigma_{\bar{t}}$

G. Watt (March 2011)

other measurements:  $y_t$ ,  $\Gamma_t$ ,  $A_{FB}$  ... mainly as test of SM (or establishing BSM)

$e_Q; T_3; \text{spin}; SU(N_c)$

test indirect constraints  
not main motivation

$t \rightarrow Wb; \quad pp \rightarrow t\bar{t}\gamma$

$m_t$  (what mass?)

input for (EW) precision  
THE measurement

$t\bar{t}$  production  
other possibilities?

Yukawa coupling  $y_t$

direct test of Higgs mech.  
important

$pp \rightarrow t\bar{t}H, \text{ ILC} ??$

CKM element  $V_{tb}$

(only) direct measurement  
nice

single top production

width  $\Gamma_t$

SM theory accurate at 1%  
(would be) really nice

only at ILC ??

anom. coupl; BSM

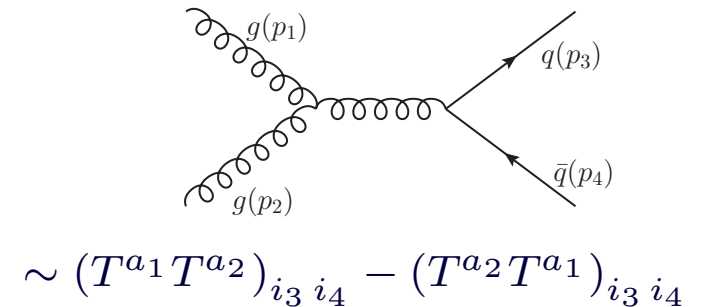
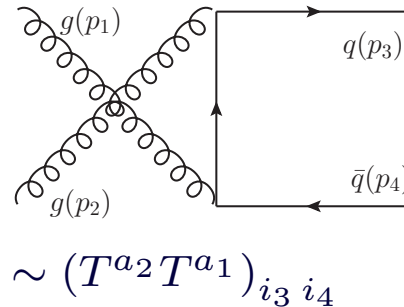
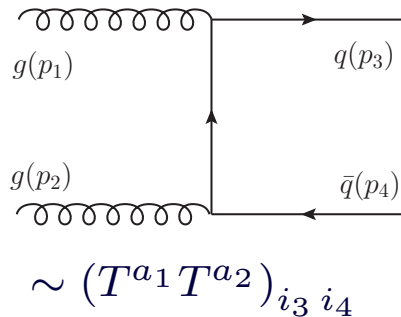
we are desperate for it  
no comment

spin correlations,  $A_{FB}$ ,  
rare decays, single top

## Part II

# Top Pair Production

Compute matrix element squared  $\mathcal{M}^{(0)} \equiv \mathcal{A}^{(0)} \mathcal{A}^{(0)*}$



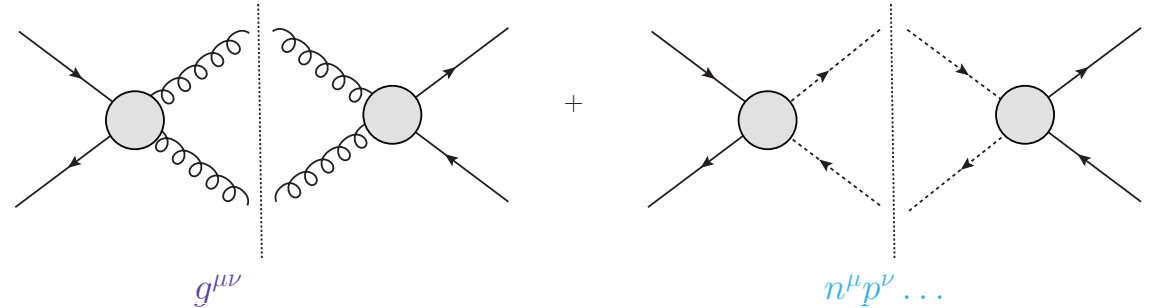
colour:

$$\mathcal{A}^{(0)} = (T^{a_1} T^{a_2})_{i_3 i_4} A_{12}(s, t, u) + (T^{a_2} T^{a_1})_{i_3 i_4} A_{21}(s, t, u)$$

$$\mathcal{M}^{(0)} = \underbrace{\frac{(N_c^2 - 1)^2}{4 N_c}}_{\text{leading colour}} \left( |A_{12}|^2 + |A_{21}|^2 \right) - \underbrace{\frac{(N_c^2 - 1)}{4 N_c}}_{\text{subleading colour}} \left( A_{12} A_{21}^* + A_{12}^* A_{21} \right)$$

Structure of (sub)amplitude:  $A_{\#\#} = \bar{u}_\alpha(p_3) v_\beta(p_4) \varepsilon^\mu(p_1) \varepsilon^\nu(p_2) (a_{\mu\nu})_{\alpha\beta}$

squaring the amplitude



conventional:

$$\sum_{\text{pols}} \varepsilon^\mu(p_i) \varepsilon^{\nu*}(p_i) \rightarrow -g^{\mu\nu} + \underbrace{\frac{n_i^\mu p_i^\nu + p_i^\mu n_i^\nu}{(n_i p_i)} - \frac{n_i^2 p_i^\mu p_i^\nu}{(n_i p_i)^2}}_{n_i^\mu \text{ arbitrary}}; \quad \sum_{\text{pols}} u_\alpha(p) \bar{u}_\beta(p) = (\not{p} + m)_{\alpha\beta};$$

QED: can drop  $n^\mu$  parts, since  $p_{3/4}^\mu a_{\mu\nu} = 0$

QCD:  $p_{3/4}^\mu a_{\mu\nu} \neq 0$ , but result independent of  $n_{3/4}^\mu$ .

alternatively, drop  $n^\mu$  parts but include ghost diagrams in squaring the amplitude.

In  $D$  dimensions we get (including mass terms) e.g.

$$|a_{12}|^2 = -\frac{2\alpha_s^2}{s^2 t^2} \left( (D-2)t(s+t) \left( (D-2)s^2 + 4st + 4t^2 \right) + 16m^4 s^2 + 16m^2 st(s+t) \right)$$

### helicity method:

fix helicities of external particles and express amplitude in terms of spinor inner products:

$$\langle ij \rangle = \langle p_i - | p_j + \rangle \equiv \bar{u}(p_i, -) u(p_j, +); \quad [ij] = \langle p_i + | p_j - \rangle \equiv \bar{u}(p_i, +) u(p_j, -) ;$$

for massive quarks:  $p = p^b + \frac{m_t^2}{2p^b \cdot \eta} \eta_p$  then  $u_{\pm}(p, m) = \frac{\not{p} + m}{\langle p^b \mp | \eta_{p\pm} \rangle} | \eta_{p\pm} \rangle$

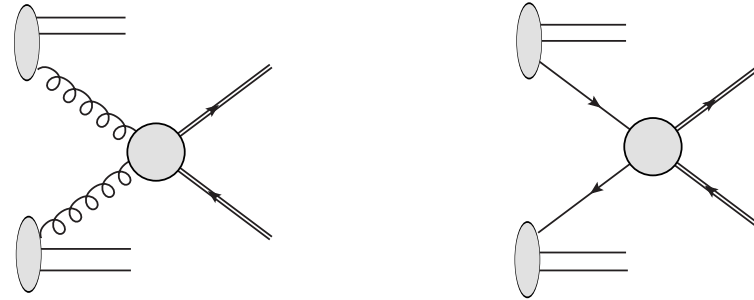
for gauge bosons use  $\varepsilon^{\mu}(p, \pm) = \pm \frac{\langle p \pm | \gamma^{\mu} | n \pm \rangle}{\sqrt{2} \langle n \mp | p \pm \rangle}$

- lightlike reference momentum  $n^{\mu}$  drops out for gauge invariant quantities
- very compact results, e.g:  $a_{12}(g_1^-, g_2^-, t_3^+, \bar{t}_4^+) = ig^2 \frac{m_t^3 \langle \eta_3 \eta_4 \rangle [12]}{\langle 12 \rangle \langle 1 | 3 | 1 \rangle \langle 3^b \eta_3 \rangle [4^b \eta_4]}$
- simplifications (due to gauge cancellations) at amplitude level
- sum over all (non-vanishing) helicity configurations

$$|a_{12}|^2 = \sum_{h_i} |a_{12}(g_1^{h_1}, g_2^{h_2}, q_3^{h_3}, \bar{q}_4^{h_4})|^2$$

- have to treat external particles in 4 dimensions

## hadronic cross section



$$d\sigma_{H_1(P_1)H_2(P_2) \rightarrow t\bar{t}} = \int_0^1 dx_1 f_{g/H_1}(x_1, \mu_F) \int_0^1 dx_2 f_{g/H_2}(x_2, \mu_F) d\hat{\sigma}_{g(x_1 P_1)g(x_2 P_2) \rightarrow t\bar{t}}(\alpha_s(\mu_R) \dots) + \dots$$

$\mu_F$ : factorization scale;       $\mu_R$ : renormalization scale

$f_{g/H_1}(x_1, \mu_F)$ : parton distribution functions

$d\hat{\sigma}$ : hard partonic cross section, at tree level  $d\hat{\sigma}^{(0)} = d\sigma^{(0)}$

there are additional partonic processes for  $H_1 H_2 \rightarrow t\bar{t}$  beyond LO ( $qg \rightarrow t\bar{t}q$ )

$$d\sigma_{H_1 H_2 \rightarrow t\bar{t}} = \int_0^1 dx_1 f_{g/H_1}(x_1) \int_0^1 dx_2 f_{g/H_2}(x_2) d\hat{\sigma}_{gg \rightarrow t\bar{t}} + \sum_{q \in \{u, d, c, s, (b)\}} \int_0^1 dx_1 f_{q/H_1}(x_1) \int_0^1 dx_2 f_{\bar{q}/H_2}(x_2) d\hat{\sigma}_{q\bar{q} \rightarrow t\bar{t}} + \{q \leftrightarrow \bar{q}\}$$

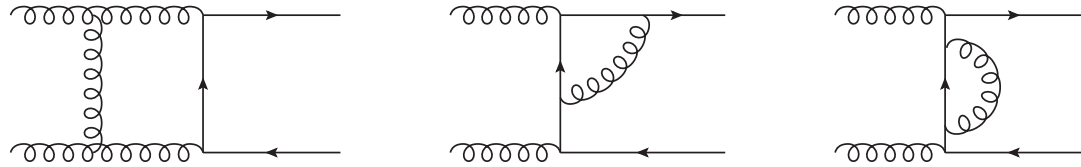


Tree-level:  $d\hat{\sigma}^{(0)} = d\sigma^{(0)}$

$$1\text{-loop: } d\hat{\sigma}^{(1)} = \underbrace{d\sigma^{(0)}}_{\mathcal{O}(\alpha_s^2)} + \underbrace{d\sigma^{\text{virt}} + d\sigma^{\text{real}} + d\sigma^{\text{coll}}}_{\mathcal{O}(\alpha_s^3)}$$

- All  $\mathcal{O}(\alpha_s^3)$  are (in general) divergent and only the sum is finite (for properly defined, i.e. infrared-safe observables).
- Regularize divergences by working in  $D = 4 - 2\epsilon$  dimensions:  $\int d^4 k \rightarrow \mu_R^{2\epsilon} \int d^D k$ ; singularities  $\rightarrow$  poles  $1/\epsilon$  (dimensional regularization).
- Other possibilities in principle, but not in practice.
- Strictly speaking, only internal momenta have to be  $D$  dimensional. There is some freedom how to treat external particles (recall helicity method needs these to be 4 dimensional)
- different schemes (variant of dimensional regularization) possible but observable is independent of this choice

virtual corrections



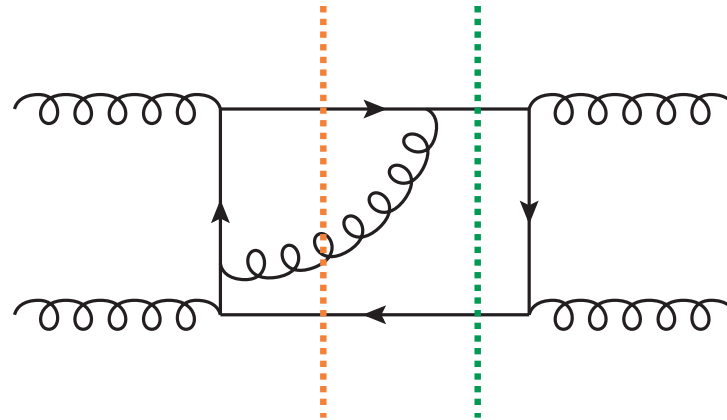
amplitude:

$$\begin{aligned} \mathcal{A}^{(1)} &= (T^{a_1} T^{a_2})_{i_3 i_4} \left( \frac{N_c}{2} A_{12}^L(s, t, u) + \frac{1}{2N_c} A_{12}^S(s, t, u) + \frac{N_F}{2} A_{12}^F(s, t, u) \right) \\ &+ \{12 \leftrightarrow 21\} \\ &+ \delta_{i_3 i_4} \frac{1}{2} \text{Tr}(T^{a_1} T^{a_2}) \left( A_{\text{tr}}(s, t, u) + \frac{N_F}{N_c} A_{\text{tr}}^F(s, t, u) \right) \end{aligned}$$

$$A_{12}^L = \frac{1}{\epsilon^2} \left[ c_s \left( \frac{-s}{\mu^2} \right)^{-\epsilon} + c_t \left( \frac{-t}{\mu^2} \right)^{-\epsilon} + \dots \right] + \frac{1}{\epsilon} \text{mess}(\log) + \text{finite mess}(\log^2, \text{Li}_2)$$

- UV singularities ( $1/\epsilon$  per loop)  $\implies$  renormalization
- soft and final-state collinear sing. ( $1/\epsilon$  per loop)  $\implies$  combine with real corrections
- soft-collinear singularities ( $1/\epsilon^2$  per loop)  $\implies$  combine with real corrections
- initial-state collinear sing. ( $1/\epsilon$  per loop)  $\implies$  combine with collinear counterterm  $d\sigma^{\text{coll}}$

virtual corrections



“squaring” the amplitude:

$$\mathcal{A}_{t\bar{t}} = \underbrace{\mathcal{A}_{t\bar{t}}^{(0)}}_{\sim \alpha_s} + \underbrace{\mathcal{A}_{t\bar{t}}^{(1)}}_{\sim \alpha_s^2} + \dots \implies \mathcal{M}^{(0)} = |\mathcal{A}_{t\bar{t}}^{(0)}|^2 \sim \alpha_s^2 \quad \text{and} \quad \mathcal{M}^{(1)} = 2 \operatorname{Re} \left( \mathcal{A}_{t\bar{t}}^{(1)} \mathcal{A}_{t\bar{t}}^{(0)*} \right) \sim \alpha_s^3$$

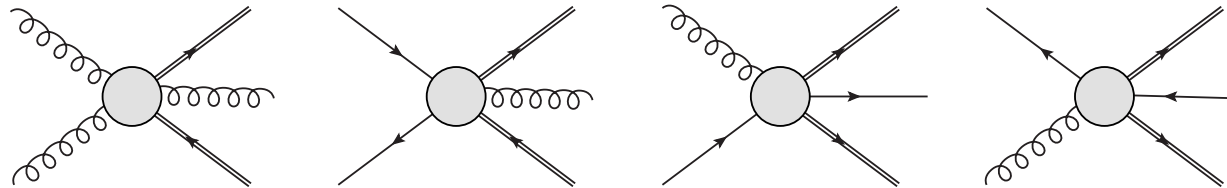
the “same” diagram with a **different cut** is part of the real corrections

$$\mathcal{M}^{(0)}(gg \rightarrow t\bar{t}g) = |\mathcal{A}_{t\bar{t}g}^{(0)}|^2 \sim \alpha_s^3$$

## Real corrections

$$d\sigma^{\text{real}} = \sum_{\bar{a}_i} \int d\Phi_3(p_1, p_2; p_3, p_4, p_5) \langle \mathcal{M}^{(0)}(a_1, a_2; \bar{a}_3, \bar{a}_4, \bar{a}_5) \rangle$$

processes:  $\mathcal{M}^{(0)}(g, g; t, \bar{t}, g)$ , but also new partonic channels  $\mathcal{M}^{(0)}(q, g; t, \bar{t}, q)$  etc.  
 calculation of  $\mathcal{M}^{(0)}$  as for tree-level.



