Phenomenology 1: QCD

Tilman Plehn

MPI für Physik & University of Edinburgh

Campos do Jordao, 2/2007

Outline

LHC — Star on the Physics Sky

QCD for New Physics Searchers

Important Standard–Model Processes

Jet Radiation

LHC basics: 1

Most powerful collider ever built

- inside 27 km LEP tunnel [used to be e^+e^- with 200 GeV, not pp with 14 TeV]
- proton–proton collisions [Tevatron: proton–antiproton at 2 TeV]
- some technical details [probably outdated]

Colliders	$\sqrt{s}$ (TeV)	$\mathcal{L}$ (cm^{-2}s^{-1})	$\delta E/E$	f (MHz)	n/bunch (10^{10})	L (km)
Tevatron	1.96	2.1×10^{32}	9×10^{-5}	2.5	p: 27, $\bar{p}$: 7.5	6.28
HERA	314	1.4×10^{31}	0.1, 0.02%	10	e: 3, p: 7	6.34
LHC	14	10^{34}	0.01%	40	10.5	26.66
SSC†	40	10^{33}	5.5×10^{-5}	60	0.8	87

- hadronic c.m. energy divided into partons [first approximation $p = (uud)$]
- luminosity: $N_{\text{ev}} = \sigma \mathcal{L}$ with $\mathcal{L} \propto f\, n_1 n_2 / a$ [measured in $\text{cm}^{-2}\text{s}^{-1} = 10^{-39}\text{fb}^{-1}\text{s}^{-1}$]
- energy spread limited by dipole magnets and cavities

Aim of the LHC

- HERA without physics beyond Standard Model [built to measure proton structure]
- LEP2 without physics beyond Standard Model [LEP1 built to study electroweak gauge sector]
- Tevatron with top quark discovery [it ain't over yet!]
- LHC built to find Higgs and other new effects!

LHC basics: 2

Computing what goes on at the LHC

– incoming protons resolved at TeV-scale energy

– to first approximation described by valence quarks (*uud*)

$$\sigma_{AB} = \sum_{a,b} \int dx_a dx_b \; f_{a/A}(x_a) f_{b/B}(x_b) \; \hat{\sigma}_{ab}$$

(1) parton density $f_{a/A}(x_a)$:
probability to find parton a of momentum fraction x_a in proton A

– parton densities measured at other experiments and at LHC

(2) partonic cross section $\hat{\sigma}_{ab}$:
just like e^+e^-, but with incoming quarks and gluons ['hard process']

– partonic cross section calculated (numerically) for any imaginable final state

LHC basics: 3

Kinematics at a hadron collider

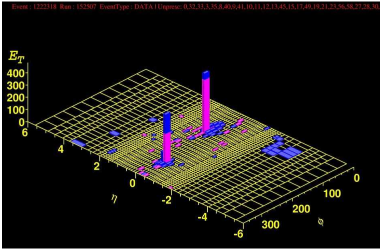
- incoming momenta in lab frame: $P_{a,b} = x_{a,b} \left(E_{A,B}, 0, 0, p_{A,B} \right)$ [partons massless]
- center-of-mass frame moving in z direction
- cylindrical coordinates $d^3\vec{p} = p_T d\phi \, dp_T \, dp_z$ [transverse momentum $p_T = \sqrt{p_x^2 + p_y^2}$]
- (1) usual event sample: ϕ symmetry

  (2) transverse momentum and mass: $E_T = \sqrt{p_T^2 + m^2}$

  (3) rapidity

$$y = \frac{1}{2} \log \frac{E + p_z}{E - p_z} \qquad y_{\text{boosted}} = y + y_{\text{z-shift}} \qquad |y_{\text{observed}}| \lesssim 5$$

- lego plot (p_T, y, ϕ) or (E_T, η, ϕ) [$\eta = \log \cot \theta/2$]
- distance between two tracks $\Delta R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}$



