SFitter: Measuring Supersymmetry

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Outline

New physics and jets

Supersymmetric parameter space

LHC measurements

Markov chains

MSSM parameters

If time allows: large extra dimensions

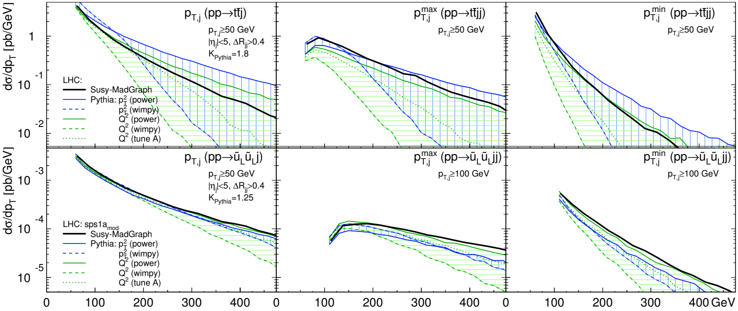
New physics and jets

Just as a side remark: jets and heavy states [Rainwater, TP, Skands]

- squarks and gluinos always with many jets
- cascade studies sensitive to jet activity? [compare to Pythia shower]
- matrix element $\tilde{g}\tilde{g}$+2j and $\tilde{u}_L\tilde{g}$+2j [$p_{T,j} > 100$ GeV]
- hard scale μ_F huge for SUSY
- obvious: $p_{T,j}$ spectra fine with jet radiation
- miracle: angular correlations better than 10%

$\Rightarrow$ QCD not a problem in new–physics signals [as long as particles heavy]

σ [pb]	$t\bar{t}_{600}$	$\tilde{g}\tilde{g}$	$\tilde{u}_L\tilde{g}$
σ_{0j}	1.30	4.83	5.65
σ_{1j}	0.73	2.89	2.74
σ_{2j}	0.26	1.09	0.85



Supersymmetric parameter space

From kinematics to SUSY parameters [Fittino; SFitter: Lafaye, TP, Rauch, Zerwas]

– complex models, including dark matter, flavor physics, low-energy physics,...

– model parameters: weak-scale Lagrangean

– measurements: masses or edges
branching fractions
cross sections

– errors: general correlation, statistics & systematics & theory

– problem in grid: huge phase space, no local maximum?
problem in fit: domain walls, no global maximum?
problem in interpretation: bad observables, secondary maxima?

First go at problem

– ask a friend how SUSY is broken ⇒ mSUGRA

– fit $m_0, m_{1/2}$ [only one best–fitting point]

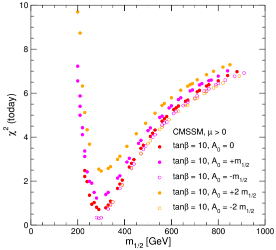
– no problem, include indirect constraints

– best-fitting pre-LHC point [Ellis,...]

– technically trivial [Minuit]

– dominated by dark matter and $(g-2)_\mu$

⇒ no theory bias, except it's mSUGRA



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Same thing for LHC

– ask same friend how SUSY is broken $\Rightarrow$ mSUGRA
– fit $m_0, m_{1/2}, A_0, \tan\beta, y_t, \ldots$
$\Rightarrow$ best–fitting point to LHC/ILC measurements

	SPS1a	$\Delta_{\text{endpoints}}$	Δ_{ILC}	$\Delta_{\text{LHC+ILC}}$	$\Delta_{\text{endpoints}}$	Δ_{ILC}	$\Delta_{\text{LHC+ILC}}$
		exp. errors			exp. and theo. errors		
m_0	100	0.50	0.18	0.13	2.17	0.71	0.58
$m_{1/2}$	250	0.73	0.14	0.11	2.64	0.66	0.59
$\tan\beta$	10	0.65	0.14	0.14	2.45	0.35	0.34
A_0	-100	21.2	5.8	5.2	49.6	12.0	11.3
m_t	171.4	0.26	0.12	0.12	0.97	0.12	0.12

LHC measurements

Simulated LHC measurements in SPS1a

– kinematic endpoints from ascade decays

– statistical error: Gaussian
systematic error (JES, LES): Gaussian [measured in parallel]
theory error: flat [no bias of higher orders]
combination: RFit scheme [same as CKMFitter]