$\mathcal{M}^{(0)}$  has no  $1/\epsilon$  poles, but has (non-integrable) singularities in some regions of phase space.

$$\underbrace{\int d\Phi_{n-1} \left( \mathcal{M}^{(0)} - \sum_{\text{sing}} \mathcal{M}^{\text{appr}} \right)}_{\text{finite}} + \underbrace{\int d\Phi_{n-1} \sum_{\text{sing}} \mathcal{M}^{\text{appr}}}_{\text{use dim reg}}$$

Real corrections naive example (e.g. gluon  $g$  soft,  $x \sim$  energy)

$$\mathcal{A}(g, g, t, \bar{t}, g) \stackrel{g \rightarrow 0}{\sim} \frac{1}{\langle pg \rangle} \mathcal{A}(g, g, t, \bar{t}) + \mathcal{A}^{\text{rem}} \sim \frac{1}{\sqrt{x}} \mathcal{A}(g, g, t, \bar{t}) + \mathcal{A}^{\text{rem}}$$

$$\mathcal{M}(g, g, t, \bar{t}, g) \sim \frac{1}{x} \mathcal{M}(g, g, t, \bar{t}) + \frac{1}{\sqrt{x}} \mathcal{M}^{\text{rem}}$$

$$\int d\Phi_3^D \mathcal{M}(g, g, t, \bar{t}, g) = \underbrace{\int d\Phi_3^4 \left( \mathcal{M}(g, g, t, \bar{t}, g) - \frac{1}{x} \mathcal{M}(g, g, t, \bar{t}) \right)}_{\text{term 1}} + \underbrace{\int d\Phi_3^D \frac{1}{x} \mathcal{M}(g, g, t, \bar{t})}_{\text{term 2}}$$

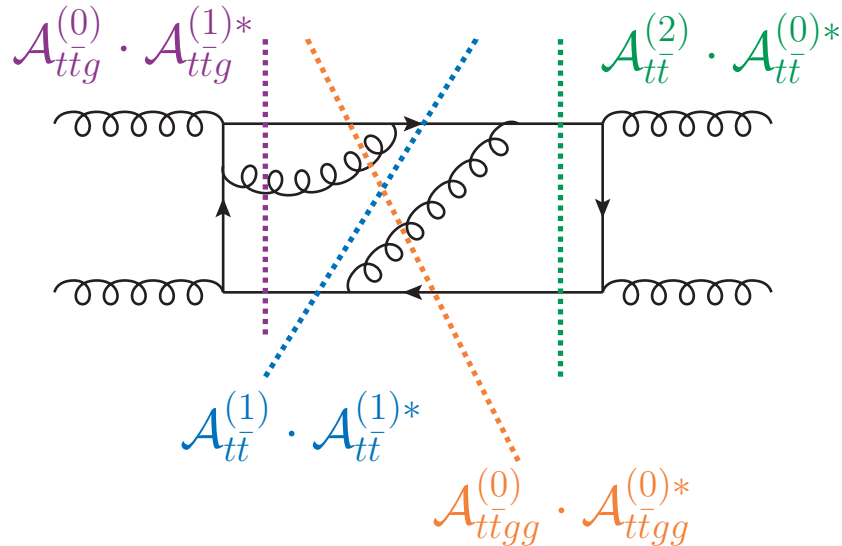
term 1: evaluate numerically in 4 dimensions, **square root singularities !**

$$\text{term 2: } \int x^{-\epsilon} \frac{1}{x} \int d\Phi_2^4 \mathcal{M}(g, g, t, \bar{t}) = -\frac{1}{\epsilon} \int d\Phi_2^4 \mathcal{M}(g, g, t, \bar{t})$$

there are several well established (and automatised) general procedures

$\implies$  FKS, Dipole subtraction . . .

## nnlo contributions



- at NNLO there are **double real**, **virtual**, **real-virtual** and **one-loop squared** contributions
- separate parts have singularities  $1/\epsilon^n$  with  $n \leq 4$
- singularities cancel in the sum of all contributions
- no general procedure yet for double-real integration, but many partial results
- $q\bar{q} \rightarrow t\bar{t}$  total cross section known (numerically) at NNLO [Czakon et al.]

- total cross section (LHC dominated by  $\hat{\sigma}_{gg}$ , beyond LO we also need  $\hat{\sigma}_{qg}$  )

$$\hat{\sigma}_{ij} = \hat{\sigma}_{ij}^{(0)} \left[ 1 + \frac{\alpha_s}{4\pi} \hat{\sigma}_{ij}^{(1)} + \frac{\alpha_s^2}{(4\pi)^2} \hat{\sigma}_{ij}^{(2)} + \dots \right]$$

- NLO QCD (and EW) corrections known [Dawson et.al.; Beenakker et.al.; Kao, Wackerroth, Bernreuther et.al; Kühn, Scharf, Uwer ...]

$$\hat{\sigma}_{ij}^{(1)} = \underbrace{\frac{a_{ij}^{(1,-1)}}{\beta}}_{\text{Coulomb}} + \underbrace{b_{ij}^{(1,2)} \log^2 \beta + b_{ij}^{(1,1)} \log \beta}_{\text{soft gluon}} + c_{ij}^{(1)}$$

- NNLO QCD corrections not (yet) fully known [Czakon et.al, Moch et.al, Beneke et.al, Ahrens et.al, Körner et.al. ... (Hathor)]

$$\hat{\sigma}_{ij}^{(2)} = \underbrace{\frac{\#}{\beta^2} + \frac{\# \log^2 \beta + \# \log \beta + \#}{\beta}}_{\text{Coulomb}} + \underbrace{\# \log^4 \beta + \# \log^3 \beta + \dots}_{\text{soft gluon}} + c_{ij}^{(2)}$$

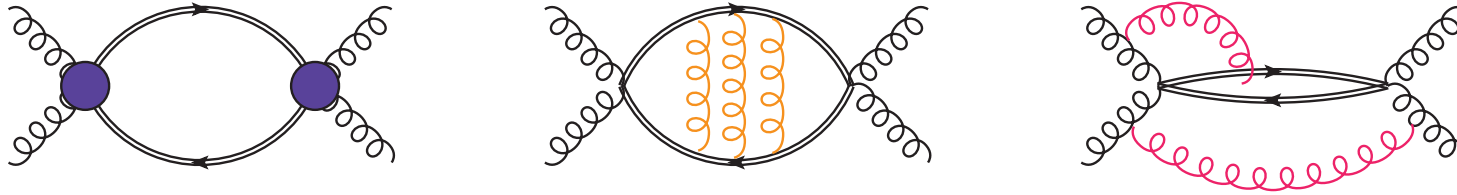
- problematic terms from threshold and soft gluon region  $\sqrt{1 - 4m_t^2/s} \equiv \beta \rightarrow 0$

enhancements from special kinematic regions  $\implies$  order by order in  $\alpha_s$  not sufficient

- in threshold region  $\sqrt{1 - 4m_t^2/s} \equiv \beta \rightarrow 0$ 
  - “bound state” effects  $(\alpha_s/\beta)^n$ , can be resummed [Fadin, Khoze; Hagiwara et.al, Kiyo et.al, Beneke et.al]
  - resummation of soft logs  $\alpha_s^n \log^{2n} \beta$ , initially to NLL now NNLL and partly NNNLL [Bonciani, Catani, Mangano, Mitov, Nason, Czakon et.al., Beneke et.al., Ahrens et.al., Kidonakis, . . . . .]
- note: cross section not necessarily dominated by small  $\beta$ , can use different resummation parameter (done at NNLL)
  - standard:  $\beta \rightarrow 0 \implies \alpha_s^n \ln^m \beta$  with  $m < 2n$
  - invariant mass:  $1 - z \equiv 1 - M^2/\hat{s} \rightarrow 0 \implies \alpha_s^n \frac{\ln^m(1-z)}{(1-z)}$  with  $m < 2n - 1$
  - SPI:  $s_4 \equiv p_X^2 - m_t^2 \rightarrow 0 \implies \alpha_s^n \frac{\ln^m(s_4/m_t)}{s_4}$  with  $m < 2n - 1$
- recover total cross section by integration
  - $\implies$  treatment of formally subleading terms are numerically relevant
- approximate “NNLO” cross section [Aliev et.al. (Hathor), Ahrens et.al, Beneke et.al, Kidonakis . . .]



structure of higher-order corrections: **hard**, **Coulomb** and **soft**



study either in **Mellin space**  $\sigma_{t\bar{t}}(N) \equiv \int_0^1 d\rho \rho^{N-1} \sigma_{t\bar{t}}(\rho)$  with  $\rho \equiv \frac{s}{4m_t^2}$

or directly in momentum space **via SCET**

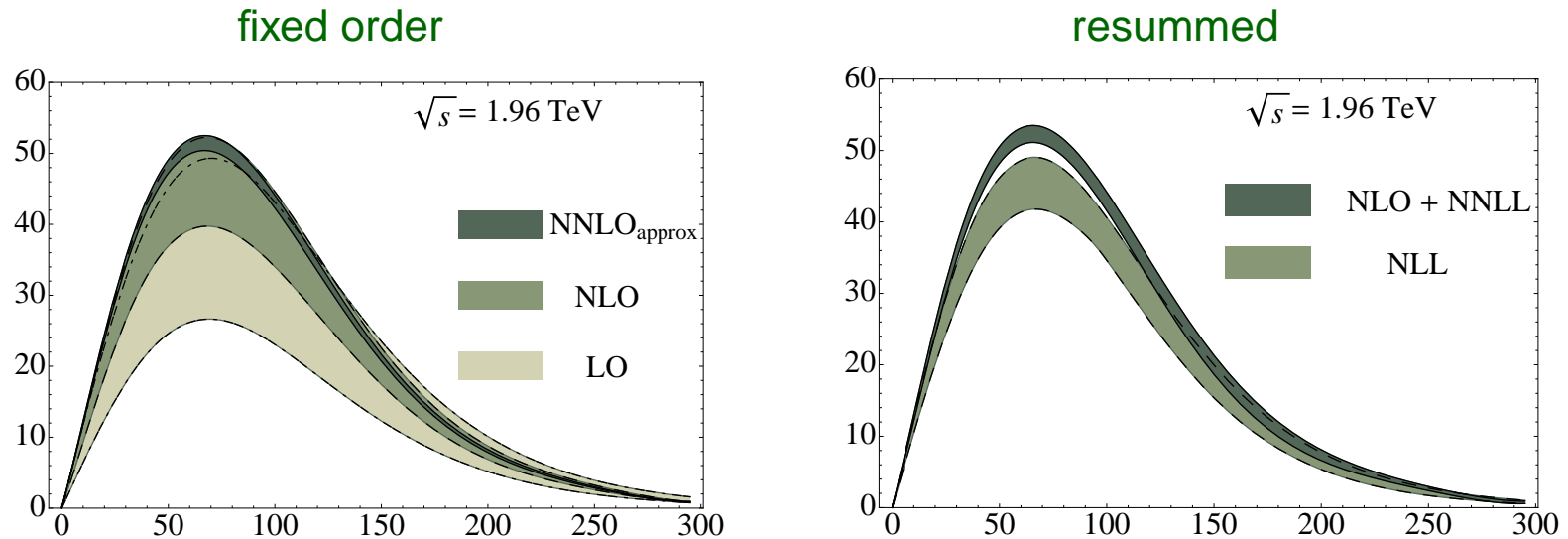
cross section factorizes (into product in Mellin space and convolution in SCET)

$$\sigma_{t\bar{t}} = \sigma_{t\bar{t}}^{(h)} \otimes \underbrace{\sigma_{t\bar{t}}^{(Coul)}}_{(\alpha_s/\beta)^n} \otimes \underbrace{\sigma_{t\bar{t}}^{(s)}}_{\log \beta}$$

$\sigma_{t\bar{t}}^{(Coul)}$  only in threshold expansion, but  $\sigma_{t\bar{t}}$  at LHC/Tev not dominated by small  $\beta$ .