LHC basics: 3

Kinematics at a hadron collider

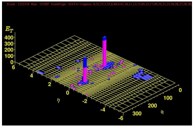
- incoming momenta in lab frame: $P_{a,b} = x_{a,b} \left(E_{A,B}, 0, 0, p_{A,B} \right)$ [partons massless]
- center-of-mass frame moving in z direction
- cylindrical coordinates $d^3\vec{p} = p_T d\phi \, dp_T \, dp_z$ [transverse momentum $p_T = \sqrt{p_x^2 + p_y^2}$]
- (1) usual event sample: ϕ symmetry

  (2) transverse momentum and mass: $E_T = \sqrt{p_T^2 + m^2}$

  (3) rapidity

$$y = \frac{1}{2} \log \frac{E + p_z}{E - p_z} \qquad y_{\text{boosted}} = y + y_{\text{z}-\text{shift}} \qquad |y_{\text{observed}}| \lesssim 5$$

- lego plot (p_T, y, ϕ) or (E_T, η, ϕ) [$\eta = \log \cot \theta/2$]
- distance between two tracks $\Delta R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}$



Useful observables

- invariant mass $(p_{\ell,1} + p_{\ell,2})^2 = m_{\ell\ell}^2 \sim m_Z^2$
- transverse mass [massless invisible neutrino]

$$\begin{aligned} m_{e\nu}^2 &= (E_e + E_\nu)^2 - (\vec{p}_{T,e} + \vec{p}_{T,\nu})^2 - (\vec{p}_{z,e} + \vec{p}_{z,\nu})^2 \\ &> (E_{T,e} + E_{T,\nu})^2 - (\vec{p}_{T,e} + \vec{p}_{T,\nu})^2 \\ &= (E_{T,e} + |\not{\vec{p}}_T|)^2 - (\vec{p}_{T,e} + \not{\vec{p}}_T)^2 = m_{T,e\nu}^2 \end{aligned}$$

LHC basics: 4

Trigger

- flood of data from each LHC crossing not manageable [event rate 1 GHz with 1 MB]
- use some prejudice to write 'interesting' events on tape [realistic 100 Hz]
- LHC is not built to test QCD! [interesting events $1/10^6$]
- soft jets, multiple interactions,... in garbage
- leptons, photons, hard jets, missing momentum,... of high priority
  open problems: invisible Higgs decays, stable particles, super-fast top jets,...

$\Rightarrow$ trigger menue $T(t, k, \$)$ [a function of time, knowledge and money]

Level-1 trigger menue from Atlas [down to 10^5 Hz, always outdated]

	Atlas	
Objects	$\lvert\eta\rvert^{\max}$	$p_T^{\min}$ (GeV)
μ inclusive	2.4	6 (20)
e/photon inclusive	2.5	17 (26)
two e's or two photons	2.5	12 (15)
1-jet inclusive	3.2	180 (290)
3 jets	3.2	75 (130)
4 jets	3.2	55 (90)
τ/hadrons	2.5	43 (65)
$\not{p}_T$	4.9	100
jets$+\not{p}_T$	3.2, 4.9	50,50 (100,100)

QCD: 1

Simple LHC process: $pp \to Z(+\text{jets})$

– Drell–Yan process: single gauge boson production [could be W, Z with $Z \to \ell\ell$]

– what is the final state in hadronic σ_{AB}?
additional jets: $\sigma(pp \to Z)$ jet-inclusive or no jets? [important later]

– to get an idea: compute $q\bar{q} \to Z$ and $q\bar{q} \to Zg$

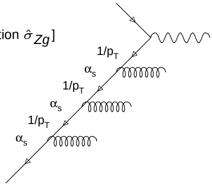
– phase space integration over gluon divergent