– 15 measurements from LHC

type		nominal	stat	LES	JES	theo
m_h		108.99	0.01	0.25		2.0
m_t		171.40	0.01		1.0	
$m_{\tilde{l}_L} - m_{\chi_1^0}$		102.45	2.3	0.1		2.2
$m_{\tilde{g}} - m_{\chi_1^0}$		511.57	2.3		6.0	18.3
$m_{\tilde{q}_R} - m_{\chi_1^0}$		446.62	10.0		4.3	16.3
$m_{\tilde{g}} - m_{\tilde{b}_1}$		88.94	1.5		1.0	24.0
$m_{\tilde{g}} - m_{\tilde{b}_2}$		62.96	2.5		0.7	24.5
$m_{ll}^{\max}$:	three-particle edge($\chi_2^0, \tilde{l}_R, \chi_1^0$)	80.94	0.042	0.08		2.4
$m_{llq}^{\max}$:	three-particle edge($\tilde{q}_L, \chi_2^0, \chi_1^0$)	449.32	1.4		4.3	15.2
m_{lq}^{low}:	three-particle edge($\tilde{q}_L, \chi_2^0, \tilde{l}_R$)	326.72	1.3		3.0	13.2
$m_{ll}^{\max}(\chi_4^0)$:	three-particle edge($\chi_4^0, \tilde{l}_R, \chi_1^0$)	254.29	3.3	0.3		4.1
$m_{\tau\tau}^{\max}$:	three-particle edge($\chi_2^0, \tilde{\tau}_1, \chi_1^0$)	83.27	5.0		0.8	2.1
m_{lq}^{high}:	four-particle edge($\tilde{q}_L, \chi_2^0, \tilde{l}_R, \chi_1^0$)	390.28	1.4		3.8	13.9
m_{llq}^{thres}:	threshold($\tilde{q}_L, \chi_2^0, \tilde{l}_R, \chi_1^0$)	216.22	2.3		2.0	8.7
m_{llb}^{thres}:	threshold($\tilde{b}_1, \chi_2^0, \tilde{l}_R, \chi_1^0$)	198.63	5.1		1.8	8.0

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– results from χ^2 fit

	LHC	ILC	LHC+ILC	SPS1a
$\tan\beta$	10.0 ± 4.5	12.1 ± 7.0	12.6 ± 6.2	10.0
M_1	102.1 ± 7.8	103.3 ± 1.1	103.2 ± 0.95	103.1
M_2	193.3 ± 7.8	194.1 ± 3.3	193.3 ± 2.6	192.9
M_3	577.2 ± 14.5	fixed 500	581.0 ± 15.1	577.9
$M_{\tilde{\tau}_L}$	$227.8 \pm \mathcal{O}(10^3)$	190.7 ± 9.1	190.3 ± 9.8	193.6
$M_{\tilde{\tau}_R}$	$164.1 \pm \mathcal{O}(10^3)$	136.1 ± 10.3	136.5 ± 11.1	133.4
$M_{\tilde{\ell}_L}$	193.2 ± 8.8	194.5 ± 1.3	194.5 ± 1.2	194.4
$M_{\tilde{\ell}_R}$	135.0 ± 8.3	135.9 ± 0.87	136.0 ± 0.79	135.8
$M_{\tilde{q}3_L}$	481.4 ± 22.0	$499.4 \pm \mathcal{O}(10^2)$	493.1 ± 23.2	480.8
$M_{\tilde{t}_R}$	$415.8 \pm \mathcal{O}(10^2)$	$434.7 \pm \mathcal{O}(10^2)$	412.7 ± 63.2	408.3
$M_{\tilde{b}_R}$	501.7 ± 17.9	fixed 500	502.4 ± 23.8	502.9
$M_{\tilde{q}_L}$	524.6 ± 14.5	fixed 500	526.1 ± 7.2	526.6
$M_{\tilde{q}_R}$	507.3 ± 17.5	fixed 500	509.0 ± 19.2	508.1
A_τ	fixed 0	$613.4 \pm \mathcal{O}(10^4)$	$764.7 \pm \mathcal{O}(10^4)$	-249.4
A_t	-509.1 ± 86.7	$-524.1 \pm \mathcal{O}(10^3)$	-493.1 ± 262.9	-490.9
A_b	fixed 0	fixed 0	$199.6 \pm \mathcal{O}(10^4)$	-763.4
m_A	$406.3 \pm \mathcal{O}(10^3)$	393.8 ± 1.6	393.7 ± 1.6	394.9
μ	350.5 ± 14.5	354.8 ± 3.1	354.7 ± 3.0	353.7
m_t	171.4 ± 1.0	171.4 ± 0.12	171.4 ± 0.12	171.4