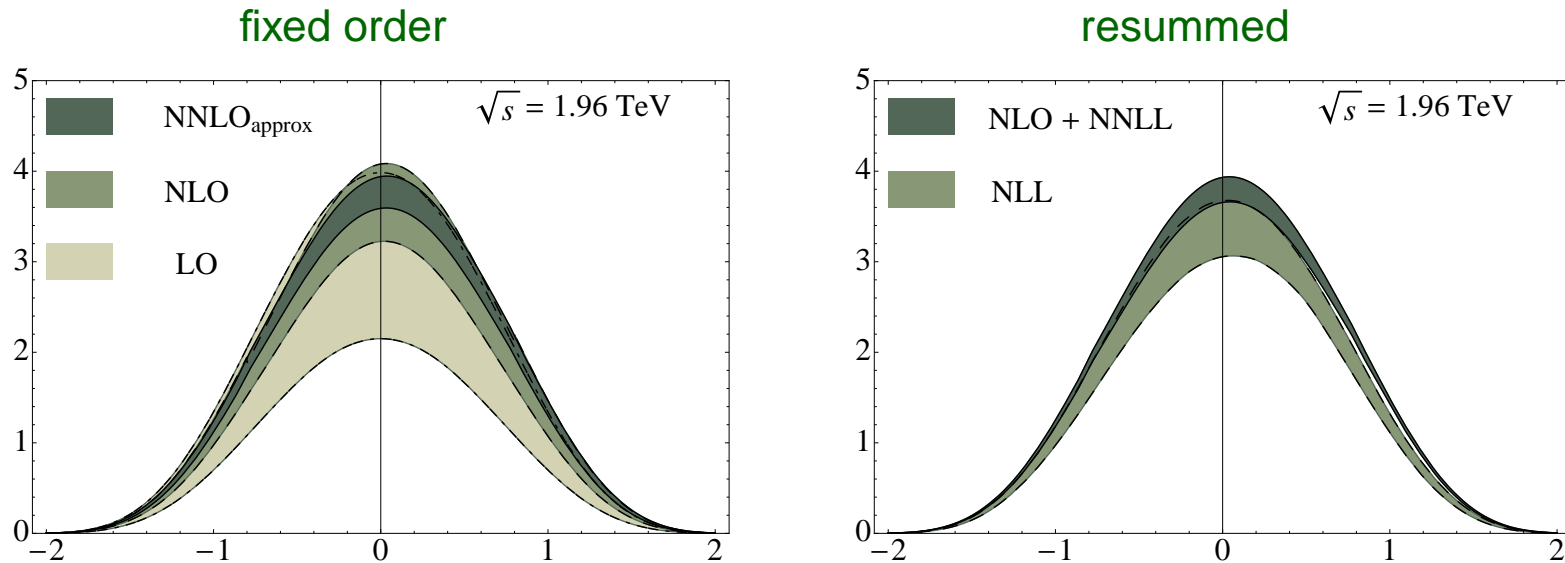
inverse Mellin transform needs prescription to avoid Landau pole, or re-expansion of resummed expression to certain order in perturbation theory

comparison fixed-order vs. resummed cross section for  $p_t$  [Ahrens et al. 1103.0550]



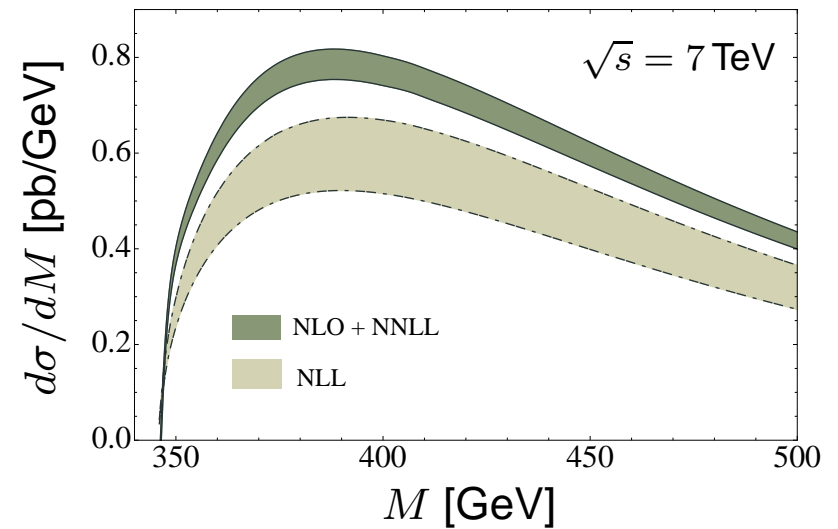
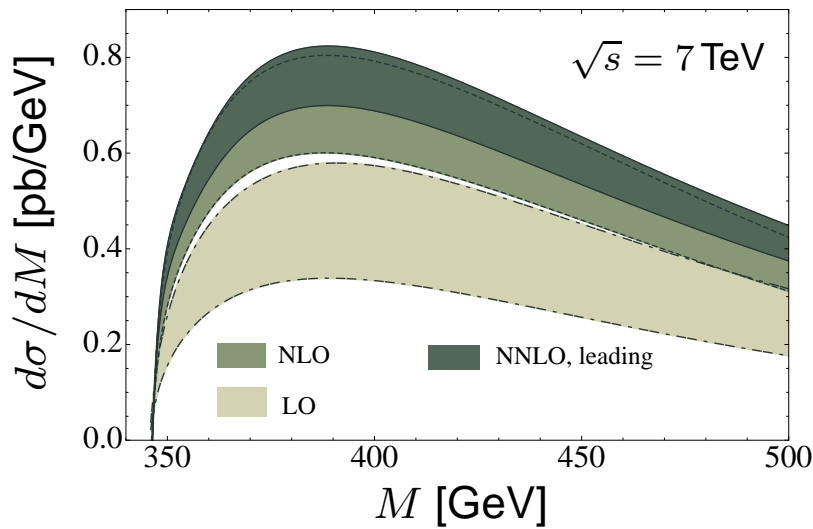
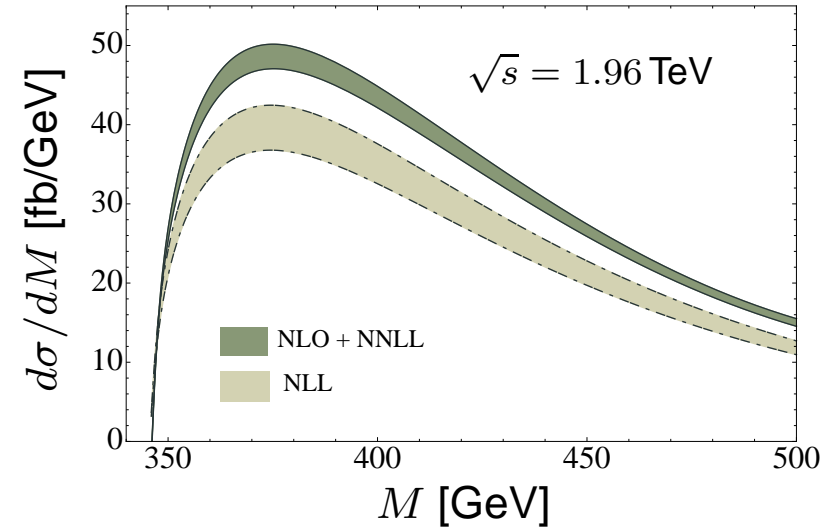
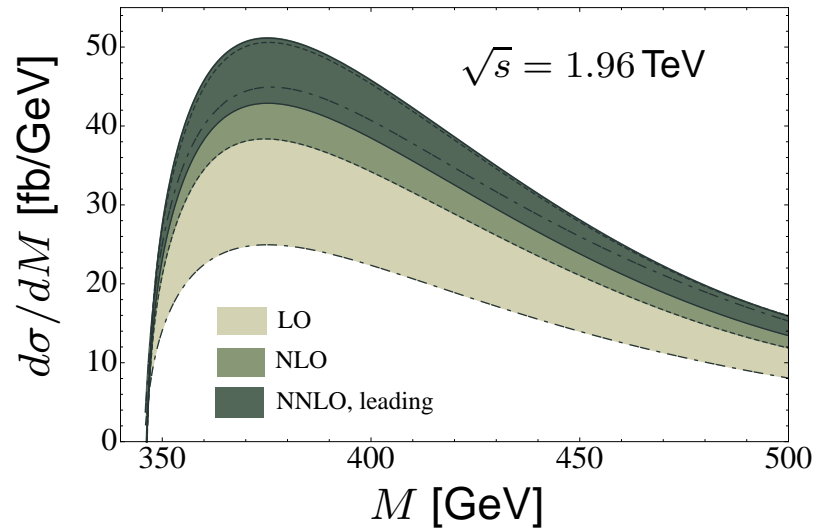
- no large numerical shift in distributions
- scale dependence substantially reduced  $\implies$  more reliable predictions
- error estimate via scale dependence more questionable than ever
  - scale dependence enters via logs, but higher-order terms also have constants
  - scale dependence is an estimate of importance of missing logs
  - higher-order logs can be predicted and resummed, but constants are still missing

comparison fixed-order vs. resummed cross section for  $y_t$  [Ahrens et al. 1103.0550]



- similar picture as for  $p_t$  distribution
- neither resummation nor approximate (!!) NNLO have a large effect
- NLO prediction seems to be fairly reliable **but full NNLO still missing!!**
- impact on  $A_{FB} \implies$  later

Resummation of logs: for invariant mass [Ahrens et.al. arXiv:1003.5827]



## bound-state effects

- near threshold Coulomb potential is dominating effect:

$$\text{colour singlet: } V(r) \simeq -\alpha_s \frac{C_F}{r} \text{ attractive}$$

$$\text{colour octet: } V(r) \simeq -\alpha_s \frac{C_F - C_A/2}{r} \text{ repulsive}$$

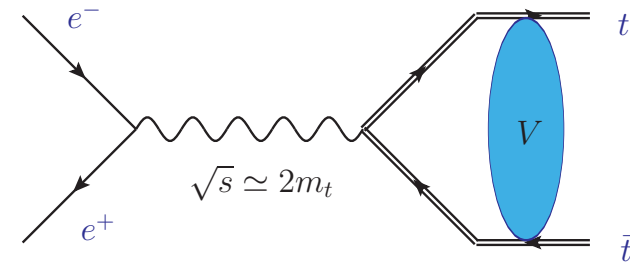
- for  $\Gamma_t \rightarrow 0$  collections of bound states (as for bottom), for  $\Gamma_t \simeq 1.4 \text{ GeV}$  a single “bump” in invariant mass remains.
- resummation of  $(\alpha/\beta)^n$  (from Coulomb potential  $\rightarrow$  “bound-state” effects) [Hagiwara et.al., Kiyo et.al.] results in modification of invariant mass spectrum
- effect small for colour octet, i.e. Tevatron ( $q\bar{q}$  is pure octet at LO), but “large” (for a theorist) at the LHC
- “bump” is impossible to be seen, but there is an effect on total cross section (threshold expansion  $\sigma_{t\bar{t}}^{(Coul)}$ )

## Top threshold scan at linear collider

top pair produced near threshold

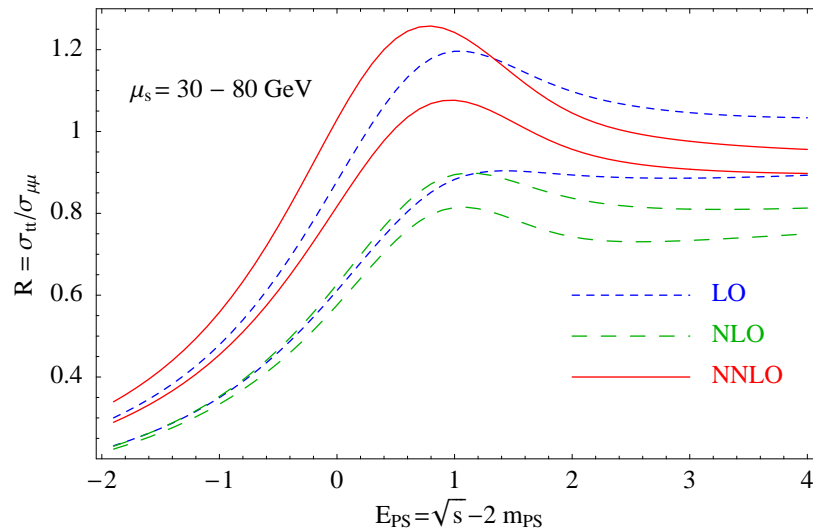
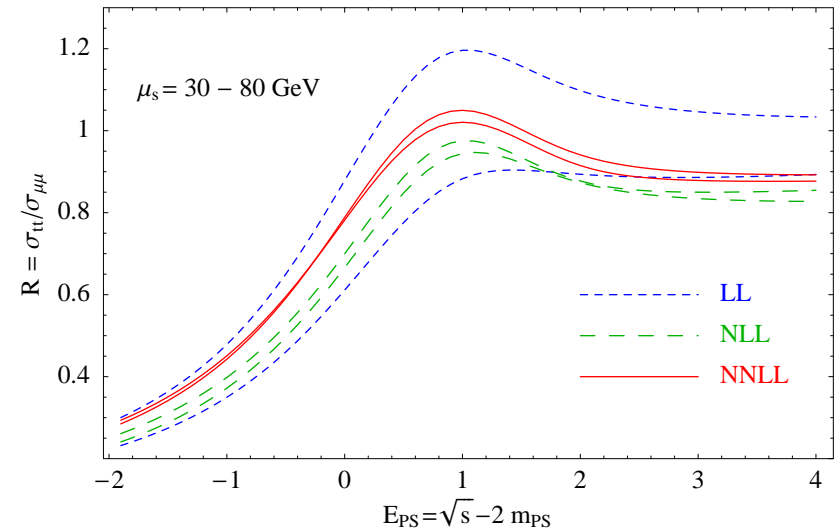
$$E \equiv \sqrt{s} - 2m \ll m$$

non-relativistic  $\rightarrow$  NRQCD



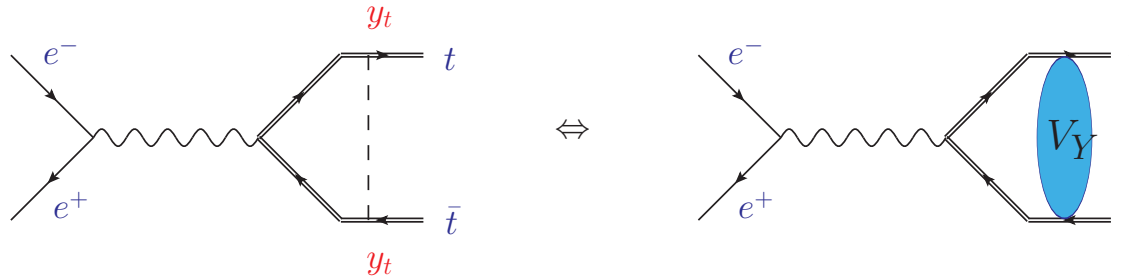
- lifetime for top  $\tau \simeq 1/\Gamma_t \simeq 5 \times 10^{-25}$  s
- typical hadronization time  $\tau_{\text{had}} \simeq 1/\Lambda_{\text{QCD}} \simeq 2 \times 10^{-24}$  s
- $\tau < \tau_{\text{had}} \Rightarrow$  top decays before it forms hadrons
- Schrödinger eq: 
$$\left( \frac{\Delta}{m^2} - \frac{\alpha_s C_F}{r} + \delta V - (E + i\Gamma_t) \right) G(\vec{r}, \vec{r}', E) = \delta(\vec{r} - \vec{r}')$$
- poles (bound states) become a bump (would-be bound state)
- position of bump  $\Rightarrow$  determination of mass
- height and width of bump  $\Rightarrow$  determination of  $\Gamma_t$
- typical scale:  $\mu \simeq 2mv \simeq 2 \left( m \sqrt{E^2 + \Gamma_t^2} \right)^{1/2} \gtrsim 30 \text{ GeV} \Rightarrow$  perturbation theory

