Phenomenology 4: Extra Dimensions

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RAL School, Oxford, 9/2007

Outline

Large extra dimensions

Large extra dimensions at the LHC

Warped extra dimensions

Warped extra dimensions at the LHC

Large extra dimensions: 1

Remember the hierarchy problem

– fundamental scalars cannot deal with a high scale in theory

– weakness of gravitational interaction means large Planck scale $G_N = 1/(16\pi M_{\text{Planck}})^2$

$\Rightarrow$ solution: there is another reason why we see a huge M_{Planck}

Large extra dimensions (ADD)

– Einstein–Hilbert action for fundamental Planck scale

$$S = -\frac{1}{2}\int d^4x\sqrt{|g|}\,M_*^2\,R \to -\frac{1}{2}\int d^{4+n}x\sqrt{|g|}\,M_*^{2+n}\,R$$

– compactify additional dimensions on torus

$$S = -\frac{1}{2}\int d^{4+n}x\sqrt{|g|}\,M_*^{2+n}\,R = -\frac{1}{2}(2\pi r)^n\int d^4x\sqrt{|g|}\,M_*^{2+n}\,R$$

– match the two theories on our brane [also: match to measurements]

$$-\frac{1}{2}(2\pi r)^n\int d^4x\sqrt{|g|}\,M_*^{2+n}\,R \equiv -\frac{1}{2}\int d^4x\sqrt{|g|}\,M_{\text{Planck}}^2\,R$$

$\Rightarrow$ express the 4D Planck scale in terms of fundamental Planck scale

$$M_{\text{Planck}} = M_*(2\pi r M_*)^{n/2}$$

Large extra dimensions: 1

Remember the hierarchy problem

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$\Rightarrow$ solution: there is another reason why we see a huge M_{Planck}

Numbers to make it work

– wanted $rM_* \gg 1$

– constraints from gravity tests above $\mathcal{O}$(mm)

– $M_* = 1$ TeV $\ll M_{\text{Planck}}$ fine for $n \gtrsim 2$

n	r
1	10^{12} m
2	10^{-3} m
3	10^{-8} m
...	...
6	10^{-11} m

$\Rightarrow$ signatures of strong gravitation in extra dimension?

Large extra dimensions: 2

Only gravitons in extra dimensions

– expand the metric in $(4+n)$ dimensions [graviton field h]

$$ds^2 = g_{MN}^{(4+n)} dx^M dx^N = \left(\eta_{MN} + \frac{1}{M_*^{n/2+1}} h_{MN} \right) dx^M dx^N$$

– include matter into Einstein's equation

$$R_{AB} - \frac{1}{2+n} g_{AB} R = -\frac{1}{M_*^{2+n}} \begin{pmatrix} T_{\mu\nu}(x)\, \delta^{(n)}(y) & 0 \\ 0 & 0 \end{pmatrix}$$

– Fourier transformation of extra dimensions [KK excitations for periodic boundary conditions]

$$h_{AB}(x;y) = \sum_{m_1=-\infty}^{\infty} \cdots \sum_{m_j=-\infty}^{\infty} \frac{h_{AB}^{(m)}(x)}{\sqrt{(2\pi r)^n}} e^{i \frac{m_j y_j}{r}}$$

– only the interacting (tensor) graviton [$h_{AB} \to G_{\mu\nu}$, QCD massless]

$$(\Box + m_k^2)\, G_{\mu\nu}^{(k)} = \frac{1}{M_{\text{Planck}}} \left[-T_{\mu\nu} + \left(\frac{\partial_\mu \partial_\nu}{\hat{m}^2} + \eta_{\mu\nu} \right) \frac{T_\lambda^\lambda}{3} \right] = \frac{-T_{\mu\nu}}{M_{\text{Planck}}}$$

– KK mass splitting [$M_* = 1$ TeV]

$$\delta m \sim \frac{1}{r} = 2\pi M_* \left(\frac{M_*}{M_{\text{Planck}}} \right)^{2/n} = \begin{cases} 0.003 \text{ eV} & (n=2) \\ 0.1 \text{ MeV} & (n=4) \\ 0.05 \text{ GeV} & (n=6) \end{cases}$$

Large extra dimensions at the LHC: 1

Gravitons for LHC phenomenologists

– tower of KK tensor gravitons $G^{(k)}_{\mu\nu}$ with mass m_k

– mass splitting $\delta m \ll$ GeV [below mass resolution]

– universal couplings to massless SM particles via $-T_{\mu\nu}/M_\text{Planck}$

$$f(k_1)-f(k_2)-G_{\mu\nu}: \quad -\frac{i}{4M_\text{Planck}}(W_{\mu\nu}+W_{\nu\mu}) \qquad \text{with} \quad W_{\mu\nu}=(k_1+k_2)_\mu\gamma_\nu$$

$\Rightarrow$ KK gravitons light and weakly coupled

Hope for collider searches

– real radiation of continuous KK tower [$dm/d|k| = 1/r$; $(d\sigma) \propto 1/M^2_\text{Planck}$]

$$d\sigma^\text{tower} = (d\sigma)\int dm\, S_{\delta-1} m^{n-1} r^n = (d\sigma)\int dm\, \frac{S_{\delta-1}\, m^{n-1}}{(2\pi M_*)^n}\left(\frac{M_\text{Planck}}{M_*}\right)^2$$

– higher-dimensional operator from virtual graviton exchange [s–channel in DY]

