Finding Supersymmetry at the LHC

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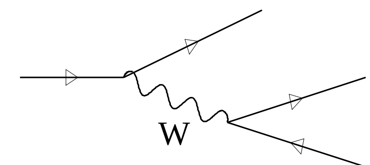
- TeV–scale supersymmetry
- Signals at Tevatron and LHC
- Measurements at LHC
- SUSY parameters at LHC (and ILC)

What's wrong with the Standard Model

A brief history of the field theory mess we are in

– Fermi 1934: weak interactions [$n \to pe^- \bar{\nu}_e$ and $\mu \to e^- \bar{\nu}_e \nu_\mu$]
  $\to$ divergent four fermion amplitude: $\mathcal{A} \propto G_F E^2$
  $\to$ unitarity violation [transition probability $\propto |\mathcal{A}|^2$]
  $\to$ 'effective theory' for $E \ll 600$ GeV

– Yukawa 1935: massive virtual particle exchange
  $\to$ Fermi's theory for $E \ll M$
  $\to$ four fermions unitary for large E: $\mathcal{A} \propto g^2 E^2/(E^2 - M^2)$
  $\to$ unitarity violation in $WW \to WW$ [$\mathcal{A} \propto G_F E^2$]



– Higgs 1964: spontaneous symmetry breaking
  $\to$ unitary gauge theory with massive W, Z [W, Z discovered 1983]
  $\to$ fermion masses linked to Yukawa couplings [top quark discoverd 1995]
  $\to$ light fundamental Higgs scalar [mass unknown]
  $\to$ Higgs couplings fixed $g_{HXX} \propto g m_X$

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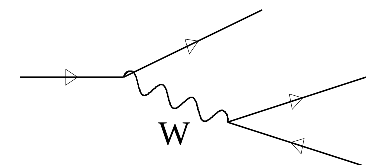

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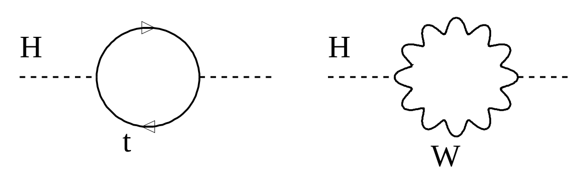
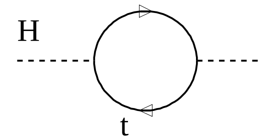


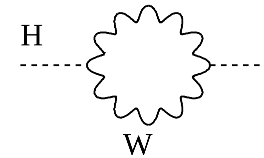
– disaster strikes: scalar mass perturbatively unstable
quadratic divergences $\delta m_H^2 \propto g^2 \Lambda^2$ [Λ loop cutoff]
all–orders Higgs mass driven to cutoff $m_H \to \Lambda$

$\Rightarrow$ new physics needed to solve hierarchy problem $m_Z \ll \Lambda$

TeV–Scale Supersymmetry: 1

Starting from data...





– ...which seem to indicate a light Higgs

– problem of light Higgs: mass driven to cutoff of theory
$\delta m_H^2 \propto g^2 (2m_W^2 + m_Z^2 + m_H^2 - 4m_t^2)\ \Lambda^2$
Veltman's condition $(\cdots) = 0$ would be fun
problem preferably solved to arbitrary loop order

$\Rightarrow$ easy solution: counter term to cancel loops $\Rightarrow$ artificial, unmotivated, ugly

$\Rightarrow$ or new physics at TeV scale: supersymmetry
extra dimensions
little Higgs (pseudo–Goldstone Higgs)
Higgsless, composite Higgs
YourFavoriteNewPhysics...

$\Rightarrow$ typically either cancellation with new particles or discussing away high scale

$\Rightarrow$ all beautiful concepts and symmetries

$\Rightarrow$ in general problematic to realize at TeV scale [data seriously in the way]

Idea of supersymmetry: cancellation of divergences through statistics factor (-1)
[scalars vs. SM fermions; fermions vs. SM gauge bosons; fermions vs. SM scalars]

TeV–Scale Supersymmetry: 2

SUSY idea: solve hierarchy problem by doubling spectrum

– stops (scalar) cancel top loop [couplings protected]

– gauginos (neutral or charged) cancel W, Z loop

– higgsinos cancel Higgs loop [mix with gauginos]

– postulate gluino for 2-loop, plus sleptons and squarks

⇒ hierarchy problem solved

⇒ rich collider and non-collider phenomenology [broken SUSY effective theory of everything]

⇒ extended Poincaré algebra, supergravity,...

Change in Higgs sector

– adjoint Higgs field not allowed in $\mathcal{L}$
  $\longrightarrow$ how to give mass to t and b?
  $\longrightarrow$ two Higgs doublets

⇒ SUSY Higgs sector interesting in itself

⇒ I hope Dave told you?

		spin	d.o.f.	
fermion	f_L, f_R	1/2	1+1	
$\longrightarrow$ sfermion	$\tilde{f}_L, \tilde{f}_R$	0	1+1	
gluon	G_μ	1	n-2	
$\longrightarrow$ gluino	$\tilde{g}$	1/2	2	Majorana
gauge bosons	γ, Z	1	2+3	
Higgs bosons	h^o, H^o, A^o	0	3	
$\longrightarrow$ neutralinos	$\tilde{\chi}_i^o$	1/2	$4 \cdot 2$	Majorana
gauge bosons	$W^\pm$	1	$2 \cdot 3$	
Higgs bosons	$H^\pm$	0	2	
$\longrightarrow$ charginos	$\tilde{\chi}_i^\pm$	1/2	$2 \cdot 4$	Dirac
graviton	G	2	2	
$\longrightarrow$ gravitino	$\tilde{G}$	3/2	2	hard to catch