## Top threshold scan at linear collider [Pineda, AS]

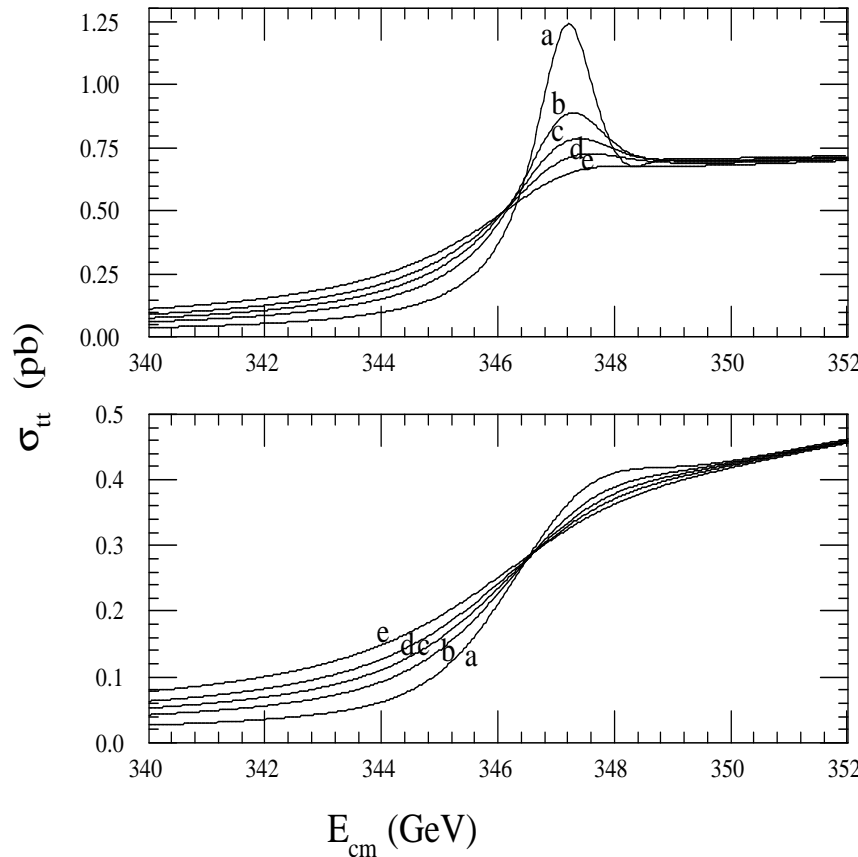
no resummation of  $\log v$ with resummation of  $\log v$ 

- normalization of cross section much more stable after resummation
- smaller scale dependence, smaller size of corrections
- potential to measure (well defined) top mass to an accuracy of  $\delta m_t \simeq 50 \text{ MeV}$
- potential for a precise measurement of  $\Gamma_t$  and maybe even the Yukawa coupling

measurement of Higgs-Yukawa potential  $\rightarrow y_t$  ?? treating Higgs as “new physics”



$$V_Y = -\frac{y_t^2}{4\pi} \frac{e^{-m_h r}}{r}$$



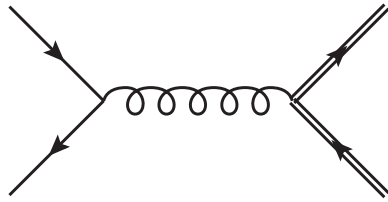
measurement of  $\Gamma_t$  [Frey et.al.]

- $\Gamma_t$  affects shape of threshold scan
- different curves correspond to  $\Gamma_t/\Gamma_t^{SM} =$  (a) 0.5, (b) 0.8, (c) 1.0, (d) 1.2, and (e) 1.5
- before (top) and after (bottom) bremsstrahlung corrections



threshold “scan” at Tevatron/LHC [Hagiwara et al. 0804.1014]

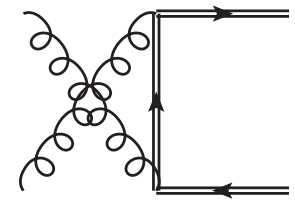
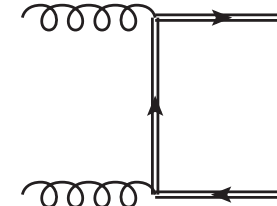
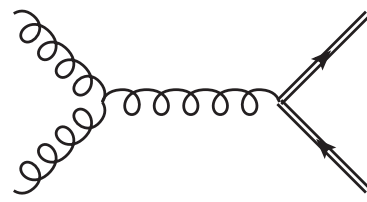
Tevatron



$$V_o = -\frac{\alpha (C_F - C_A/2)}{r}$$

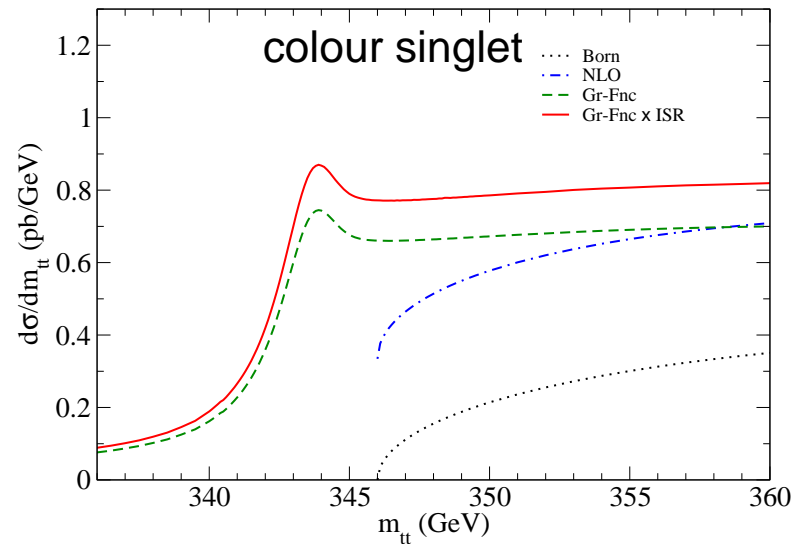
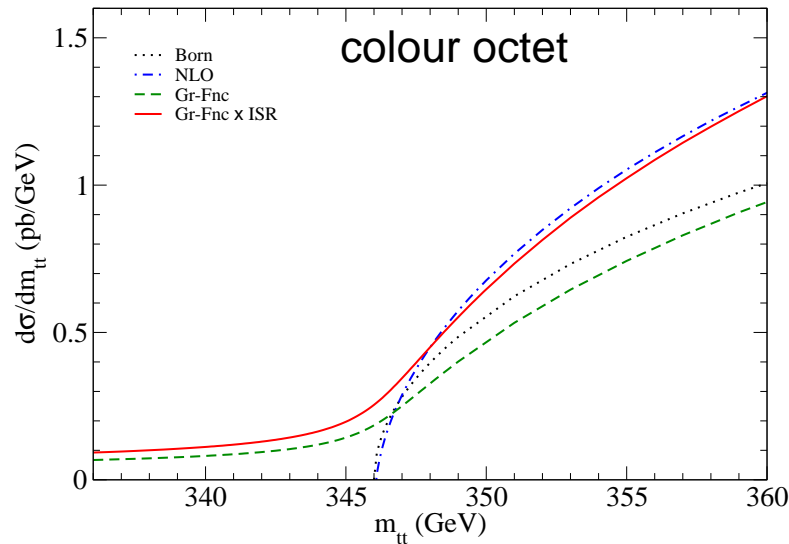
repulsive

LHC



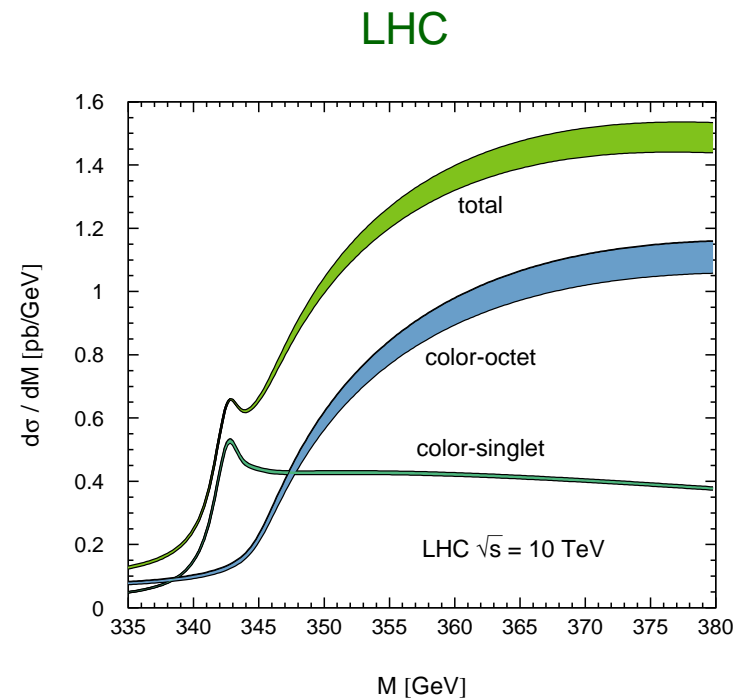
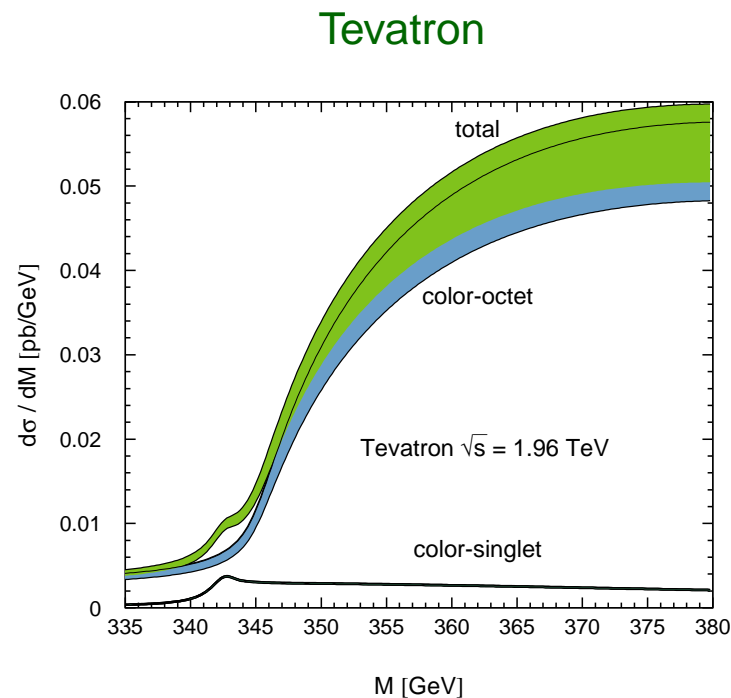
$$V_s = -\frac{\alpha C_F}{r}$$

attractive



Top “threshold scan” at LHC [Kiyo et al. 0812.091]

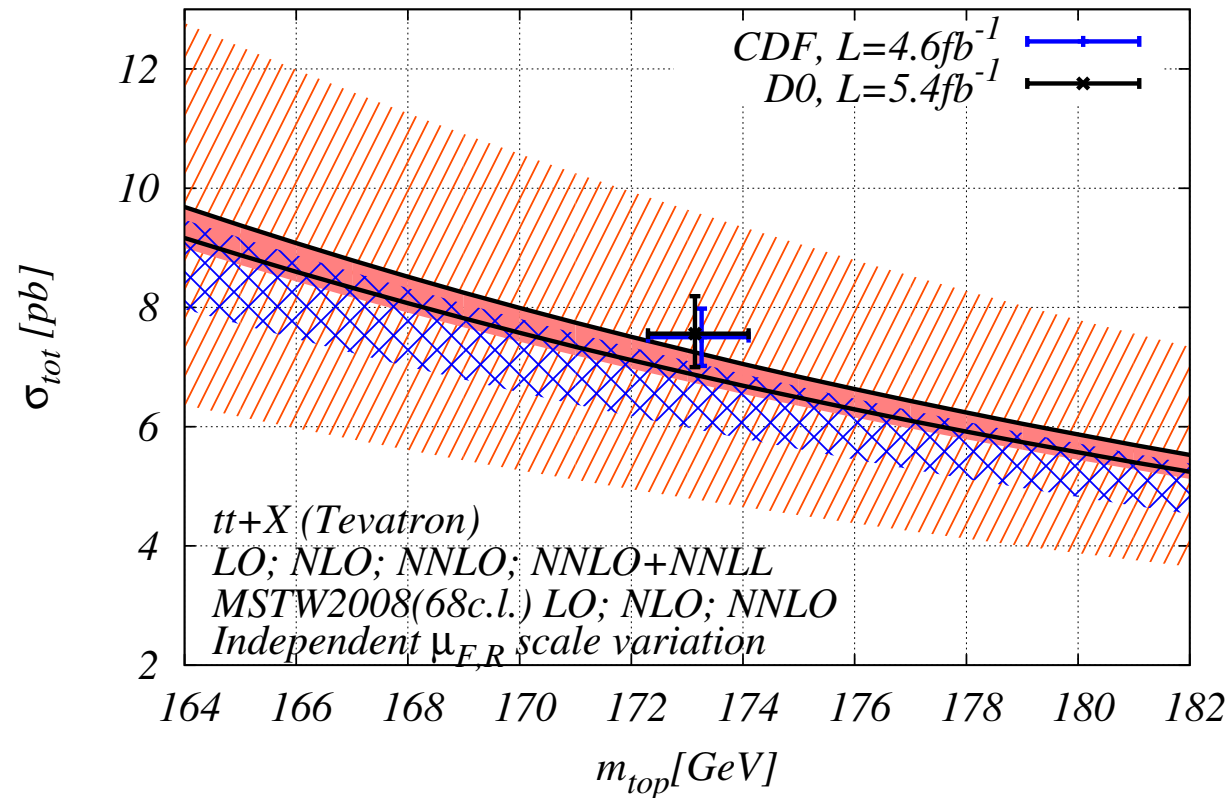
including all channels and parton-distribution functions:



this bump cannot be seen directly but has some (small) impact on the total cross section

total cross section,  $\sigma_{q\bar{q}}^{(2)}$  computed numerically [Bärnreuther, Czakon, Mitov]