$\Rightarrow$ works for MSSM

Markov chains

From kinematics to weak–scale parameters [Fittino; SFitter: Lafaye, TP, Rauch, Zerwas]

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Probability maps of new physics [Baltz,...; Roszkowski,...; Allanach,...; SFitter]

– starting point: probability measure for each continuous model hypothesis
fully exclusive likelihood map $p(d|m)$ over m [hard part]

– LHC problem: remove pathetic directions [e.g. endpoints or dark matter vs rates]

(1) Bayesian: $p(m|d) \sim p(d|m)\, p(m)$ with theorists' bias $p(m)$ [cosmology, BSM]
advantage: proper probability distribution
problem: integration measure needed: $p(m)$
problem: noise from integration over flat directions [volume effects]

(2) frequentist: best–fitting point $\max_m p(d|m)$ [flavor]
advantage: no measure in profile likelihood
advantage: high resolution without noise
problem: size of likelihood peaks arbitrary

– LHC era: (1) compute high-dimensional map $p(d|m)$
(2) find and rank local maxima in $p(d|m)$
(3) Bayesian–frequentist dance to reduce dimensions

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Bayesian or frequentist?

- toy potential $V(\vec{x})$ in 5 dimensions [2 spheres, cigar, 2 cubes]
- best–fitting point: small sphere
  most likely scenatio: large sphere [water in spoon/cloud]
- two-fold SFitter output: list & map

V=74.929 @(655.00,253.72,347.83,348.57,349.59)
V=59.972 @(850.04,224.99,650.00,649.99,654.56)
V=58.219 @(849.97,225.01,587.08,650.01,650.02)
V=25.110 @(750.00,749.99,450.00,450.01,450.01)
V=16.042 @(245.45,253.44,552.51,542.58,544.75)
V=12.116 @(350.70,650.40,650.36,650.40,650.38)
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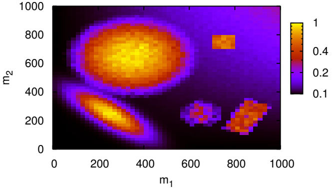

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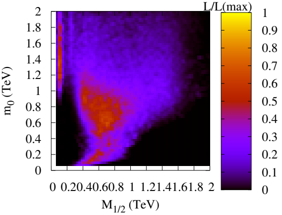
– same for MSUGRA today [Allanach, Cranmer, Lester, Weber]

– 'Which is the most likely parameter point?'
'How does dark matter annihilate/couple?'



Markov chains

Weighted Markov chains [SFitter, Ferrenberg & Swendsen]

– classical: produce representative set of spin states
compute average energy based on this reduced sample

⇒ map (chain) based on probability of a state
expensive energy function on sample

– BSM physics: produce map $p(m|d)$ of parameter points
evaluate same probability from (binned) density
typical problem: two bins with probability 10% : 90%

⇒ weighted Markov chains [like weighted Monte Carlo]

– binning weighted events without double counting

$$P_{\text{bin}}(p \neq 0) = \frac{N}{\sum_{i=1}^{N} 1/p}$$

– MSUGRA: error dominated by weighted events

– MSSM: error dominated by zero region? [at some point...]

– already for mSUGRA: MCMC resolution not sufficient

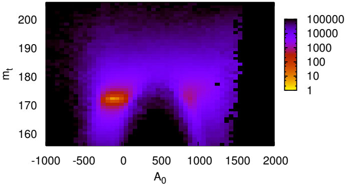
⇒ use additional probability maximization to rank maxima

MSSM parameters

Toy model: MSUGRA map from LHC [LHC endpoints with free y_t]

– SFitter output #1: fully exclusive likelihood map
  SFitter output #2: ranked list of local maxima

– strong correlation e.g. of A_0 and y_t [including all errors]



χ^2	m_0	$m_{1/2}$	$\tan\beta$	A_0	μ	m_t
0.3e-04	100.0	250.0	10.0	-99.9	+	171.4
27.42	99.7	251.6	11.7	848.9	+	181.6
54.12	107.2	243.4	13.3	-97.4	-	171.1
70.99	108.5	246.9	13.9	26.4	-	173.6
88.53	107.7	245.9	12.9	802.7	-	182.7
...						