TEV–SCALE SUPERSYMMETRY: 3

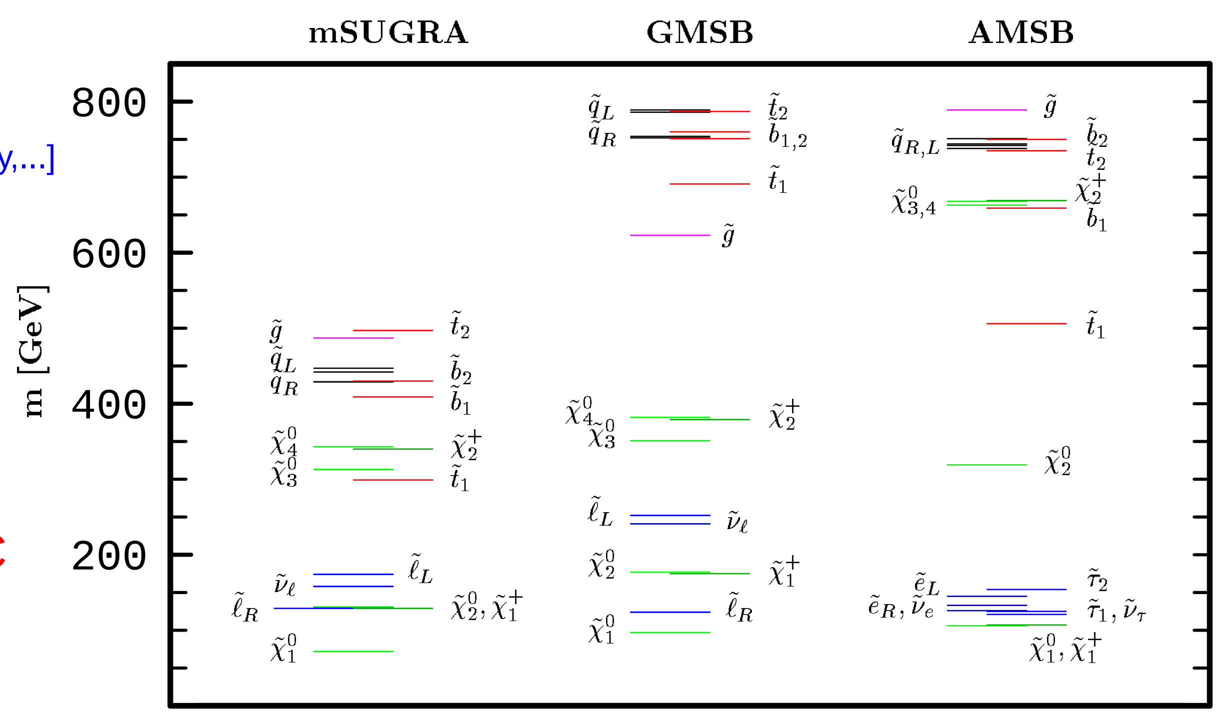
SUSY breaking: heavy partners

- mechanism for partner masses unknown [soft breaking keeps away quadratic divergences]
- link to flavor physics and baryogenesis/leptogenesis unknown
- SUSY breaking unknown [hidden sector, assume mediation to visible sector]
- maximally blind mediation at high scale: mSUGRA [not the LHC paradigm anymore!]
  scalars: m_0, fermions: $m_{1/2}$, tri-scalar term: A_0
  plus $\mathrm{sign}(\mu)$ and $\tan\beta$ in Higgs sector [Higgs masses free: NUHM]

Alternatives to mSUGRA [SoftSusy,...]

- gauge mediation
- anomaly mediation
- gaugino mediation
- $\cdots$?

$\Rightarrow$ measure spectrum at LHC



Instead of SUSY breaking — structures in the MSSM spectrum

– gauginos–higgsinos mixing: $m_{\tilde{\chi}_2^0} \sim m_{\tilde{\chi}_1^+}$ or $m_{\tilde{\chi}_1^0} \sim m_{\tilde{\chi}_1^+}$ in MSSM

$$\begin{pmatrix} m_{\tilde{B}} & 0 & -m_Z s_w c_\beta & m_Z s_w s_\beta \\ 0 & m_{\tilde{W}} & m_Z c_w c_\beta & -m_z c_w s_\beta \\ -m_Z s_w c_\beta & m_Z c_w c_\beta & 0 & -\mu \\ m_Z s_w s_\beta & -m_Z c_w s_\beta & -\mu & 0 \end{pmatrix} \quad \begin{pmatrix} m_{\tilde{W}} & \sqrt{2} m_W s_\beta \\ \sqrt{2} m_W c_\beta & -\mu \end{pmatrix}$$

– off-diagonal elements the SUSY-protected Yukawas

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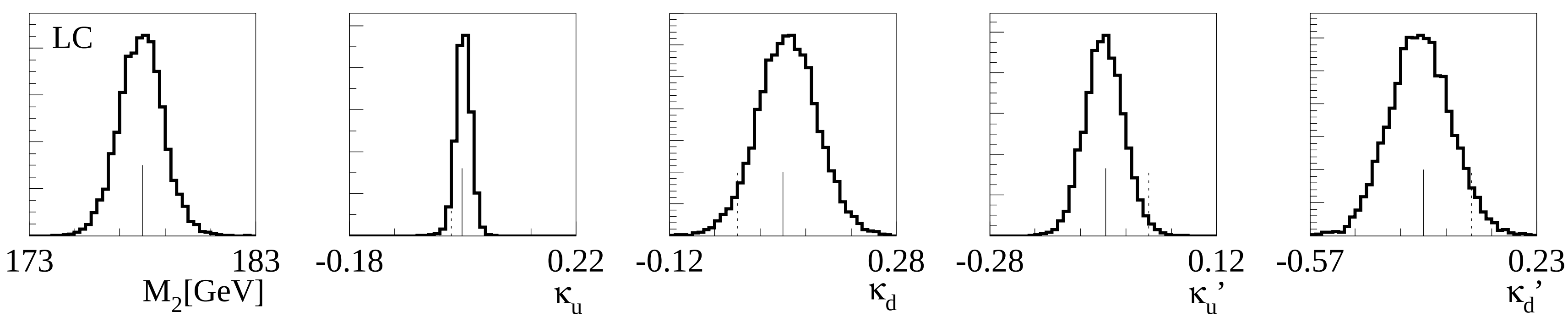
– off-diagonal elements the SUSY-protected Yukawas

$\Rightarrow$ test of SUSY at Linear Collider: $m_Z s_w c_\beta \to m_Z s_w c_\beta (1 + \kappa_d) \ldots$



[Kilian, TP, Richardson, Schmidt]

Instead of SUSY breaking — structures in the MSSM spectrum

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– stop and sbottom mixing in MSSM

$$\begin{pmatrix} m_Q^2 + m_t^2 + \left(\frac{1}{2} - \frac{2}{3} s_w^2\right) m_Z^2 c_{2\beta} & -m_t \left(A_t + \mu \cot\beta\right) \\ -m_t \left(A_t + \mu \cot\beta\right) & m_U^2 + m_t^2 + \frac{2}{3} s_w^2 m_Z^2 c_{2\beta} \end{pmatrix}$$