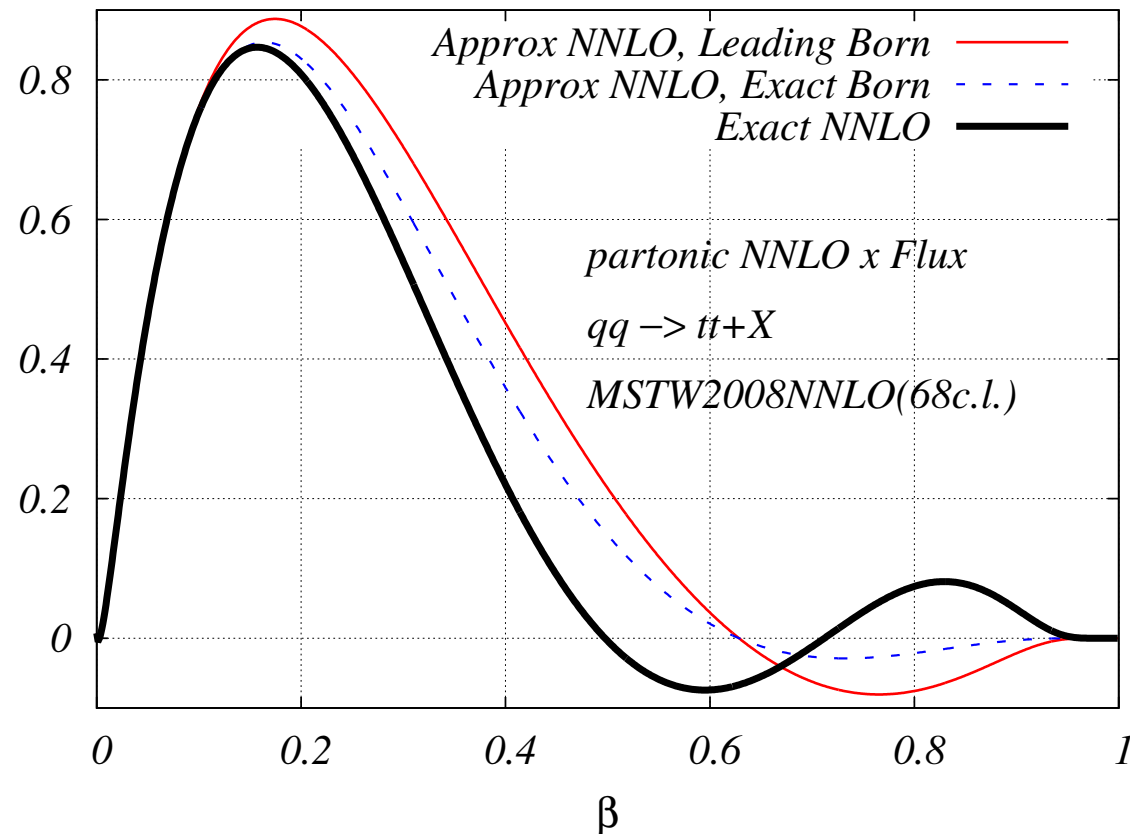
$$\hat{\sigma}_{ij} = \alpha_s^2 \left[ \sigma_{ij}^{(0)} + \alpha_s \left( \sigma_{ij}^{(1,0)} + \sigma_{ij}^{(1,1)} \log(\mu^2/m^2) \right) + \alpha_s^2 \left( \sigma_{ij}^{(2,0)} + \sigma_{ij}^{(2,1)} \log(\mu^2/m^2) + \sigma_{ij}^{(2,2)} \log^2(\mu^2/m^2) \right) \right]$$



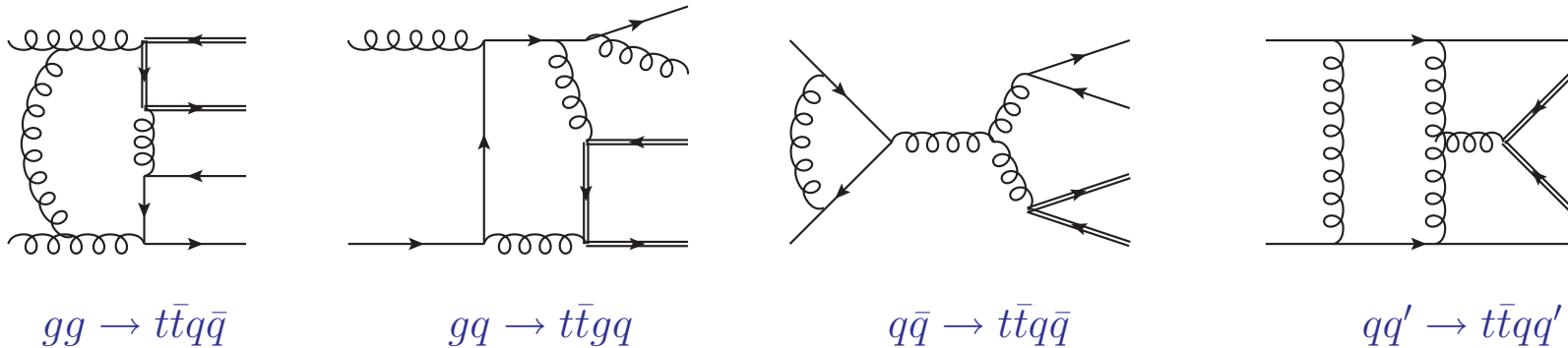
total cross section [Bärnreuther, Czakon, Mitov]

$\sigma_{ij}^{(2,i)}$  expanded in  $\beta$  corresponds to threshold expansion [Beneke et.al.]

$$\sigma_{q\bar{q}}^{(2,0)} = \sigma_{q\bar{q}}^{(0)} \left[ \frac{k^{(2,0)}}{\beta^2} + \sum_{n=0}^2 \frac{k^{(1,n)}}{\beta} \log^n \beta + \sum_{n=0}^4 k^{(0,n)} \log^n \beta \right]$$

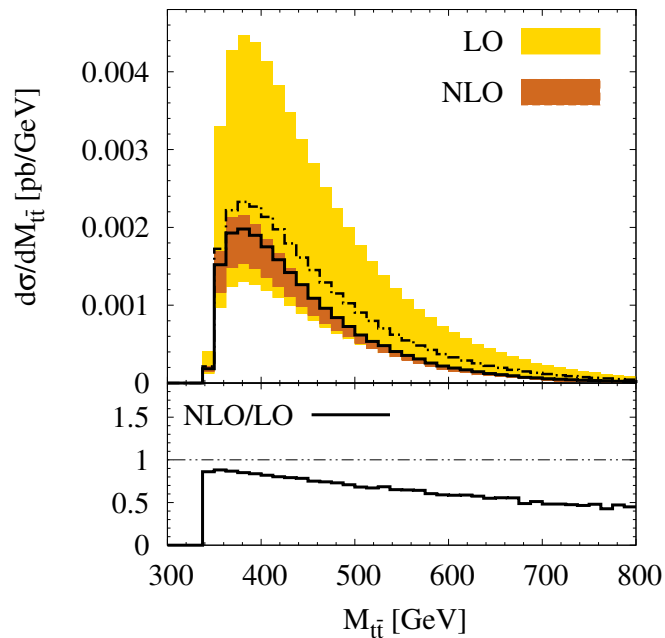


many partonic processes, up to 6-point integrals: (tree level  $\sim \alpha_s^4(\mu)$  !!)

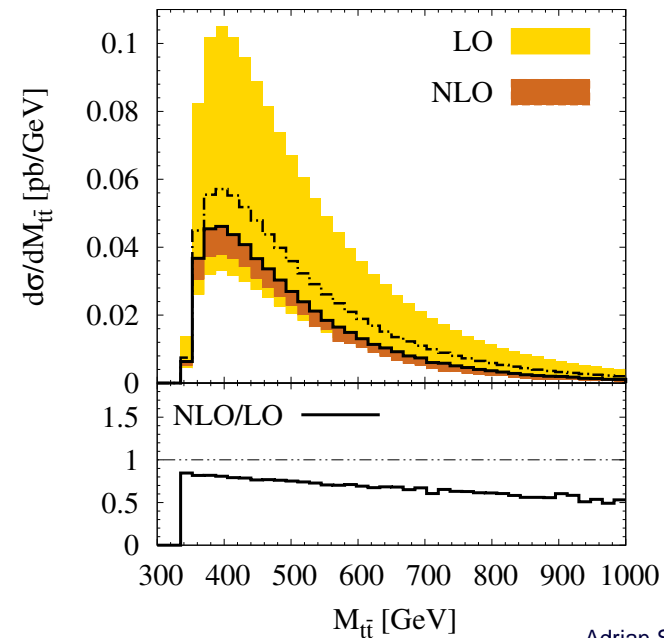


e.g: invariant mass of top pair [Bevilacqua et al. 1108.2851]

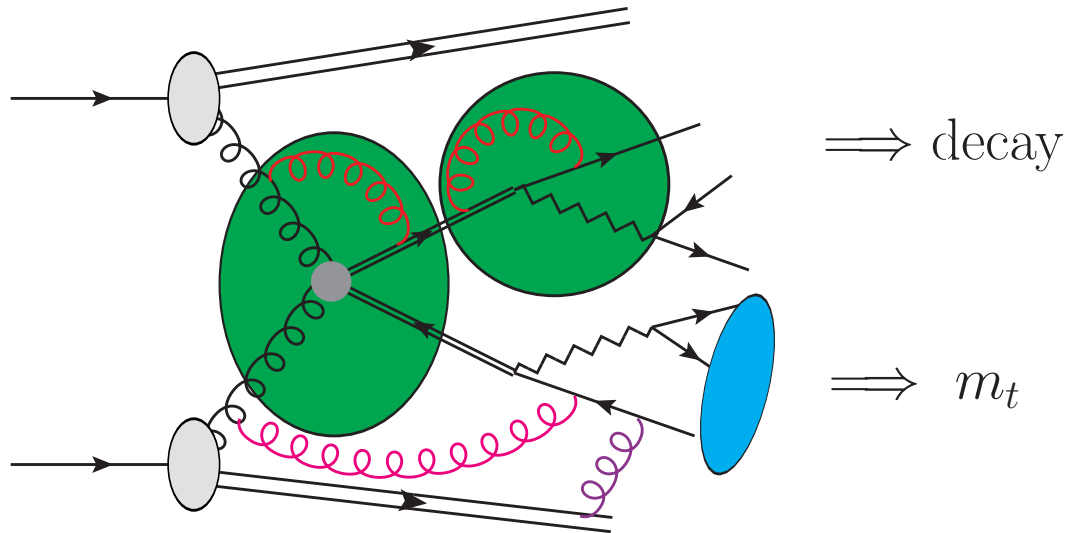
Tevatron



LHC

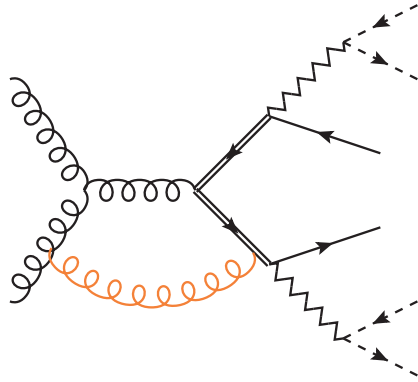


## more detailed questions

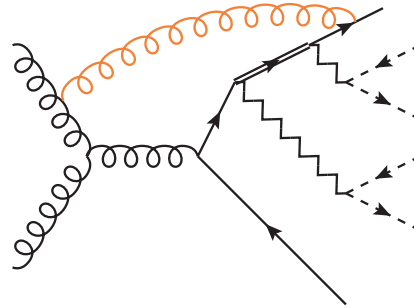


- cuts on decay products (missing  $E_T$ , rapidity and  $p_t$  of leptons etc. )
- testing decay of top (spin correlations)
- non-factorizable corrections (off-shell effects)
- colour connection between decay products and proton remnants

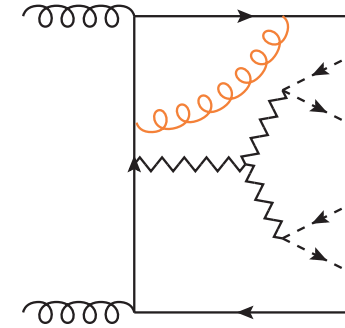
include decay of top and  $W$ ,  $gg \rightarrow W^+ b W^- \bar{b}$



double resonant



single resonant



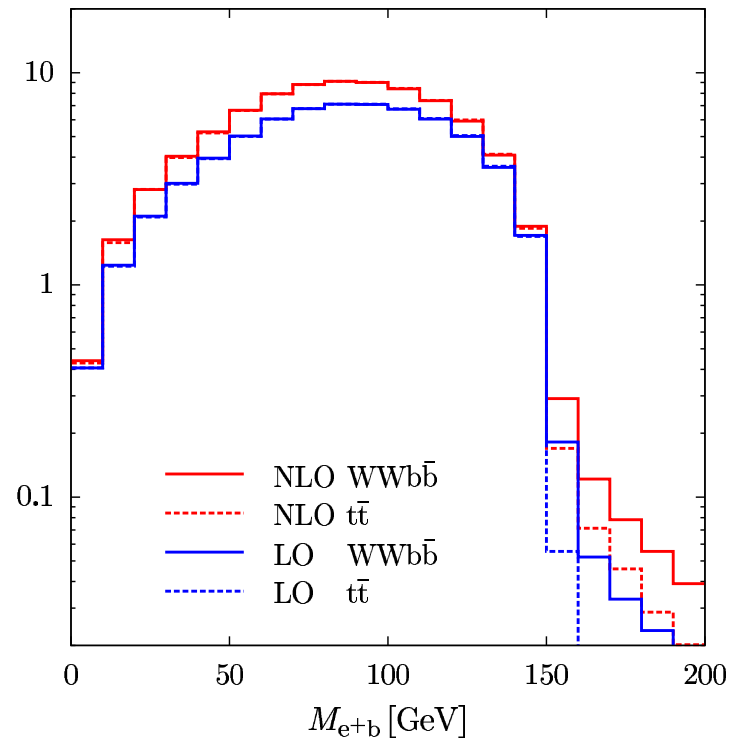
non-resonant

- calculation available by two groups [[Bevilacqua et al](#); [Denner et. al](#)]
- complex mass scheme for treatment of intermediate unstable particles  
 $m_t^2 \rightarrow \mu_t^2 \equiv m_t^2 - im_t\Gamma_t$
- requires integrals with complex masses
- treatment of  $W$  (with leptonic decay): also resonant or non-resonant