Initial state radiation

– gluon off incoming quark with p_{Tj}

$\Rightarrow$ finite–energy jet spectrum IR-divergent [already partonic cross section $\hat{\sigma}_{Zg}$]

$$\frac{d\sigma_{Zg}}{d\,p_{Tj}} \propto \frac{p_{Tj}}{p_{Tj}^2 + m_j^2} = \frac{1}{p_{Tj}} \quad \Rightarrow \quad \sigma_{Zg} \propto \log \frac{p_{Tj}^{\max}}{p_{Tj}^{\min}}$$



QCD: 1

Simple LHC process: $pp \to Z(+\text{jets})$

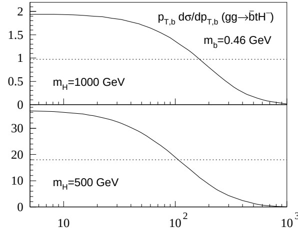
- Drell–Yan process: single gauge boson production [could be W, Z with $Z \to \ell\ell$]
- what is the final state in hadronic σ_{AB}?
  additional jets: $\sigma(pp \to Z)$ jet-inclusive or no jets? [important later]
- to get an idea: compute $q\bar{q} \to Z$ and $q\bar{q} \to Zg$
- phase space integration over gluon divergent

Initial state radiation

- gluon off incoming quark with p_{Tj}

$\Rightarrow$ finite–energy jet spectrum IR-divergent [already partonic cross section $\hat{\sigma}_{Zg}$]

$$\frac{d\sigma_{Zg}}{d\,p_{Tj}} \propto \frac{p_{Tj}}{p_{Tj}^2 + m_j^2} = \frac{1}{p_{Tj}} \quad \Rightarrow \quad \sigma_{Zg} \propto \log \frac{p_{Tj}^{\text{max}}}{p_{Tj}^{\text{min}}}$$



QCD: 2

Reminder: divergences, regularization and scales in QCD

– NLO process $q\bar{q} \to b\bar{b}$ via s-channel gluon

– including chain of (reducible) gluon self energies

– regularize gluon self–energy bubbles [$n = 4 - 2\epsilon$]

$$\int \frac{d^4k}{(2\pi)^4} \;\to\; r_\epsilon(\mu_R) \int \frac{d^nk}{(2\pi)^n} \sim \mu_R^{2\epsilon} \int \frac{d^nk}{(2\pi)^n}$$

rewrite $\mu_R^\epsilon = \exp(\log \mu_R^\epsilon) = \exp(\epsilon \log \mu_R) = 1 + \epsilon \log \mu_R + ...$

– naked divergent gluon self energy

$$\frac{\Sigma(p^2)}{p^2} = \frac{\alpha_s}{4\pi} \frac{r_\epsilon(\mu_R)}{\epsilon} \left(\frac{4T_R N_F}{3} - \frac{13C_A}{6} \right) \left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2} \right)$$

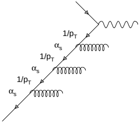
– add chains of one, two, three,.... one–loop bubbles
call the resulting coupling $\alpha_s(p^2)$ [$\overline{\text{MS}}$ scheme]

$$\frac{1}{\alpha_s(p^2)} = \frac{1}{\alpha_s(\mu_R^2)} + \frac{1}{4\pi} \left(\frac{11C_A}{3} - \frac{4T_R N_F}{3} \right) \log \frac{p^2}{\mu_R^2}$$

$$\Leftrightarrow \frac{\partial \alpha_s(\mu^2)}{\partial \log \mu^2} = -\frac{\alpha_s^2}{4\pi} \left(\frac{11C_A}{3} - \frac{4T_R N_F}{3} \right) + ...$$