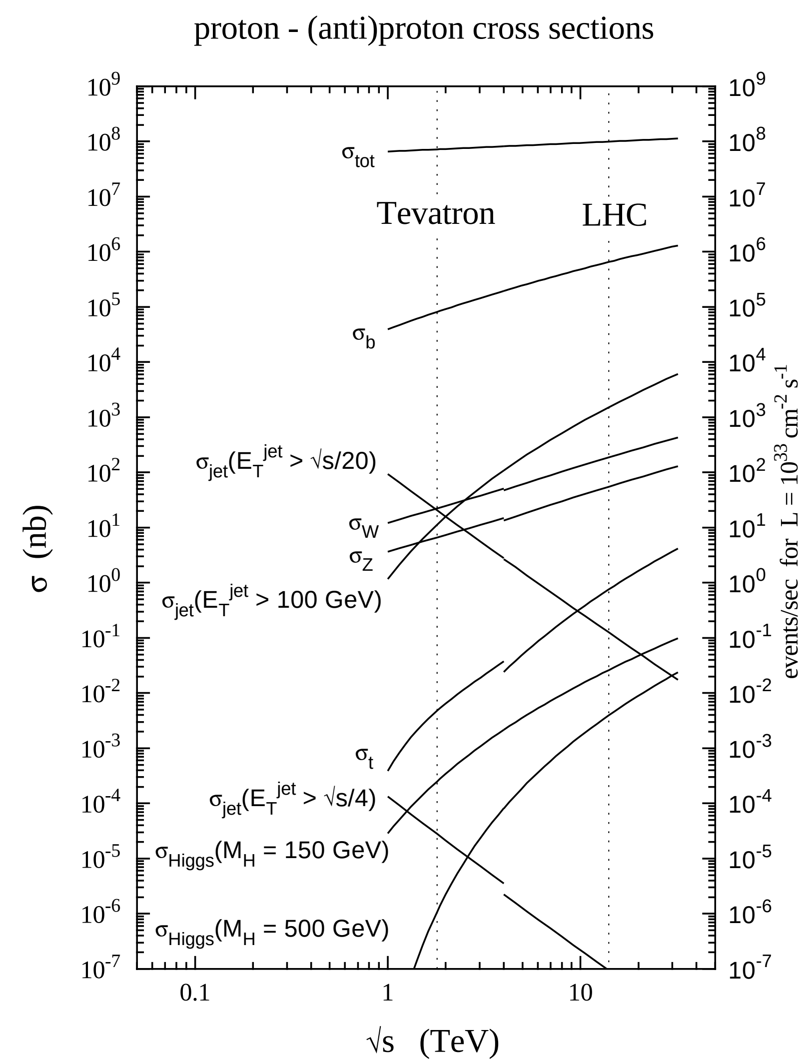
– heavy gluinos, squarks through unification: $m_{\tilde B, \tilde W, \tilde g}/m_{1/2} \sim 0.4, 0.8, 2.6$

$m_{\tilde\ell, \tilde q}/m_{1/2} \sim 0.7, 2.5$ $[m_0 \ll m_{1/2}]$

[mass and coupling unification independent]

Conversion of beam energy into particle mass

- search for new particles easier if particle produced
  $\rightarrow$ highest possible energies required
- clean e^+e^- colliders:
  LEP: Z pole
  LEP2: 206 GeV for e.g. ZH
  ILC/CLIC: 1...4 TeV in future
- powerful hadron colliders:
  Tevatron: $p\bar{p}$ with 2 TeV [valence quarks]
  LHC: pp with 14 TeV [gluons]
- LHC mass reach $\sim$ 3 TeV [win by luminosity]



New physics at hadron colliders

- what is a jet and what is inside? [b, τ tag]
- trigger: 'no leptons — no data'
- huge backgrounds $pp \rightarrow b\bar{b}, WZ+\text{jets},...$
- statistics: $S/\sqrt{B} > 5$ called discovery

Inclusive: squarks and gluinos at Tevatron

– squarks, gluinos strongly interacting
  $p\bar{p} \to \tilde{q}\tilde{q}^*, \tilde{q}\tilde{g}, \tilde{g}\tilde{g}$ [best if $m(\tilde{q}) \sim m(\tilde{g})$]

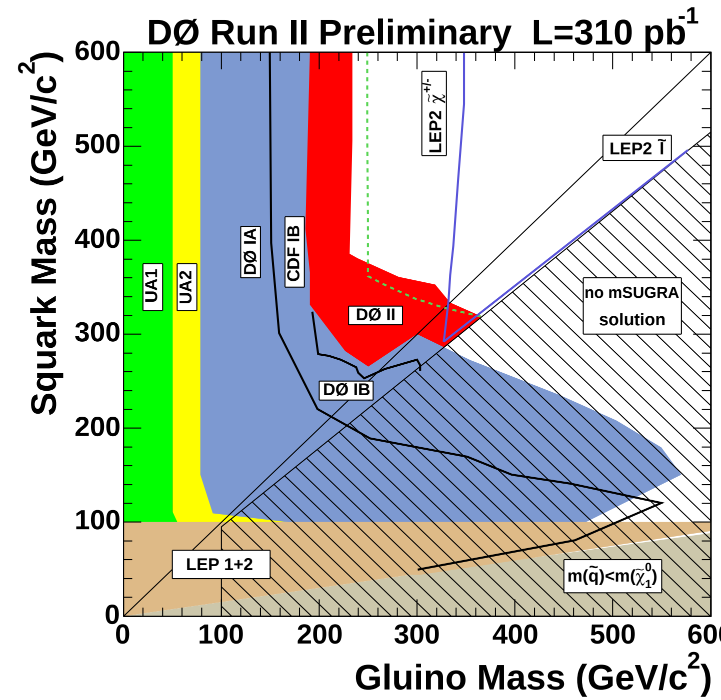
– cross sections large at Tevatron [and LHC]

– decays to jets and LSP

  [additional jets and leptons possible]

– gaugino mass unification only for efficiency

$\Rightarrow$ we know search for inclusive jets plus LSP



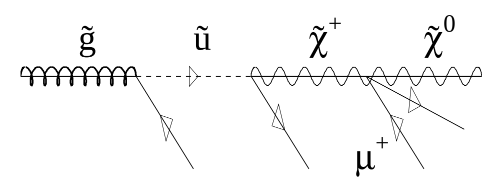
How do we know it is SUSY?

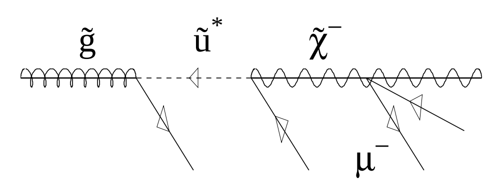
– remember: gluinos Majorana fermions

– jet in gluino decay q or $\bar{q}$

$\Rightarrow$ final–state leptons with both charges

$\Rightarrow$ like–sign dileptons from $\tilde{g}\tilde{g}$ [Barnett, Gunion, Haber]