⇒ correlations and secondary maxima significant

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MSSM map from LHC

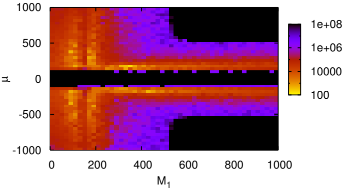
– shifting from 6D to 19D parameter space [killing grids, Minuit, laptop–style fits...]

– SFitter outputs #1 and #2 still the same [weighted Markov chain plus hill climber]

– 1. Markov chain + Minuit over entire parameter space [flat proposal]
2. high–res Markov chain + Minuit over $M_i, \mu \tan\beta, m_t$ [flat proposal]
3. high–res Markov chain + Minuit over orthogonal space [Breit–Wigner proposal]
4. Minuit over all parameters

– three neutralinos observed
4 solutions for M_1, M_2, μ
2 solutions for $\pm|\mu|$
2 solutions for $\pm|A_t|$

$\Rightarrow$ secondary maxima degenerate in MSSM



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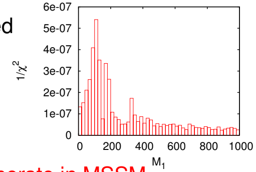

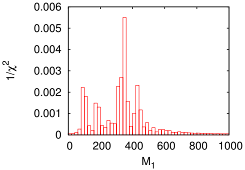

$\Rightarrow$ secondary maxima degenerate in MSSM

$\Rightarrow$ no perfect statistical approach

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Theorists' goal [SFitter + Kneur]

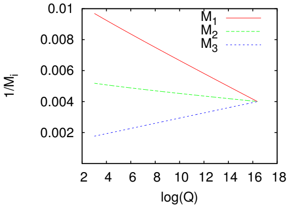
– unification and supersymmetry

– test mass unification with errors [Cohen, Schmalz]

– properly: RGE running bottom–up

– error analysis yet missing

⇒ LHC: fundamental physics from weak scale



If time allows: large extra dimensions

Also solving the hierarchy problem [Arkani-Hamed, Dimopoulos, Dvali]

– weak gravity = large Planck scale $G_N \sim 1/M_{\text{Planck}}^2$ [$M_{\text{Planck}} \sim 10^{19}$ GeV]

– Einstein–Hilbert action in $4+n$ dimensions [on torus — periodic boundaries]

$$\int d^4x \sqrt{|g|}\, M_{\text{Planck}}^2 R \to \int d^{4+n}x \sqrt{|g|}\, M_*^{2+n} R = (2\pi r)^n \int d^4x \sqrt{|g|}\, M_*^{2+n} R$$

$$M_{\text{Planck}} = M_* \; (2\pi r M_*)^{n/2} \gg M_* \sim 1 \text{ TeV}$$

– to get numbers right: $r = 10^{12}, 10^{-3}, \dots 10^{-11}$ m for $n = 1, 2, \dots 6$

$\Rightarrow$ fundamental Planck scale at TeV

Kaluza–Klein gravitons

– Fourier–transform extra dimensions [QCD massless]

$$(\Box + m_k^2)\; G_{\mu\nu}^{(k)} = -\frac{T_{\mu\nu}}{M_{\text{Planck}}} \qquad \delta m \sim \frac{1}{r} = 2\pi M_* \left(\frac{M_*}{M_{\text{Planck}}}\right)^{2/n} \lesssim 0.05 \text{ GeV}$$

– graviton couplings to quarks and gluons

$$f(k_1) - f(k_2) - G_{\mu\nu}: \quad -\frac{i}{4M_{\text{Planck}}}\left(W_{\mu\nu} + W_{\nu\mu}\right) \qquad \text{with} \quad W_{\mu\nu} = (k_1 + k_2)_\mu \gamma_\nu$$

$\Rightarrow$ single gravitons tightly spaced and coupled as $1/M_{\text{Planck}}$