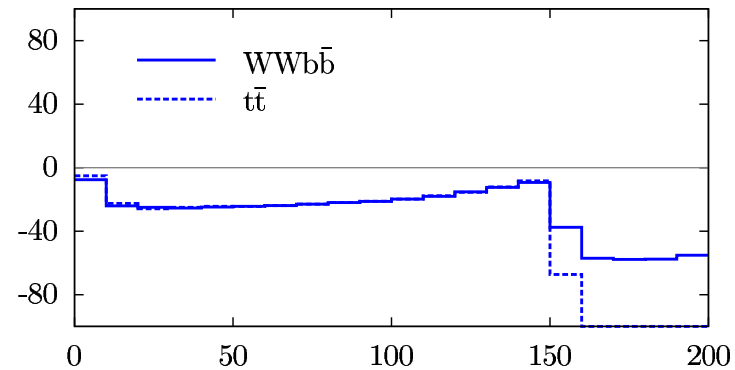
top quark  $M_{eb}$  distribution distribution for 8 TeV LHC [Denner et al. 1203.6803]

$$\frac{d\sigma}{dM_{e+b}} \left[ \frac{\text{fb}}{\text{GeV}} \right] \quad pp \rightarrow \nu_e e^+ \mu^- \bar{\nu}_\mu b \bar{b} + X$$

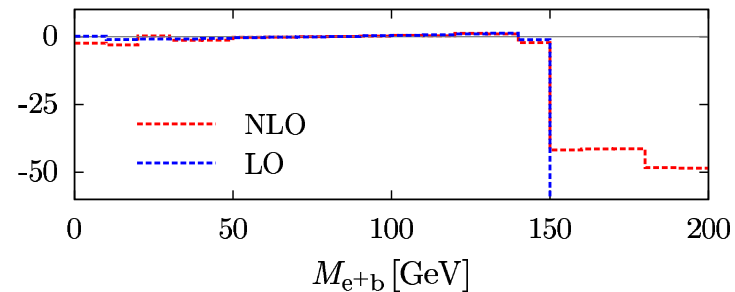
$$\sqrt{s} = 7 \text{ TeV}$$



$$\text{LO/NLO} - 1 \text{ [\%]}$$

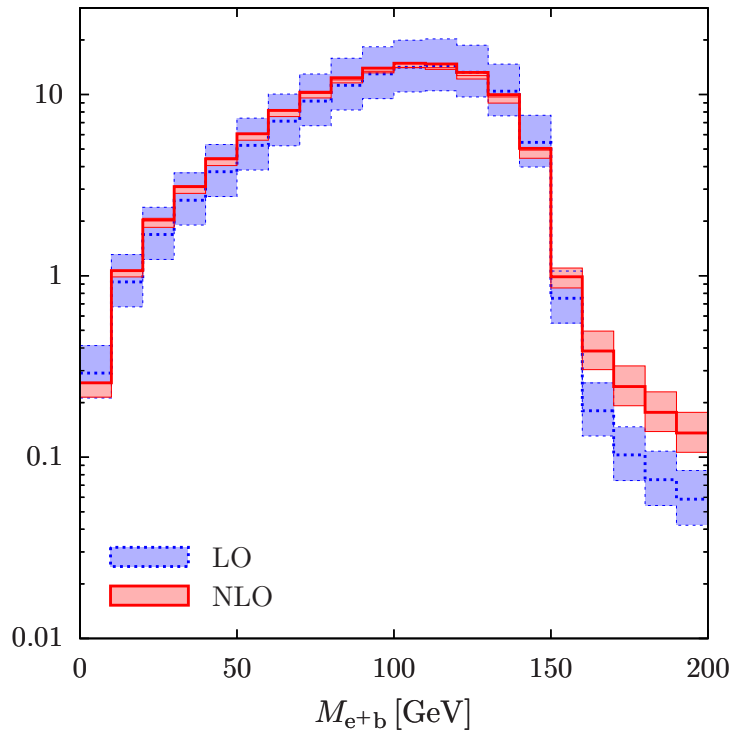
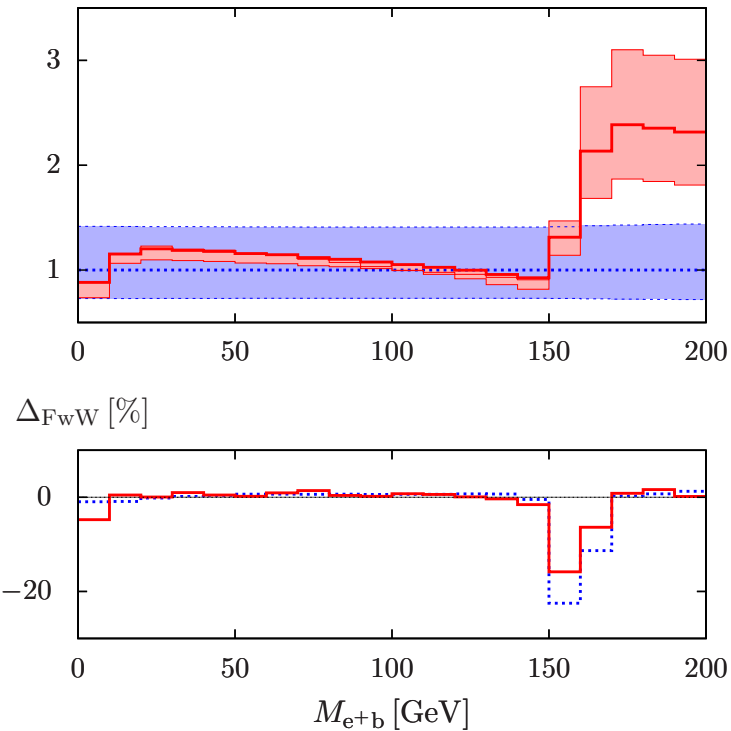


$$t\bar{t}/WWb\bar{b} - 1 \text{ [\%]}$$



- off-shell effects (from top) small in general
- can be enhanced at kinematic boundaries ( at LO:  $M_{eb}^2 < m_t^2 - M_W^2$  )



$M_{eb}$  distribution for 8 TeV LHC [Denner et al. 1207.5018] $d\sigma/dM_{e+b}$  [fb/GeV] $K$   $pp \rightarrow \nu_e e^+ \mu^- \bar{\nu}_\mu b \bar{b} + X @ \sqrt{s} = 8 \text{ TeV}$ 

- off-shell effects (from  $W$ ) small except in special (but possibly important) kinematic regions ( $m_t$  measurement)

# Part III

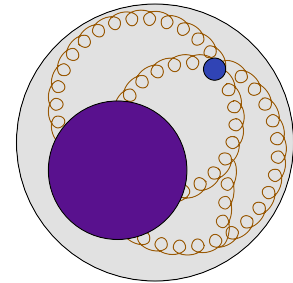
# Top Mass

**Problem 1:** conceptual problem with pole mass;  $\mathcal{O}(\Lambda_{\text{QCD}})$

The pole mass has an intrinsic uncertainty of order  $\Lambda_{\text{QCD}}$  in perturbation theory (infrared sensitivity, renormalon ambiguity)

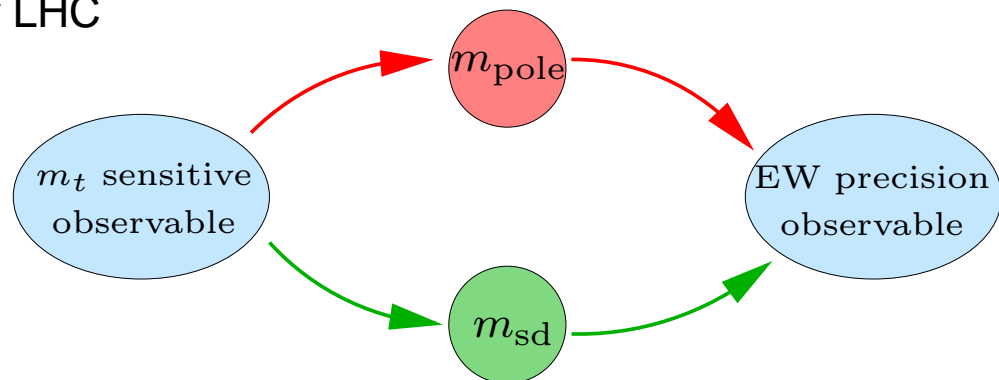
consider (fictitious) meson:

$$\underbrace{M}_{\text{well def. pole mass}} = \underbrace{m_Q}_{\text{pert. ambiguity}} + m_q + \underbrace{V(q^2)}_{\text{pert. ambiguity}}$$



There is a principal limitation of the usefulness of the pole mass:  $\delta m_t > \Lambda_{\text{QCD}}$

- can be solved in principle by using other (short-distance) mass definitions
- highly relevant for  $m_t$  determinations at linear collider [Beneke et.al, Hoang et.al]
- probably (??) not relevant for LHC



## Problem 2: scheme dependence

- $m_t$  has no meaning, unless you precisely specify what you mean by it
- quark mass definition is **not unique**, it is simply a theoretical parameter
- different definitions (schemes) are possible and widely used e.g.  $m_{\text{pole}}, \overline{m}, m_{\text{PS}}, m_{1\text{S}}, \overline{m}_{\text{DR}} \dots$
- for each (**acceptable**) scheme  $s_1$  the mass  $m_{s_1}$  can be related to the bare mass  $m_0$  by divergent relations to any order in perturbation theory

$$m_{s_1}^{(i)} = m_0 (1 + \alpha_s d_{s_1}^{(1)} + \alpha_s^2 d_{s_1}^{(2)} + \dots + \alpha_s^i d_{s_1}^{(i)})$$

- the masses in two (**acceptable**) schemes  $s_1$  and  $s_2$  are related by finite relations

$$m_{s_1}^{(i)} = m_{s_2}^{(i)} (1 + \alpha_s f_{s_1, s_2}^{(1)} + \alpha_s^2 f_{s_1, s_2}^{(2)} + \dots + \alpha_s^i f_{s_1, s_2}^{(i)})$$

- at tree level, all mass definitions are equal, but the higher-order coefficients can be **numerically large**, e.g. relating  $m_{\text{pole}}^{(3)}$  to  $\overline{m}^{(3)}$ :

$$172.5 \text{ GeV} \simeq (162.0 + 8.0 + 1.9 + 0.6) \text{ GeV}$$

observable  $O$ , mass scheme  $s_1$

$$O_{\text{exp}} = \underbrace{O_{s_1}^{(0)}(m_{s_1} \dots)}_{\text{determination of } m_{s_1}^{(0)}} + \alpha_s O_{s_1}^{(1)}(m_{s_1} \dots) + \alpha_s^2 O_{s_1}^{(2)}(m_{s_1} \dots) + \dots$$

$$\underbrace{\hspace{10em}}_{\text{determination of } m_{s_1}^{(1)} = m_{s_1}^{(0)}(1 + c_{s_1}^{(1)} \alpha_s)}$$

$$\underbrace{\hspace{15em}}_{\text{determination of } m_{s_1}^{(2)} = m_{s_1}^{(0)}(1 + c_{s_1}^{(1)} \alpha_s + c_{s_1}^{(2)} \alpha_s^2)}$$

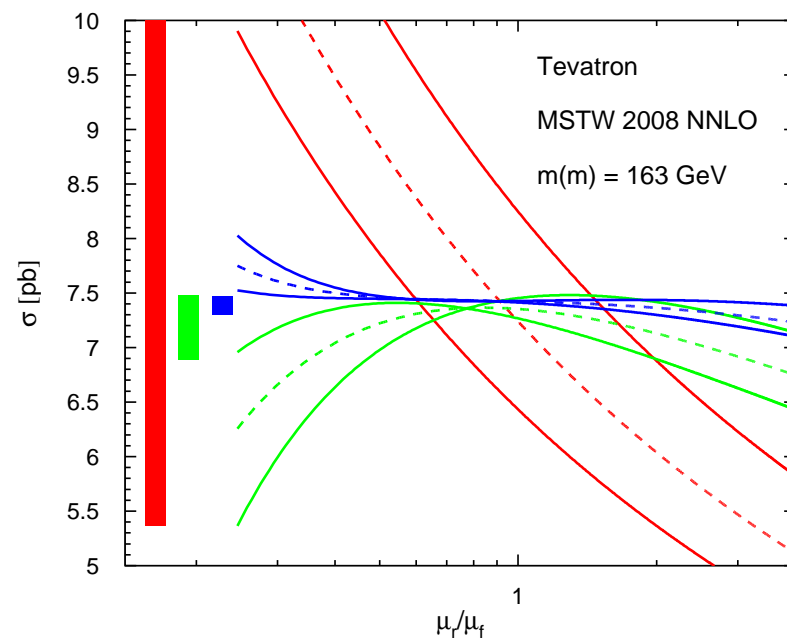
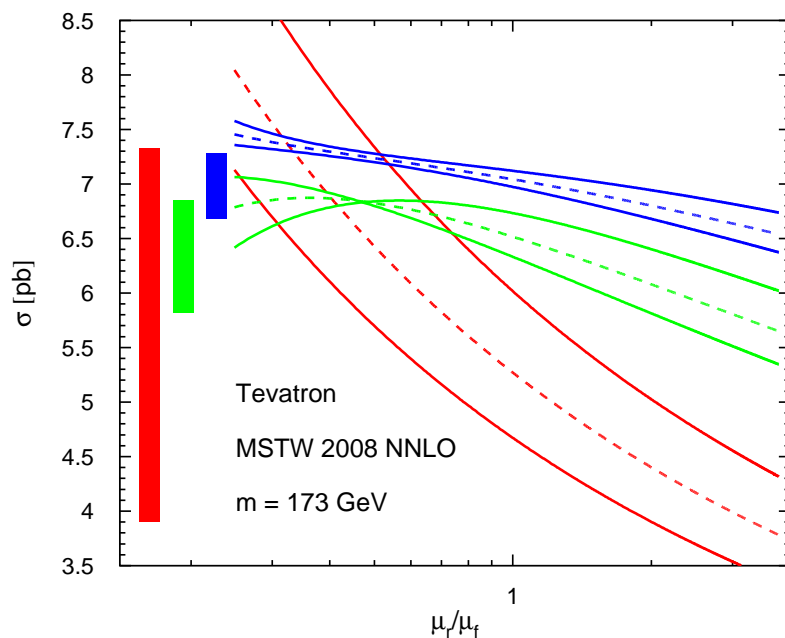
- working at order  $\alpha_s^n$ , the determinations of  $m_{s_2}$  by
  - using mass scheme  $s_2$  directly in determination above
  - using mass scheme  $s_1$  as above and then converting  $m_{s_1}$  to  $m_{s_2}$
 are different at order  $\alpha_s^{n+1}$
- to get a reliable top-mass determination we either have to work to very high order in perturbation theory or use a scheme where the corrections are not large.