$\Rightarrow$ renormalization group sums chains of divergent diagrams

QCD: 3

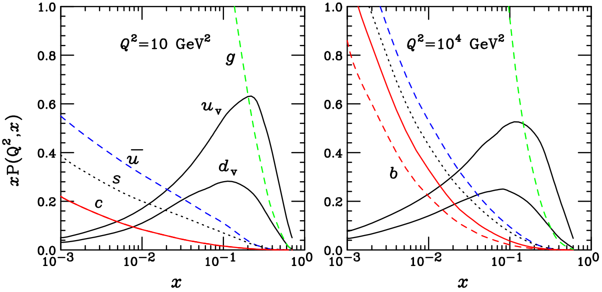


Initial–state gluon radiation

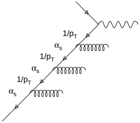
- back to multiple gluon radiation off incoming quark
- each splitting $\alpha_s \log p_{Tj}^{\max}/p_{Tj}^{\min}$
- chain just like multiple gluon self energies [IR problem instead of UV problem]
- IR–divergent in collinear limit [soft–collinear and hard–collinear]

⇒ sum diagrams and absorb in running of parton density [DGLAP]

$$\frac{df_{a/A}(x,\mu_F)}{d\,\log\mu_F^2} = \sum_B \int_x^1 \frac{dz}{z} P_{a\leftarrow b}\left(\frac{x}{z},\alpha_s\right) f_{b/A}(z,\mu_F)$$



QCD: 3



Initial–state gluon radiation

– back to multiple gluon radiation off incoming quark

– each splitting $\alpha_s \log p_{Tj}^{\max}/p_{Tj}^{\min}$

– chain just like multiple gluon self energies [IR problem instead of UV problem]

– IR–divergent in collinear limit [soft–collinear and hard–collinear]

$\Rightarrow$ sum diagrams and absorb in running of parton density [DGLAP]

$$\frac{df_{a/A}(x,\mu_F)}{d\ \log \mu_F^2} = \sum_B \int_x^1 \frac{dz}{z} P_{a\leftarrow b}\left(\frac{x}{z},\alpha_s\right) f_{b/A}(z,\mu_F)$$

– with universal splitting kernels [means factorization]

$$P_{q\leftarrow q}(x) = C_F\frac{1+x^2}{1-x} \qquad P_{q\leftarrow g}(x) = T_R\left(x^2+(1-x)^2\right)$$

$$P_{g\leftarrow q}(x) = C_F\frac{1+(1-x)^2}{x} \qquad P_{g\leftarrow g}(x) = C_A\left(\frac{x}{1-x}+\frac{1-x}{x}+x(1-x)\right)$$

– initial-state radiation treated in collinear limit [no jet distributions for $p_{Tj} < \mu_F$]

– divergence $p_{Tj}^{\min} \to 0$ removed, $p_{Tj}^{\max} \equiv \mu_F$ artifical parameter [smart choice?]

$\Rightarrow$ jet radiation below factorization scale ($p_{Tj} < \mu_F$) included in pdf

QCD: 4

Error estimates in QCD

– QCD at the LHC always perturbative in α_s [plus possibly logs]

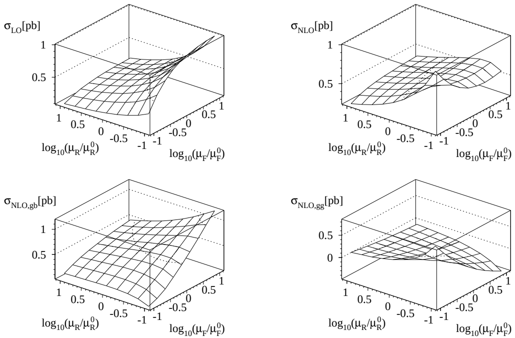
– let's hope, large logarithms identified and summed

– all observables function of scales $O = O(\mu_F, \mu_R)$

$\Rightarrow$ scales artifact of perturbation theory, so vanish at all orders

$\Rightarrow$ scale dependence (only?) measure of theoretical uncertainty

Example: $pp \to tH^-$



QCD: 4

Error estimates in QCD

– QCD at the LHC always perturbative in α_s [plus possibly logs]