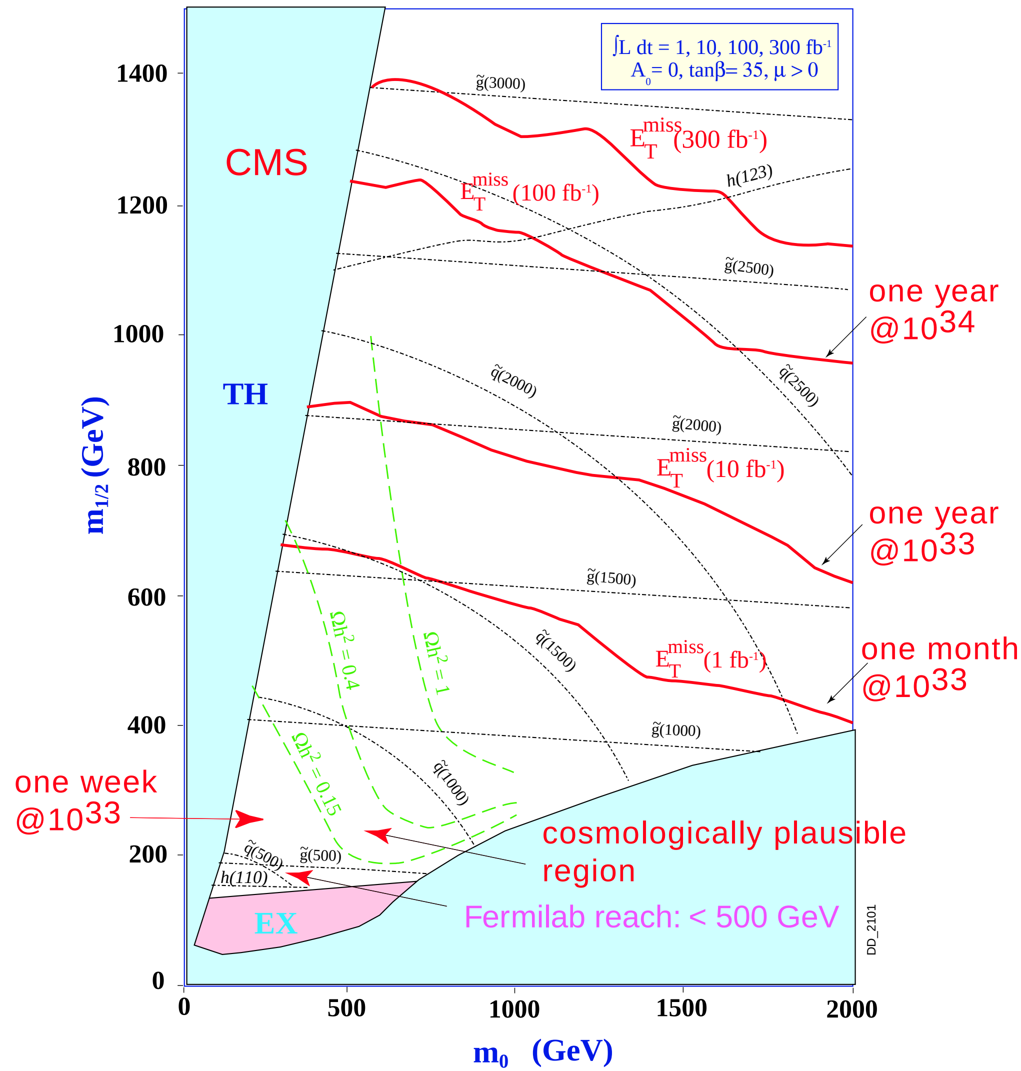


Supersymmetry at the LHC

(1) possible discovery — signals for new physics, exclusion of parameter space

(2) measurements

(3) parameter studies



Supersymmetry at LHC: 1

Supersymmetry at the LHC

(1) possible discovery — signals for new physics, exclusion of parameter space

(2) measurements — masses, cross sections, decays

(3) parameter studies — MSSM Lagrangean, SUSY breaking

SUSY signals include [NLO: Prospino2]

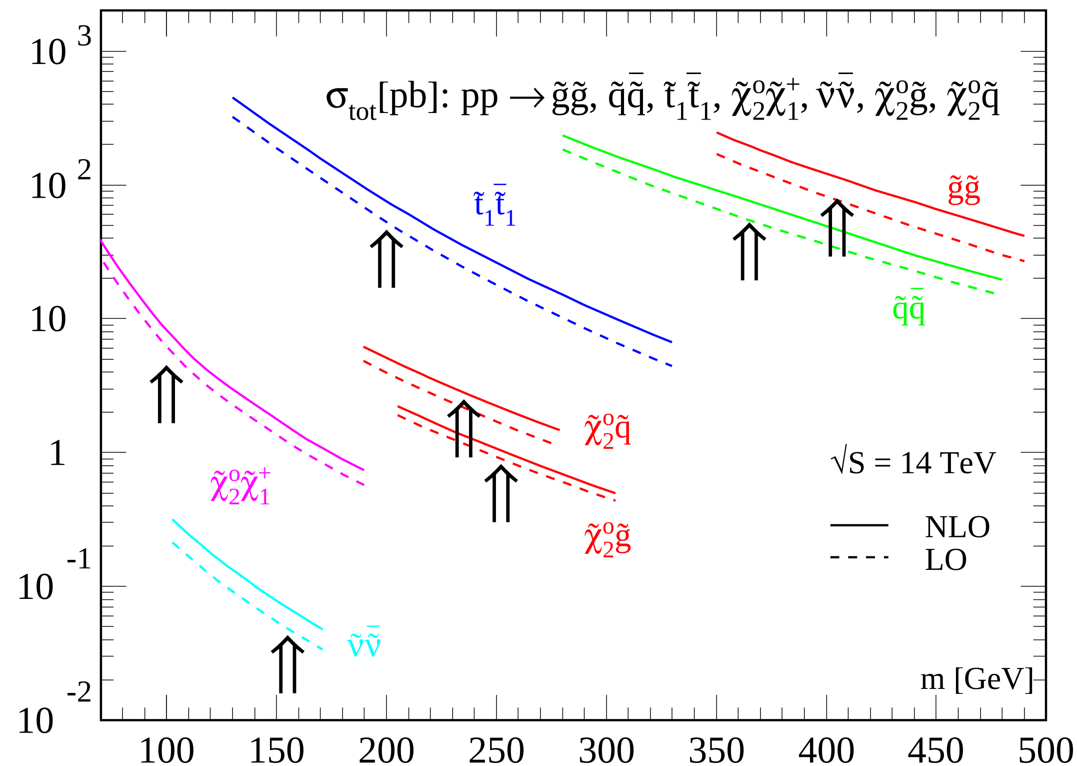
– QCD coupling $g\tilde{q}\tilde{q}, q\tilde{g}\tilde{q}, g\tilde{g}\tilde{g}$

– jets and $\not{E}_T$: $pp \to \tilde{q}\tilde{q}^*, \tilde{g}\tilde{g}, \tilde{q}\tilde{g}$

– funny tops: $pp \to \tilde{t}_1\tilde{t}_1^*$

– tri-leptons: $pp \to \tilde{\chi}_2^0\tilde{\chi}_1^-$

$[\tilde{\chi}_2^0 \to \tilde{\ell}\bar{\ell} \to \tilde{\chi}_1^0\ell\bar{\ell};\ \tilde{\chi}_1^- \to \tilde{\chi}_1^0\ell\bar{\nu}]$