**Problem 2:** how to relate  $m_{\text{exp}}$  to pole mass;  $\mathcal{O}(\Gamma_t)$

- $m_X$  determination by requiring  $O^{\text{th}}(m_X) \stackrel{!}{=} O^{\text{exp}}$ , in principle for any scheme  $X$  and any (mass sensitive and well measurable) observable  $O$
- in practice limitation through lack of higher-order terms in  $O^{\text{th}}$
- $m_t$  measurements through kinematics of decay products are basically tree-level determinations
- pick a scheme where higher-order corrections are small, i.e. pole scheme  $\implies m_t$  extracted using decay products is “something like” the pole mass
- the issue is **not** (and never was) whether this mass  $m_{\text{exp}}$  is the pole mass or  $\overline{\text{MS}}$  mass, but what the precise relation between  $m_{\text{exp}}$  and  $m_{\text{pole}}$  is
- care has to be taken when interpreting  $m_{\text{exp}} \stackrel{??}{=} m_{\text{pole}}$   
however  $m_{\text{exp}} \stackrel{!!}{=} m_{\text{pole}} + \mathcal{O}(\Gamma_t)$  is fine. (**Note:** non-factorizable corrections have been computed and seem to be small [Denner et.al., Bevilacqua et.al.]
- alternative ways to measure  $m_t$ , using different  $O$ , where higher-order corrections are known, e.g. total cross section [Langenfeld et.al] or other choices [Melnikov et.al.]
- the ultimate  $m_t$  determination with  $\delta m_t \sim 100 \text{ MeV}$  from threshold scan at ILC.

determination of  $\overline{m}(\overline{m})$  through cross section [Langenfeld, Moch, Uwer]

compare  $\sigma_{\text{tot}}$  expressed in terms of pole and  $\overline{\text{MS}}$  mass (for  $\mu_F \in \{0.5, 1, 2\} \times m_t$ )



- $\overline{\text{MS}}$  scheme more reliable (bands overlap, smaller uncertainty)
- direct extraction of  $\overline{\text{MS}}$  mass  $\overline{m}(\overline{m})$  with  $\delta m \simeq 3 \text{ GeV}$
- PDF uncertainties etc... ??

Compare direct vs. indirect determination of pole mass [Alekhin, Djouadi, Moch]

### Tevatron

CDF&D0	ABM11	JR09	MSTW08	NN21
$m_t^{\overline{\text{MS}}}(m_t)$	162.0 <sup>+2.3 +0.7</sup> <sub>-2.3 -0.6</sub>	163.5 <sup>+2.2 +0.6</sup> <sub>-2.2 -0.2</sub>	163.2 <sup>+2.2 +0.7</sup> <sub>-2.2 -0.8</sub>	164.4 <sup>+2.2 +0.8</sup> <sub>-2.2 -0.2</sub>
$m_t^{\text{pole}}$	171.7 <sup>+2.4 +0.7</sup> <sub>-2.4 -0.6</sub>	173.3 <sup>+2.3 +0.7</sup> <sub>-2.3 -0.2</sub>	173.4 <sup>+2.3 +0.8</sup> <sub>-2.3 -0.8</sub>	174.9 <sup>+2.3 +0.8</sup> <sub>-2.3 -0.3</sub>
$(m_t^{\text{pole}})$	169.9 <sup>+2.4 +1.2</sup> <sub>-2.4 -1.6</sub>	171.4 <sup>+2.3 +1.2</sup> <sub>-2.3 -1.1</sub>	171.3 <sup>+2.3 +1.4</sup> <sub>-2.3 -1.8</sub>	172.7 <sup>+2.3 +1.4</sup> <sub>-2.3 -1.2</sub>

### LHC

ATLAS&CMS	ABM11	JR09	MSTW08	NN21
$m_t^{\overline{\text{MS}}}(m_t)$	159.0 <sup>+2.1 +0.7</sup> <sub>-2.0 -1.4</sub>	165.3 <sup>+2.3 +0.6</sup> <sub>-2.2 -1.2</sub>	166.0 <sup>+2.3 +0.7</sup> <sub>-2.2 -1.5</sub>	166.7 <sup>+2.3 +0.8</sup> <sub>-2.2 -1.3</sub>
$m_t^{\text{pole}}$	168.6 <sup>+2.3 +0.7</sup> <sub>-2.2 -1.5</sub>	175.1 <sup>+2.4 +0.6</sup> <sub>-2.3 -1.3</sub>	176.4 <sup>+2.4 +0.8</sup> <sub>-2.3 -1.6</sub>	177.4 <sup>+2.4 +0.8</sup> <sub>-2.3 -1.4</sub>
$(m_t^{\text{pole}})$	166.1 <sup>+2.2 +1.7</sup> <sub>-2.1 -2.3</sub>	172.6 <sup>+2.4 +1.6</sup> <sub>-2.3 -2.1</sub>	173.5 <sup>+2.4 +1.8</sup> <sub>-2.3 -2.5</sub>	174.5 <sup>+2.4 +2.0</sup> <sub>-2.3 -2.3</sub>

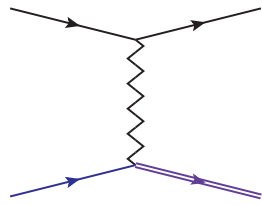
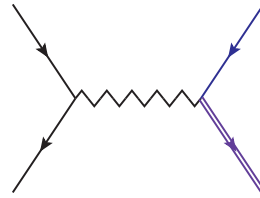
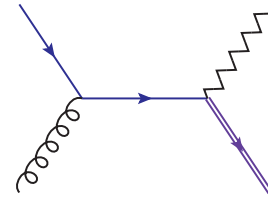
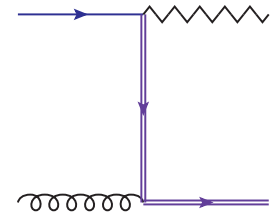
- with errors  $\delta m_t \sim 2 - 3 \text{ GeV}$  renormalon problems are not main issue.
- if  $\delta m_t \lesssim 1 \text{ GeV}$  **must not** use pole mass



# Part IV

# Single Top

## basic processes

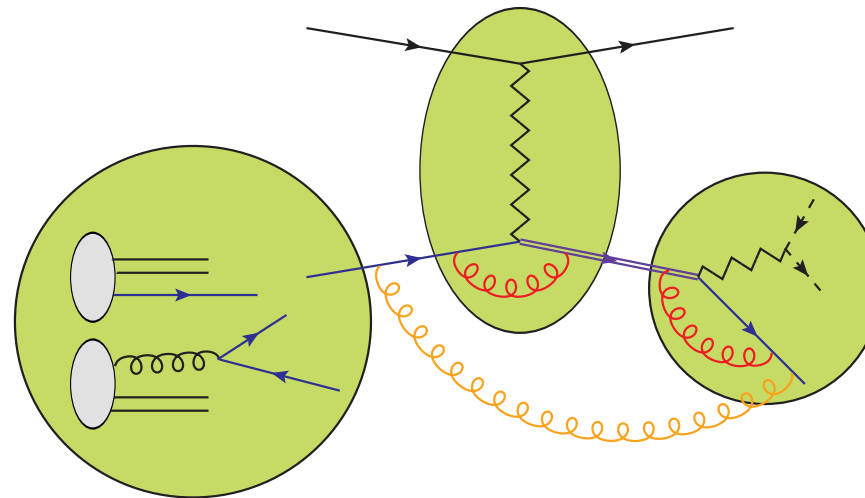
 $t$  - channel $s$  - channel $W t$  (or  $H^- t$ ) production

classification of physical processes is not that straightforward

approximate (!) expected / measured SM cross sections in pb

	Tevatron	7 TeV LHC	14 TeV LHC
$t(\bar{t})$ "t"-ch	1.2	40 (20)	150 (100)
$t(\bar{t})$ "s"-ch	0.55	2.5 (1.4)	7 (4)
$t W^-$	0.15	8	45

more detailed questions



- NLO corrections in production
- resummation of soft logs  $\rightarrow$  “N”NLO corrections
- top decay, at LO/NLO, spin correlations
- off-shell effects / non-factorizable corrections
- initial  $b$  quark and  $m_b$  effects : 5 flavour scheme vs. 4-flavour scheme
- matching to parton showers

- fully differential NLO QCD corrections for  $t$ -,  $s$ -channel and  $Wt$  known [Harris et.al; Sullivan; Zhu ...]
- resummation at NNLL of inclusive cross section [Kidonakis; Wang et.al.]
  - “poor man’s” NNLO corrections
- top decay added, with NLO corrections in production and decay [Campbell et.al; Cao et.al]
  - issues with definition of channel
  - spin correlations
- EW corrections known in SM and MSSM [Beccaria et.al; Macorini et.al]
  - effect small, a few %
- non-factorizable corrections known [Falgari et.al]
  - effects small, except at kinematic boundaries
- 4-flavour vs. 5-flavour scheme [Campbell et.al]
  - generally good agreement at NLO
- all channels (including  $t H^-$ ) included in MC@NLO and POWHEG [Frixione, Frederix, Laenen, Motylinski, Alioli, Nason, Re, Webber, White .....]
- BSM effects (e.g. anomalous trilinear couplings) included in WHIZARD
  - interference with background diagrams on its way [Bach, Kilian, Ohl. ...]

## s-channel: Kidonakis [1001.5034]

- resummation in moment space
- $s_4 \equiv (p_a + p_b - p_1)^2 - m_t^2 = s + t + u - m_t^2$  for  $s_4 \rightarrow 0 \Rightarrow$   

$$\alpha_s^n L^{2n-1} \equiv \alpha_s^n [\log^{2n-1}(s_4/m_t^2)/s_4]_+$$
- NLL  $\rightarrow$  NNLO:  $\alpha_s^2 L^3$  and  $\alpha_s^2 L^2$       NLLLO<sub>approx</sub>/NLO  $\sim 10\%$  increase  
 NNLL  $\rightarrow$  NNLO: also  $\alpha_s^2 L^1$  and  $\alpha_s^2 L^0$       NLLLO<sub>approx</sub>/NLO further 3-4% increase
- soft limit good approximation for Tevatron and LHC
- damping factors (to limit soft gluon contributions away from threshold) improve soft approximation
- “best” predictions, MSTW2008 NNLO pdf:

Kidonakis  $m_t = 173$  GeV

$$\sigma_{\text{TeV}} = 0.523_{-0.005}^{+0.001} {}_{-0.028}^{+0.030} \text{ pb}$$

$$\sigma_{\text{LHC } 7} = 3.17_{-0.06}^{+0.06} {}_{-0.10}^{+0.13} \text{ pb}$$

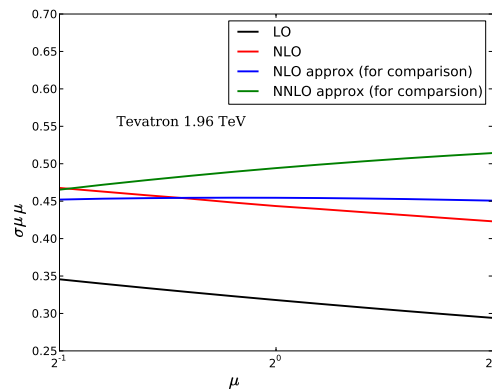
Zhu et.al.  $m_t = 173.2$  GeV

$$\sigma_{\text{TeV}} = 0.467_{-0.01}^{+0.01} \text{ pb}$$

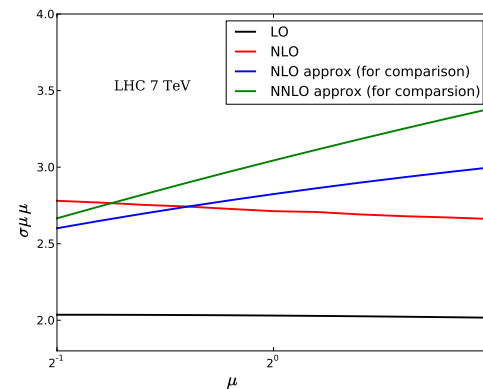
$$\sigma_{\text{LHC } 7} = 2.81_{-0.10}^{+0.16} \text{ pb}$$

s-channel: Zhu, Li, Wang, Zhang [1006.0681]

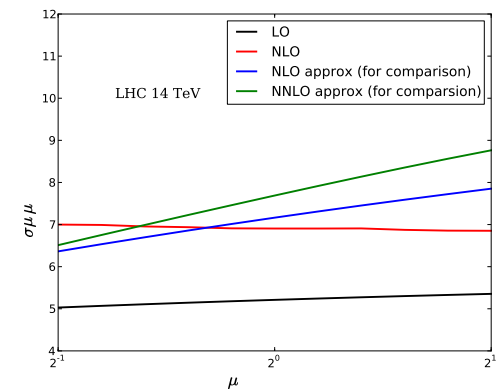
- resummation via SCET
- **different** definition of resummation variable  $s_4 \equiv (p_a + p_b - p_t)^2$   
also includes hard-collinear logarithms
- soft/coll limit good approximation for Tevatron, not very good for LHC



Tevatron



LHC @ 7 TeV



LHC @ 14 TeV

t-channel: Kidonakis [1103.2792] vs Wang, Li, Zhu, Zhang [1010.4509]

- similar technical (moments vs SCET) and physical (resummation kinematics and virtual contribution) differences as for s-channel
- soft gluon approximation not considered reliable
- results for  $m_t = 173$  GeV and MSTW2008 NNLO pdf

Kidonakis

$$\sigma_{\text{TeV}} = 1.04_{-0.02}^{+0.00} \pm 0.06 \text{ pb}$$

$$\sigma_{\text{LHC } 7} = 41.7_{-0.2}^{+1.6} \pm 0.8 \text{ pb}$$

$$\sigma_{\text{LHC } 14} = 151_{-1}^{+4} \pm 3 \text{ pb}$$

Wang et.al.

$$\sigma_{\text{TeV}} = 0.982 \text{ pb}$$

$$\sigma_{\text{LHC } 7} = 40.9_{-0.1}^{+0.1} \text{ pb}$$

$$\sigma_{\text{LHC } 7} = 152.4_{-1.0}^{+0.4} \text{ pb}$$

- better numerical agreement than for s-channel
- resummation effects decrease scale dependence

*W t* and *H<sup>-</sup> t*: Kidonakis [1005.4451]