Large extra dimensions

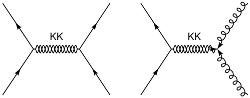
Hope for collider searches

– real radiation of continuous KK tower $[dm/d|k| \sim 1/r]$

$$\sigma^{\text{tower}} \sim \sigma^{\text{graviton}} \int dm\, S_{n-1} m^{n-1} r^n = \sigma^{\text{graviton}} \int dm\, \frac{S_{n-1}\, m^{n-1}}{(2\pi M_*)^n} \left(\frac{M_{\text{Planck}}}{M_*} \right)^2$$

– higher-dimensional operator from virtual gravitons

$$\mathcal{A}(s; m) = \frac{1}{M^2_{\text{Planck}}} T_{\mu\nu} T^{\mu\nu} \frac{1}{s - m^2} \to \frac{S_{n-1}}{2M_*^4} \left(\frac{\Lambda}{M_*} \right)^{n-2}$$



⇒ $1/M_*$ coupling for KK tower

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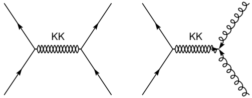

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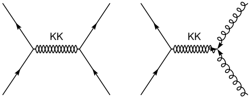


⇒ $1/M_*$ coupling for KK tower

Virtual gravitons at LHC

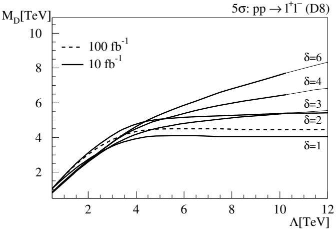
– s–channel $gg \to \mu^+\mu^-$

– LHC rates (or reach) dependent on cut-off Λ

– effective theory not useful at LHC

⇒ UV completion necessary

[Antoniadis, Benakli, Laugier; Cullen, Perelstein, Peskin,...]



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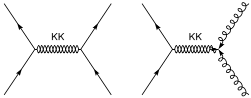

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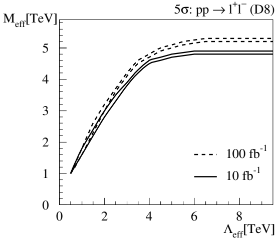


Large extra dimensions

Renormalization flow of gravity [Reuter,...; Litim,...]

– dimensionless coupling $g(\mu) = G(\mu)\mu^{2+n} = G_0 Z_G^{-1}(\mu)\mu^{2+n}$

– UV fixed point [anomalous dimension: $\eta = -\mu\partial_\mu \log Z_G \propto g$]

$$\mu\frac{\partial}{\partial\mu} g(\mu) = (2+n+\eta(g))\; g(\mu) = 0 \qquad \text{for} \qquad g \neq 0 \qquad \eta(g) = -2-n$$

– asymptotic safety $G(\mu) \sim Z_G^{-1} \sim \mu^{-(2+n)} \to 0$ [Weinberg]

$\Rightarrow$ gravity weak enough for LHC predictions?

Graviton propagator [Litim, TP; Hewett & Rizzo]

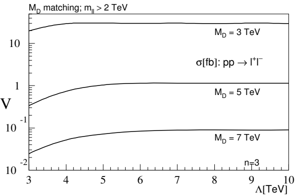
– iterative approach: start with anomalous dimension [similar to QCD analyses]

– UV: dressed scalar propagator [$1/(Z_G(|p|)\; p^2) \sim 1/p^{4+n}$]

$$P(s,m) = \begin{cases} \dfrac{1}{s-m^2} & m < \Lambda_\text{trans} \sim M_* \\[2ex] \dfrac{M_*^{n+2}}{(s-m^2)^{n/2+2}} & m > \Lambda_\text{trans} \sim M_* \end{cases}$$

– fixed point regularizing integrated $\mathcal{A} = \mathcal{A}_\text{IR} + \mathcal{A}_\text{UV}$

$\Rightarrow$ LHC sensitive to UV completions



Supersymmetry at the LHC

TeV–scale new physics

- know there is BSM physics
- trust solution of hierarchy problem
- explain dark matter

Theory/Phenomenology in the LHC era

(1) look for solid new–physics signals [missing energy?]

(2) measure weak–scale Lagrangian [highD parameter spaces?]

(3) determine fundamental physics

- test discrete new–physics properties
- construct sensible new–physics hypotheses
- avoid getting killed by QCD
- never talk about CMSSM analyses again

⇒ LHC more than a discovery machine!

Jets

Parameters

Measurements

Markov chains

MSSM

Extra dimensions