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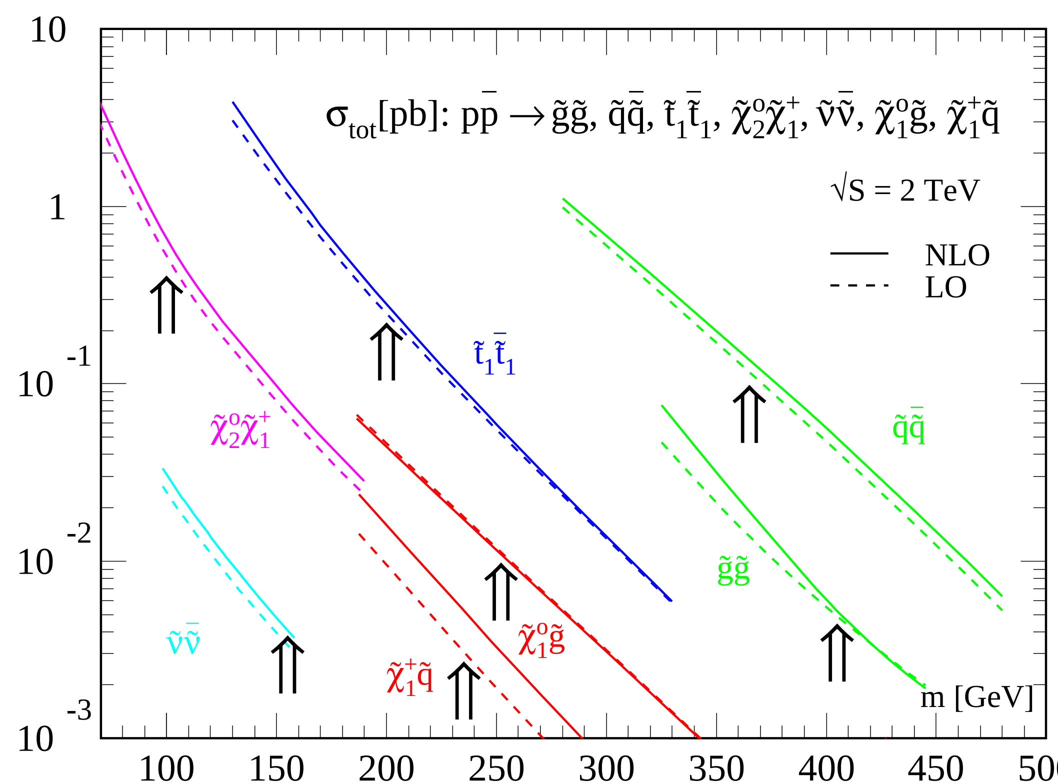

Supersymmetry at the LHC

(1) possible discovery — signals for new physics, exclusion of parameter space

(2) measurements — masses, cross sections, decays

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⇒ at least 10% precision to be matched at LHC [theorist's nightmare, yet unsolved]

Hadron collider observables with errors

⋆ masses from σ_{tot}

⋆ branching fractions from σ_{tot}

– renormalization scale in $\alpha_s, y_{b,t}$

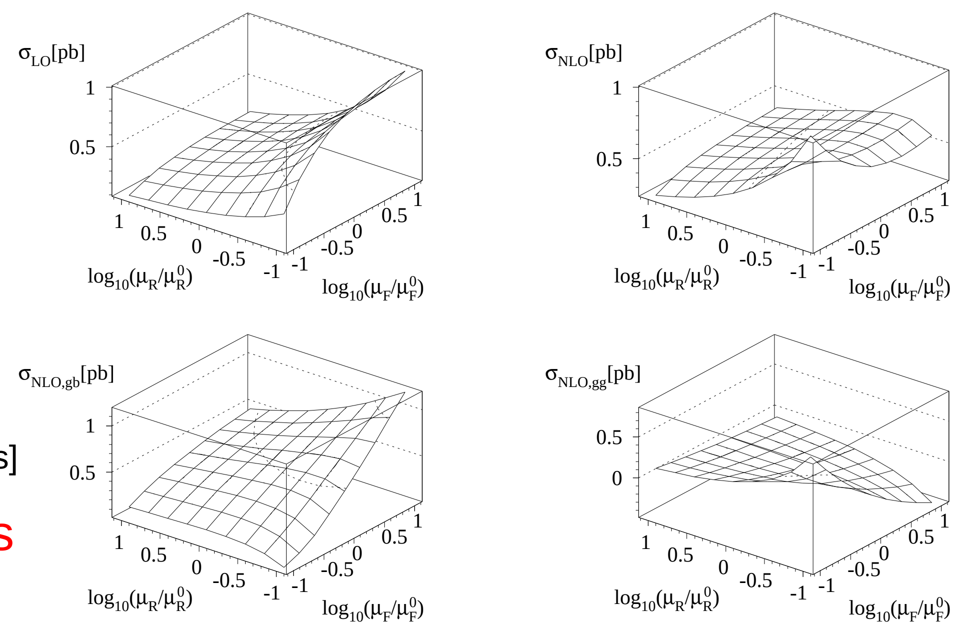
– factorization scale in pdf's

– perturbative series $N_c \alpha_s / \pi \sim 10\%$

– finite terms larger [LO-NLO-NNLO: DY, Higgs]

⇒ NLO errors: 15...40 % for SUSY pairs

[Tevatron limits not possible without NLO]