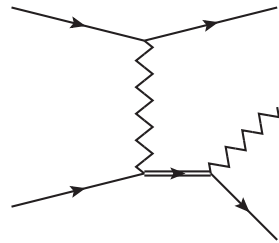
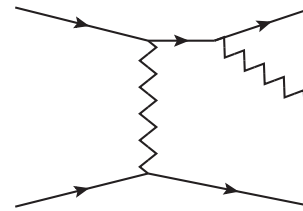
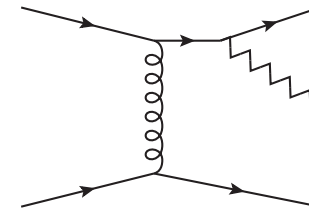
- resummed cross section re-expanded:

$$\sigma^{(2)} = \sigma^{(0)} \alpha_s^2 \left( \underbrace{c_3 L^3 + c_2 L^2}_{\text{NLL}} + \underbrace{c_1 L^1 + c_0 L^0}_{\text{NNLL}} \right)$$

- soft gluons claimed to be dominant
- damping factors applied
- NLO → 'N'NLO: 8% increase at 7 TeV LHC
- $m_t = 173$  GeV, MSTW2008 NNLO pdf:  $\sigma(t W^-) = 7.8 \pm 0.2_{-0.6}^{+0.5}$  pb
- scale variation error < pdf error
- similar analysis for *H<sup>-</sup> t*: corrections NLO → 'N'NLO: 15-20%, depending on  $m_H$



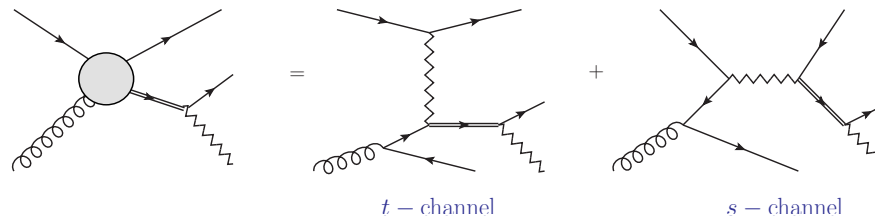
- new issue: definition of process, e.g t-channel

 $\mathcal{A}_{\text{res}}$  $\mathcal{A}_{\text{EWbg}}$  $\mathcal{A}_{\text{QCDbg}}$ 

- it is an “irrelevant coincidence” at LO that

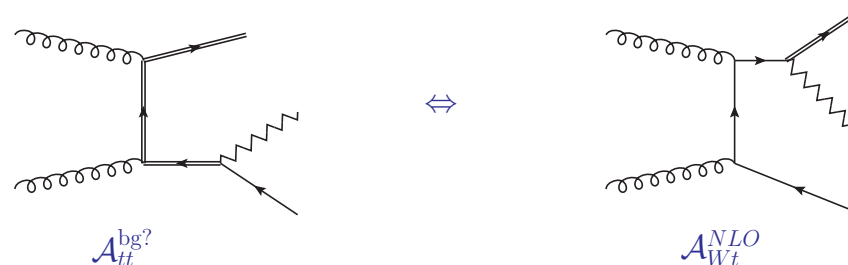
$$|\mathcal{A}_{\text{res}} + \mathcal{A}_{\text{EWbg}} + \mathcal{A}_{\text{QCDbg}}|^2 = |\mathcal{A}_{\text{res}} + \mathcal{A}_{\text{EWbg}}|^2 + |\mathcal{A}_{\text{QCDbg}}|^2$$

- shouldn't we define a proper observable (to which  $\mathcal{A}_{\text{QCDbg}}$  contributes) with proper final states (e.g. b-jets), rather than try to subtract  $|\mathcal{A}_{\text{QCDbg}}|^2$  ?
- similar comment regarding distinction between s-channel and t-channel

 $t$  - channel $s$  - channel

- mixing but no interference at NLO (another “irrelevant coincidence”), beyond NLO there is interference

- this issue is particularly acute for  $Wt$  and has been studied extensively [Kersevan et.al; Tait; Belyaev et.al; Campbell et.al; Frixione et.al]



- possible remedies
  - invariant mass (anti-) cut  $|M_{Wb} - m_t|^2 \gg \Gamma_t$
  - $p_T^b < p_T^{\text{veto}}$  (hard  $b$  tend to come from  $t$  decay)
  - Diagram removal  $\mathcal{A}_{(Wt)} + \mathcal{A}_{(tt)} \rightarrow \mathcal{A}_{(Wt)}$
  - Diagram subtraction

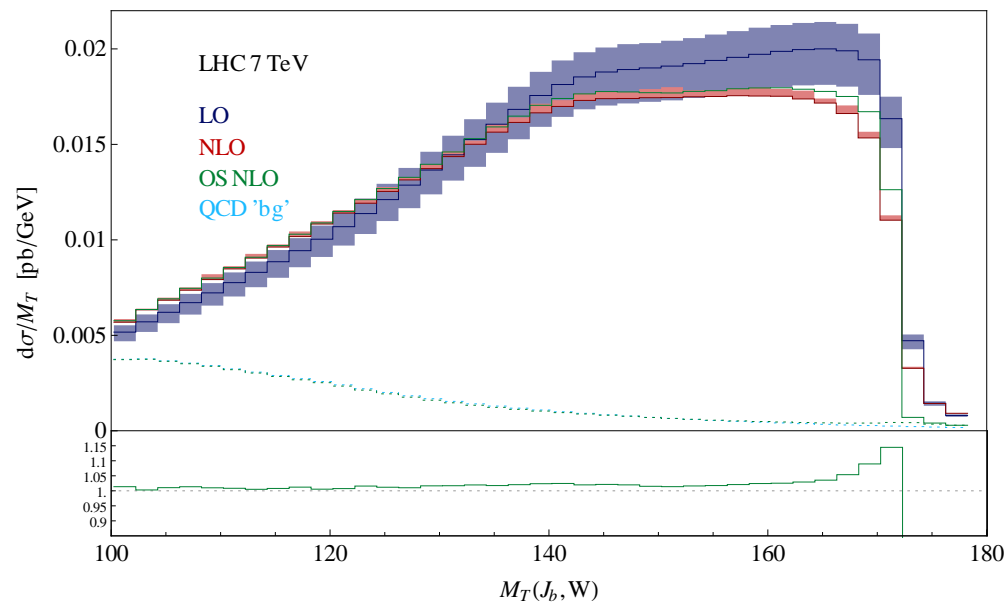
$$|\mathcal{A}_{(Wt)} + \mathcal{A}_{(tt)}|^2 \rightarrow |\mathcal{A}_{(Wt)}|^2 + 2\text{Re}(\mathcal{A}_{(Wt)}\mathcal{A}_{(tt)}^*) + |\mathcal{A}_{(tt)}|^2 - \widetilde{|\mathcal{A}_{(tt)}|^2}$$

- using  $b$ -jet rather than  $b$ -parton allows to define (at least theoretically) clean observables

non-factorizable corrections have been extensively studied [Fadin et.al; Melnikov et.al; Beenakker et.al; Denner et.al.; Jadach et.al; . . .] but usually neglected at hadron colliders:

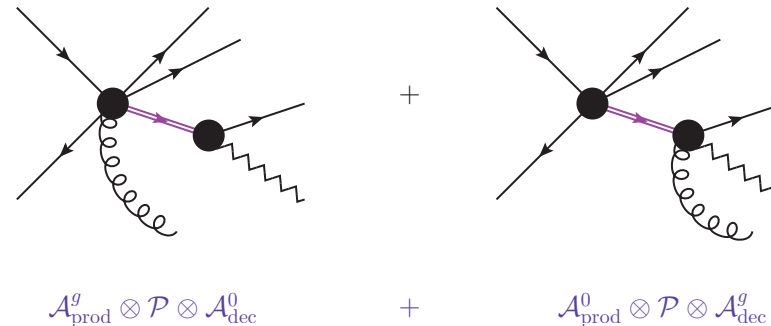
- they seem to be more difficult to compute (not really)
- they are generally small [Beenakker et.al; Pittau]
  - resonant  $\rightarrow$  non-resonant propagator unless  $E \lesssim \Gamma$  is small (soft)
  - cancellations for “inclusive” observables [Fadin, Khoze, Martin]
- include off-shell effects: consistently combine non-factorizable with propagator corrections:

[Falgari et.al] e.g. transverse mass: 
$$M_T = \sqrt{\sum_{J_b, \ell, \nu} |p_T|^2 - \left( \sum_{J_b, \ell, \nu} \vec{p}_T \right)^2}$$



effective-theory inspired calculation (hard/soft through method of region)

real amplitude:



corrections to production (soft and coll singularities):

$$\int d\Phi_{n+1} \left| \mathcal{A}_{\text{prod}}^g \otimes \mathcal{P} \otimes \mathcal{A}_{\text{dec}}^0 \right|^2 \text{ plus (hard) virtual corrections for } t\text{-production is IR finite}$$

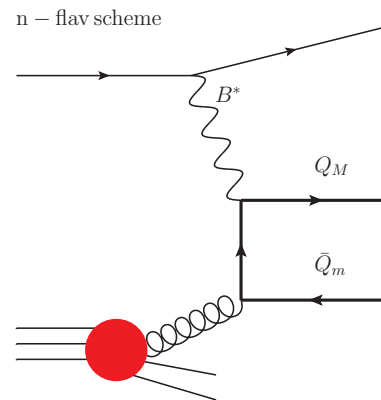
corrections to decay (soft and coll singularities):

$$\int d\Phi_{n+1} \left| \mathcal{A}_{\text{prod}}^0 \otimes \mathcal{P} \otimes \mathcal{A}_{\text{dec}}^g \right|^2 \text{ combined with (hard) virtual correction for decay is IR finite}$$

non-factorizable corrections (soft singularities only):

$$\int d\Phi_{n+1} 2 \text{Re} \left( \mathcal{A}_{\text{prod}}^0 \otimes \mathcal{P} \otimes \mathcal{A}_{\text{dec}}^g \right) \left( \mathcal{A}_{\text{prod}}^g \otimes \mathcal{P} \otimes \mathcal{A}_{\text{dec}}^0 \right)^* \text{ plus soft virtual is IR finite}$$

## 4-flavour scheme vs. 5-flavour scheme

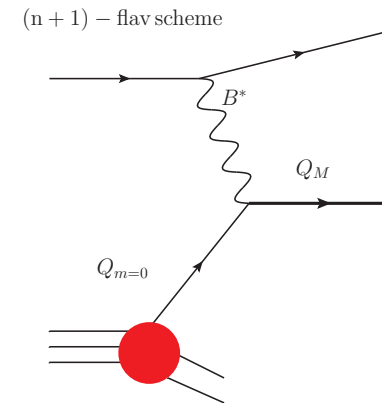


$b \notin p$ : 4 flavour scheme

$\exists \bar{b}$  @ LO

only 1  $\log \mu_f^2/m_b^2$  @ NLO

$m_b$  effects can be included



$b \in p$ : 5 flavour scheme

$\nexists \bar{b}$  @ LO

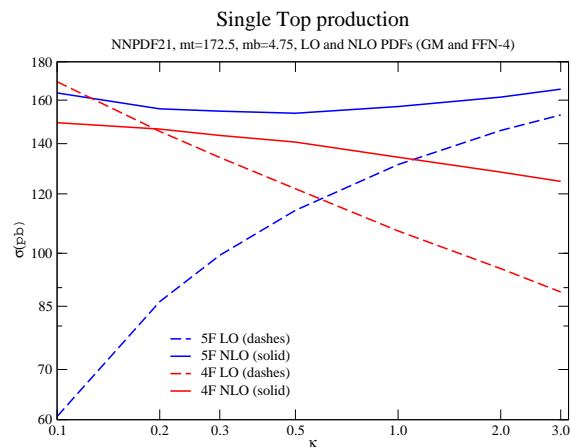
$\log \mu_f^2/m_b^2$  resummed

$m_b = 0$  for initial state

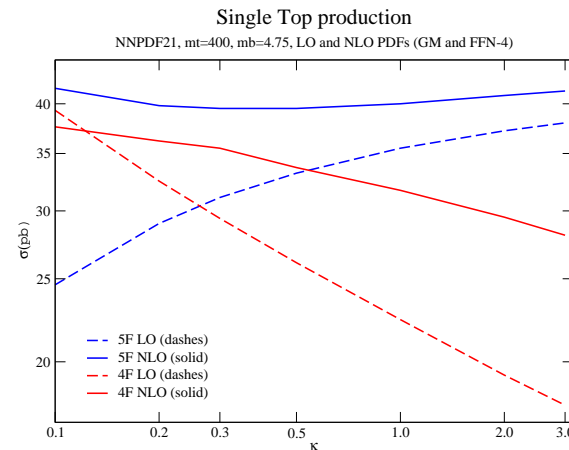
- Comparison 4F vs 5F for single top at NLO [[Campbell et.al](#)]:
- Generally good agreement already at NLO
- A detailed single-top analysis POWHEG vs aMC@NLO in 4F (and 4F vs 5F including parton showers) is under way [[Frederix, Re, Torrielli](#)]

## 4-flavour scheme vs. 5-flavour scheme

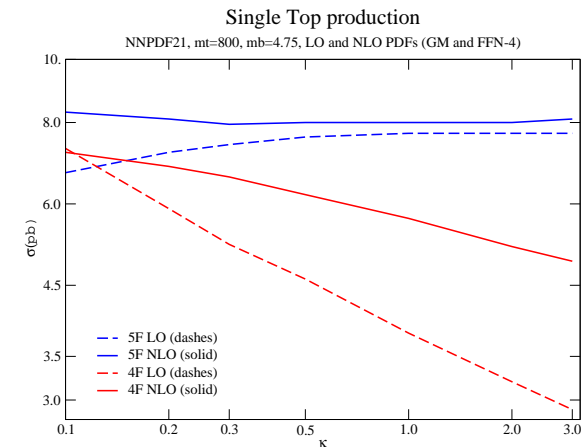
- general analysis 4F vs 5F [Maltoni, Ridolfi, Ubiali (1203.6393)]
- resummation of  $\log \mu_f^2 / m_x^2$  numerically not very important (except for  $x$  large)
- scale in  $\log$  suppressed through phase space



$$m_t = 172.5 \text{ GeV}$$



$$m_t = 400 \text{ GeV}$$



$$m_t = 800 \text{ GeV}$$

## tools (no claim for completeness!)

- resummed total cross sections available
  - for s- and t-channel by two groups
  - for  $W t$ ,  $H t$  by one group
- several fixed-order NLO calculations (including decay and spin correlations) available
- off-shell effects at NLO available
- all channels (s-, t-,  $W t$ ,  $H t$ ) implemented in POWHEG and MC@NLO
- t-channel in 4 flavour scheme (very soon) available in POWHEG and (a)MC@NLO
- all channels (s-, t-,  $W t$ ,  $H t$ ) available in WHIZARD
  - up to 6 final state partons at LO
  - including “background” diagrams
  - BSM models implemented
  - including interface to shower