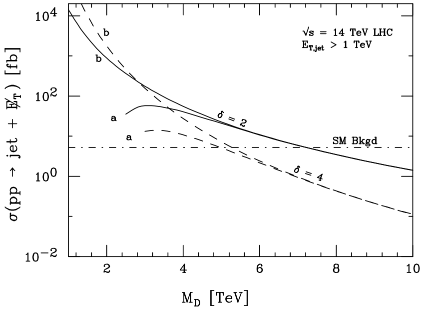
$$\mathcal{A} = \frac{1}{M^2_\text{Planck}} T_{\mu\nu}T^{\mu\nu}\frac{1}{s-m^2_\text{KK}} \sim \frac{S_{\delta-1}}{2}\frac{\Lambda^{n-2}}{M_*^{n+2}}$$

$\Rightarrow$ $1/M_*^2$ interactions after integration over KK tower

Large extra dimensions at the LHC: 2

Gravitons radiation

- off single–jet production [huge rate]
- off DY production [precise knowledge]
- background: radiation of $Z \to \nu\bar{\nu}$ [measure $Z \to \ell\ell$ and subtract]



$\Rightarrow$ jet channel no challenge at LHC

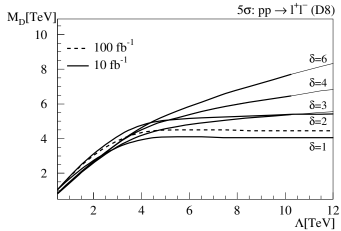
Large extra dimensions at the LHC: 2

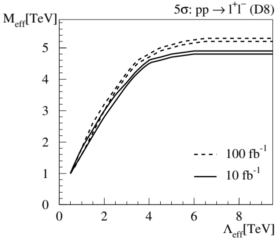
Gravitons radiation

– off single–jet production [huge rate]
– off DY production [precise knowledge]
⇒ jet channel no challenge at LHC

Virtual gravitons

– s channel $gg \to \mu^+\mu^-$ new at LHC
– s channel $gg \to$ jets useless [QCD background uncertainty huge]
– effective field theory in M_{Planck} [$1/M_{\text{eff}}^4$ better]





Warped extra dimensions: 1

Alternative Solution

– try one extra dimension, but not flat [TeV brane at $y = b$]

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 \quad \Leftrightarrow \quad g_{AB} = \begin{pmatrix} e^{-2k|y|}\eta_{\mu\nu} & 0 \\ 0 & \eta_{jk} \end{pmatrix}$$

– integration measure in our usual Lagrangian $d^4\tilde{x} e^{-4kb}, \tilde{g}_{\mu\nu} = \eta_{\mu\nu}$

$$S = \int dy \delta(y)\, d^4\tilde{x}\, e^{-4kb}\,\mathcal{L} = \int d^4\tilde{x}\, e^{-4kb} \left[|D_\mu H|^2 - \lambda(|H|^2 - v^2)^2 + ... \right]$$

– write effecive 4D theory on TeV brane scaling all fields

$$\tilde{H} = e^{-kb} H \qquad \text{scalars}$$

$$\tilde{A}_\mu = e^{-kb} A_\mu \qquad \text{or } \tilde{D}_\mu = e^{-kb} D_\mu$$

$$\tilde{\Psi} = e^{-3kb/2}\Psi \qquad \text{fermions}$$

$$\tilde{m} = e^{-kb} m$$

$$\tilde{v} = e^{-kb} v$$

– assume $kb \sim 35$ and large $M^* \sim k \sim M_\text{Planck}$

$\Rightarrow$ mass scale on TeV brane shifted

$$\tilde{v} \sim 0.1\, e^{-kb}\, M_\text{Planck} \sim 0.1\ \text{TeV}$$

Warped extra dimensions: 2

Gravitons in one warped extra dimension

– re-write the metric including 4D graviton

$$ds^2 = \frac{1}{(1+kz)^2} \left(\eta_{\mu\nu} + h_{\mu\nu}(x,z)\, dx^\mu dx^\nu - dz^2 \right)$$

– solve Einstein's equations separating variables $\tilde{h}_{\mu\nu}(x,z) = \hat{h}_{\mu\nu}(x)\Phi(z)$

$$\partial_\mu \partial^\mu \hat{h}_{\mu\nu} = m^2 \hat{h}_{\mu\nu}$$

$$-\partial_z^2 \Phi + \frac{15}{4} \frac{k^2}{(k|z|+1)^2} \Phi = m^2 \Phi$$

⇒ Bessel functions, masses given by roots $J_1(x_j) = 0$ [Neumann boundary conditions]

$$m_j = x_j\, k\, e^{-kb} \qquad x_j = 3.8, 7.0, 10.2, 16.5, ...$$

– couplings via wave-function overlap in z [approximately, neglect Bessel functions]

$$\frac{\Phi(z)\big|_{\text{TeV}}}{\Phi(z)\big|_{\text{Planck}}} \sim \frac{\sqrt{kz+1}\big|_{\text{Planck}}}{\sqrt{kz+1}\big|_{\text{TeV}}} \sim \frac{1}{\sqrt{e^{ky}}\big|_{\text{TeV}}} \sim \frac{1}{e^{kb/2}}$$

⇒ universal couplings except for zero mode graviton

$$\mathcal{L} \sim \frac{1}{M_{\text{Planck}}} T^{\mu\nu} h^{(0)}_{\mu\nu} + \frac{1}{M_{\text{Planck}} e^{-kb}} T^{\mu\nu} \sum h^{(m)}_{\mu\nu}$$

Warped extra dimensions at the LHC

TeV–scale resonances to e.g. leptons, revisited...

Extra Dimensions at the LHC

Extra dimensions alternative scenario for LHC

- interesting new model
- signal: missing energy or resonances
- no challenge for LHC trigger
- identification of model parameters?