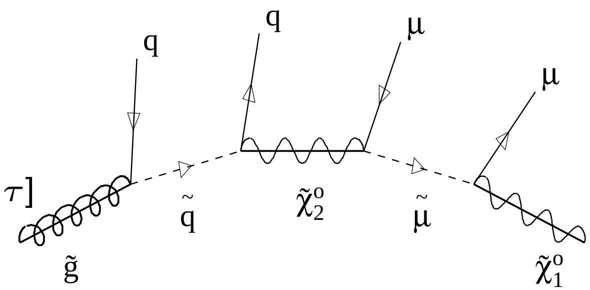


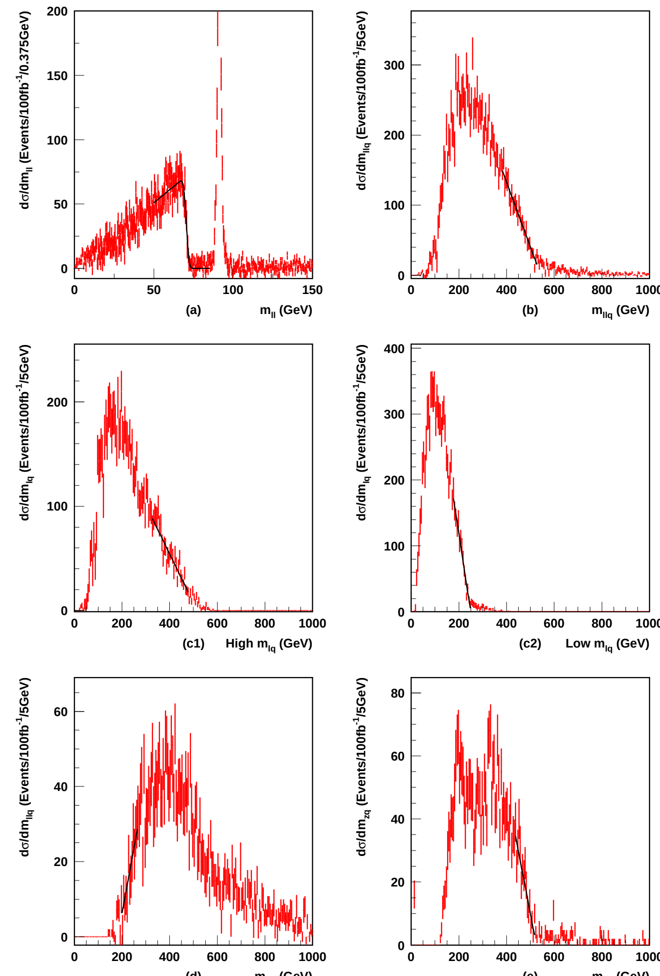
Spectra from cascade decays

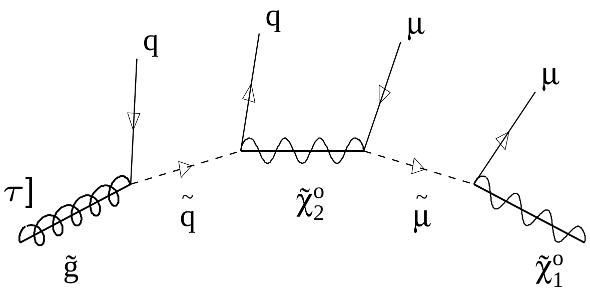
- decay $\tilde{g} \to \tilde{q}\bar{q} \to \tilde{\chi}_2^0 q\bar{q} \to \mu^+\mu^- q\bar{q}\tilde{\chi}_1^0$ [better not via Z or to τ]
- cross sections some 100 pb [more than 3×10^5 events]
- thresholds & edges $m_{\ell\ell}^2 < (m_{\tilde{\chi}_2^0}^2 - m_{\tilde{\ell}}^2)(m_{\tilde{\ell}}^2 - m_{\tilde{\chi}_1^0}^2)/m_{\tilde{\ell}}^2$
- detector resolution, calibration, systematic errors, shape analysis, cross sections as input?

⇒ $\tilde{q}_L$ cascade reconstruction established

[Hinchliffe, Paige,...;Allanach, Parker, Webber,...]







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– detector resolution, calibration, systematic errors, shape analysis, cross sections as input?

⇒ $\tilde{q}_L$ cascade reconstruction established [mass difference even better]

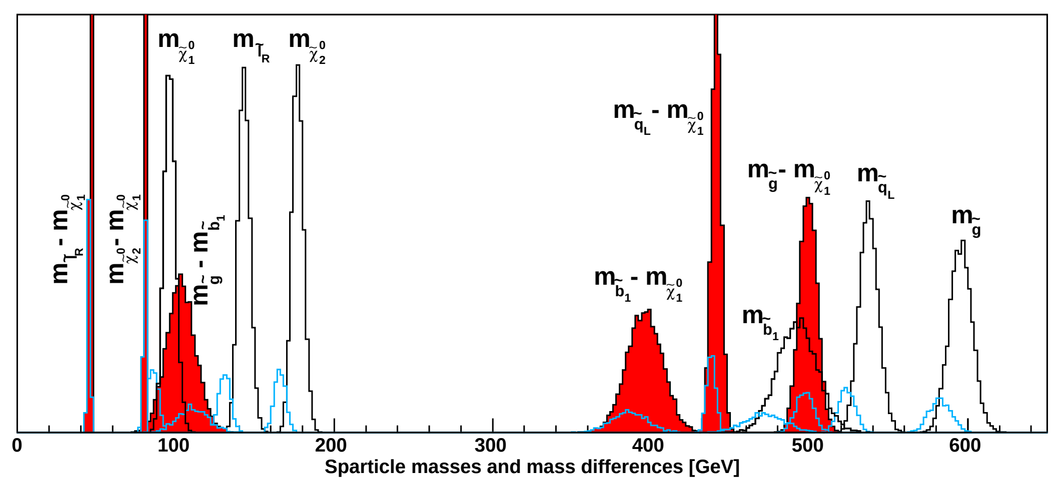
Gluino mass [Gjelsten, Miller, Osland]

– now four jets instead of two

– $\tilde{b}_L$ instead, all jets b-tagged

– most of time: cascade correct

⇒ gluino mass to $\sim$ 1%



Complex final states with Smadgraph [Cho, Hagiwara, Kanzaki, TP, Rainwater, Stelzer]

– Majoranas and fermion number violation in tools like Madgraph

– complete set of Feynman rules [400+ processes compared: Madgraph - Whizard - Sherpa]

Squarks and gluinos always with many jets [TP, Rainwater, Skands]

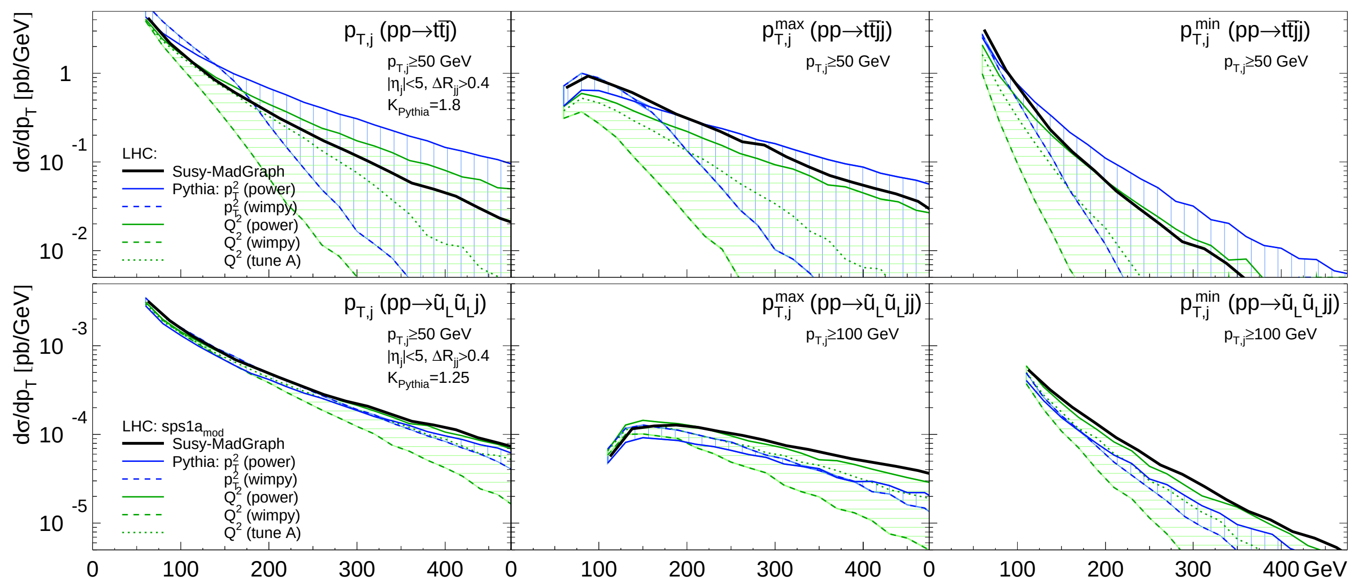
– cascade studies sensitive to jets?

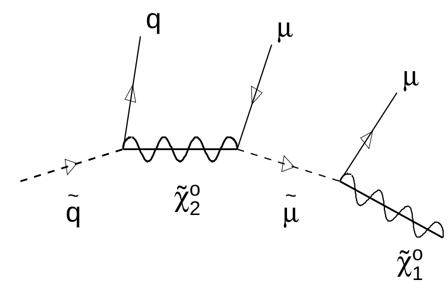
– matrix element $\tilde{g}\tilde{g}$+2j and $\tilde{u}_L\tilde{g}$+2j [$p_{T,j} > 100$ GeV]

– Pythia shower tuned at Tevatron

$\Rightarrow$ SUSY easier than tops?

σ [pb]	$t\bar{t}_{600}$	$\tilde{g}\tilde{g}$	$\tilde{u}_L\tilde{g}$
σ_{0j}	1.30	4.83	5.65
σ_{1j}	0.73	2.89	2.74
σ_{2j}	0.26	1.09	0.85





How to make sure it is SUSY

– assume squark is found in cascades

⇒ strongly interacting scalar? [first stop towards ‘neutralino’]

⇒ straw-man model where squark is a fermion: universal extra dimensions

[Cheng, Dobrescu,...; mass spectra degenerate —ignore this information; cross section factor 10 larger —ignore this as well]

Squark–slepton cascade [Smillie, Webber]

– decay chain $\tilde\chi_2^0 \to \ell\tilde\ell^* \to \ell\bar\ell\tilde\chi_1^0$

– compare with first KK Z and ℓ

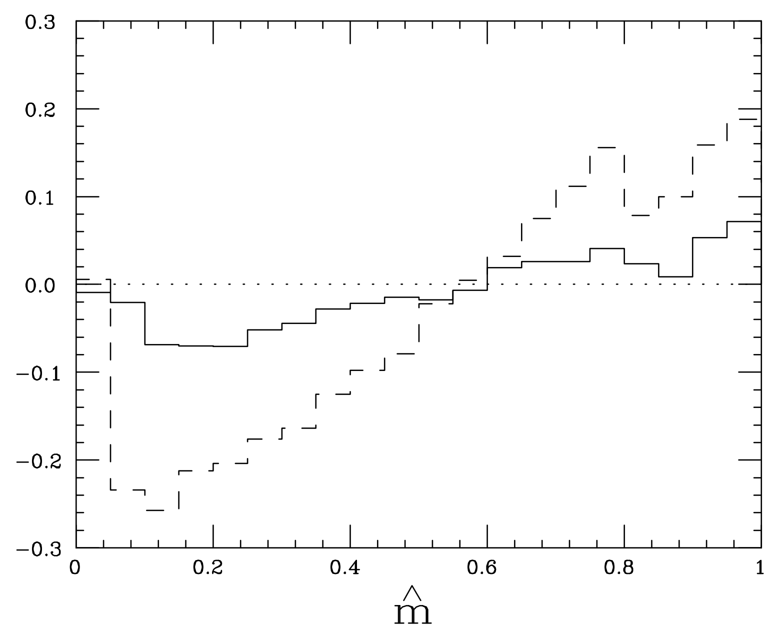
– typically largest pp $\to \tilde q\tilde g$ [$\tilde q : \tilde q^* \sim 1 : 2$]

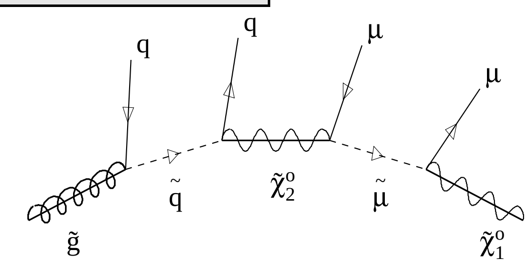
– trick: mass variables, ‘normalized angles’ [Barr]

⇒ $\widehat{m} = m_{j\ell}/m_{j\ell}^{max}$ most promising

$$\mathcal{A} = [\sigma(j\ell^+) - \sigma(j\ell^-)]/[\sigma(j\ell^+) + \sigma(j\ell^-)]$$

⇒ more than proof of feasibility [dashed SUSY]





Show it is SUSY–QCD

- given like–sign dileptons, gluino would be better

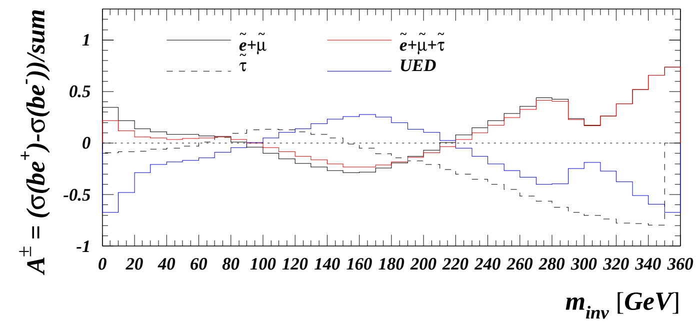
⇒ if fermion, then definitely Majorana [call it gluino]

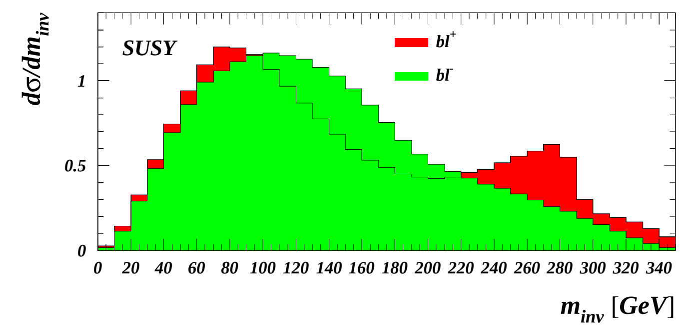
⇒ compare with our straw man

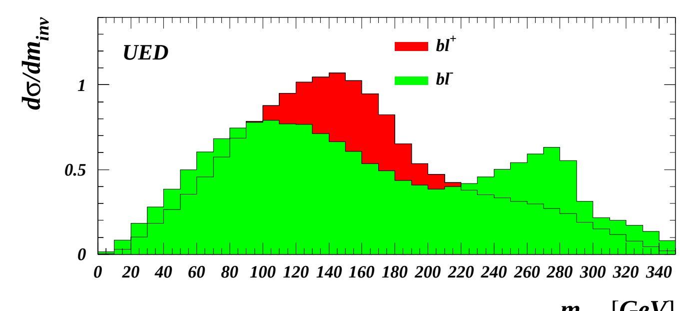
Gluino–bottom cascade [Alves, Eboli, TP]

- decay chain as for gluino mass measurement
- compare with first KK g, q, Z, and ℓ
- replace initial–state asymmetry by b vs. $\bar{b}$
- $\widehat{m} = m_{b\ell}/m_{b\ell}^{max}$
  $\mathcal{A} = [\sigma(b\ell^+) - \sigma(b\ell^-)]/[\sigma(b\ell^+) + \sigma(b\ell^-)]$
- very preliminary

⇒ gluino spin accessible at LHC







SUPERSYMMETRIC PARAMETERS

SUSY parameters from observables [Lafaye, TP, Zerwas; Fittino; Arkani-Hamed,...]

– parameters: weak-scale MSSM Lagrangean

– measurements: masses or edges
  branching fractions
  cross sections

– errors: general correlation, statistics & systematics & theory

– problem in grid: huge phase space, local minimum?
  problem in fit: domain walls, starting values, global minimum?

First go at problem

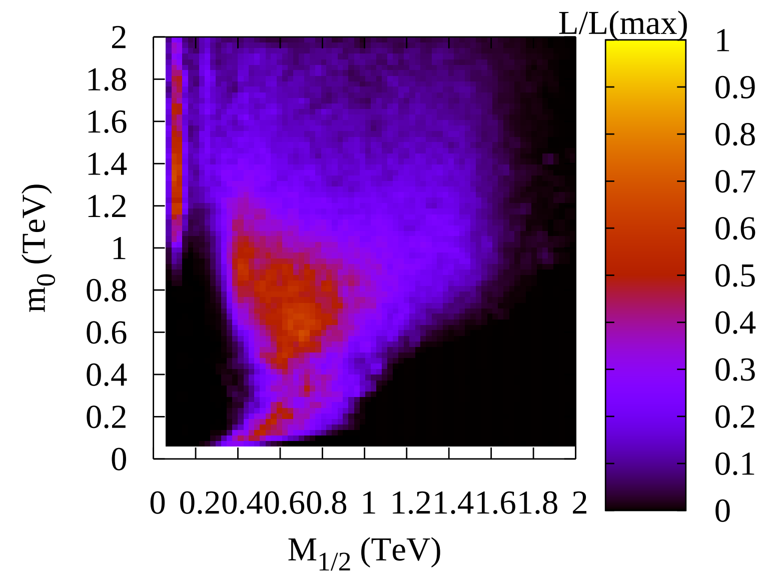
– ask a friend who knows how SUSY is broken

⇒ mSUGRA

– fit $m_0, m_{1/2}, A_0, \tan\beta, \text{sign}(\mu)$

– no problem, include indirect constraints

⇒ likelihood map today [Allanach, Lester]



Supersymmetric Parameters

SUSY parameters from observables [Lafaye, TP, Zerwas; Fittino; Arkani-Hamed,...]

– parameters: weak-scale MSSM Lagrangean

– measurements: masses or edges
  branching fractions
  cross sections

– errors: general correlation, statistics & systematics & theory

– problem in grid: huge phase space, local minimum?
  problem in fit: domain walls, starting values, global minimum?

First go at problem [TP, Lafaye, Zerwas]

– ask a friend who knows how SUSY is broken

⇒ mSUGRA

– fit $m_0, m_{1/2}, A_0, \tan\beta, \mathrm{sign}(\mu)$

– LHC edges or masses?

⇒ edges much more powerful

	SPS1a	ΔLHC masses	ΔLHC edges	ΔILC	ΔLHC+ILC
m_0	100	3.9	1.2	0.09	0.08
$m_{1/2}$	250	1.7	1.0	0.13	0.11
$\tan\beta$	10	1.1	0.9	0.12	0.12
A_0	-100	33	20	4.8	4.3

SUPERSYMMETRIC PARAMETERS

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Combination of methods [TP, Lafaye, Zerwas]

- (1) grid for closed subset
  (2) fit of remaining parameters
  (3) complete fit
- more modern alternaives:
  simulated annealing
  Markov Chains

⇒ LHC+ILC with no assumptions

	LHC	ILC	LHC+ILC	SPS1a
$\tan\beta$	10.22±9.1	10.26±0.3	10.06±0.2	10
M_1	102.45±5.3	102.32±0.1	102.23±0.1	102.2
M_3	578.67±15	fix 500	588.05±11	589.4
$M_{\tilde{\tau}_L}$	fix 500	197.68±1.2	199.25±1.1	197.8
$M_{\tilde{\tau}_R}$	129.03±6.9	135.66±0.3	133.35±0.6	135.5
$M_{\tilde{\mu}_L}$	198.7±5.1	198.7±0.5	198.7±0.5	198.7
$M_{\tilde{q}3_L}$	498.3±110	497.6±4.4	521.9±39	501.3
$M_{\tilde{t}_R}$	fix 500	420±2.1	411.73±12	420.2
$M_{\tilde{b}_R}$	522.26±113	fix 500	504.35±61	525.6
A_τ	fix 0	-202.4±89.5	352.1±171	-253.5
A_t	-507.8±91	-501.95±2.7	-505.24±3.3	-504.9
A_b	-784.7±35603	fix 0	-977±12467	-799.4

OUTLOOK

LHC phenomenology beyond the Standard Model

– Tevatron perfect training ground for SUSY at LHC
– many new ideas: QCD matching, parameter extraction, spin measurement,...
– many new tools: Prospino2, Smadgraph, Sfitter,...
– lots of more work to be done

$\Rightarrow$ experiment and theory have to work together

$\Rightarrow$ LHC will be the coolest experiment ever!

SUPERSYMMETRY AT LHC: 4

Complex final states [Cho, Hagiwara, Kanzaki, TP, Rainwater, Stelzer]

– Majoranas and fermion number violation in tools like Madgraph

– complete set of Feynman rules [400+ processes compared: Madgraph - Whizard - Sherpa]

Squarks and gluinos always with many jets [TP, Rainwater, Skands]

– cascade studies sensitive to jets?

– matrix element $\tilde{g}\tilde{g}$+2j and $\tilde{u}_L\tilde{g}$+2j [$p_{T,j} > 100$ GeV]

– Pythia shower tuned at Tevatron

⇒ SUSY easier then tops

σ [pb]	$t\bar{t}_{600}$	$\tilde{g}\tilde{g}$	$\tilde{u}_L\tilde{g}$
σ_{0j}	1.30	4.83	5.65
σ_{1j}	0.73	2.89	2.74
σ_{2j}	0.26	1.09	0.85