– let's hope, large logarithms identified and summed

– all observables function of scales $O = O(\mu_F, \mu_R)$

⇒ scales artifact of perturbation theory, so vanish at all orders

⇒ scale dependence (only?) measure of theoretical uncertainty

Caveats

– total cross section with jets $\sigma(\mu_F, \mu_R)$

(1) asymptotic freedom: $\sigma(\mu_R \to 0) = \infty$ and $\sigma(\mu_R \to \infty) = 0$
(2) p_T size of proton: $\sigma(\mu_F \to 0) = 0$ and $\sigma(\mu_F \to \infty) = \infty$

⇒ scale dependences can cancel [more on DY process later]

– there is no 'correct' scale, but there are idiotic scales [introducing large logs]

QCD: 4

Error estimates in QCD

– QCD at the LHC always perturbative in α_s [plus possibly logs]

– let's hope, large logarithms identified and summed

– all observables function of scales $O = O(\mu_F, \mu_R)$

$\Rightarrow$ scales artifact of perturbation theory, so vanish at all orders

$\Rightarrow$ scale dependence (only?) measure of theoretical uncertainty

Physics of wrong scales

– μ_F defining p_T-size of parton, perturbative part $f_{a/A} \propto \log \mu_F$

– large μ_F means (too) many collinear jets included LO
corrected by correct NLO jet radiation: $K = \sigma_{\text{NLO}}/\sigma_{\text{LO}} \ll 1$

small μ_F means (too) few jets included in LO
corrected by correct NLO jets: $K \gg 1$

– μ_R defining scale in coupling constants or masses $\propto \log \mu_1/\mu_2$

– $\alpha_s(\mu_R)$ leading higher–order contributions, but NLO with explicit argument $\log Q/\mu_R$ explicitely contributing to K

– known trick: mimick soft jet radiation using $\alpha_s(p_{Tj})$

QCD: 5

Definition of jets

– jet–parton duality $\Leftrightarrow$ partons in detector
– jet-rich environment at LHC [mostly but not only soft or collinear]
$\Rightarrow$ need jet algorithm to decide what comes from one parton
– stable w.r.t inclusion of soft or collinear jet [IR save]
– crucial for m_{qq} searching for $Z' \to q\bar{q}$, etc
– even more crucial if likelihood $|\mathcal{M}|^2$ of interest

LHC standard: k_T algorithm [experts: exclusive with $R = 1$]

– define a distance jet–jet d_{kl} and jet–beam d_{kB}
only requirement: $d_{kl} \sim k^2_{T,kl}$ for $\theta_{kl} \to 0$ [e.g. $d_{kB} = E_k^2 \theta_k^2$]
– (0) define resolution d_cut [or number of jets]
(1) find minimum $d_\text{min} = \min_{kl}(d_{kl}, d_{kB})$
(2a) if $d_\text{min} > d_\text{cut}$ all objects are jets
(2b) if $d_\text{min} = d_{kl} < d_\text{cut}$ combine k and l, go to (1)
(2c) if $d_\text{min} = d_{kB} < d_\text{cut}$ remove k from list, go to (1)
$\Rightarrow$ d_cut separating hard from soft–collinear physics
– if pheno analysis requires jet algorithms, you'll likely get it wrong

Processes: 1

Example processes everyone will need

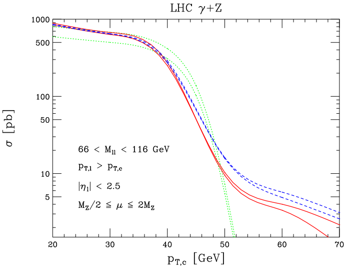
– total cross section (coupling) measurements need luminosity

– effectively: normalization to standard candle

– simple (leptonic) final state
well-controlled QCD corrections
high–precision Monte–Carlo [NNLO if possible]

⇒ Drell–Yan production $pp \to \mu^+\mu^-$

Example: Drell-Yan at LO, NLO, NNLO [error from scales?]



Processes: 2

Example processes everyone will need

– new physics usually including bottom jets, lepton, missing momentum

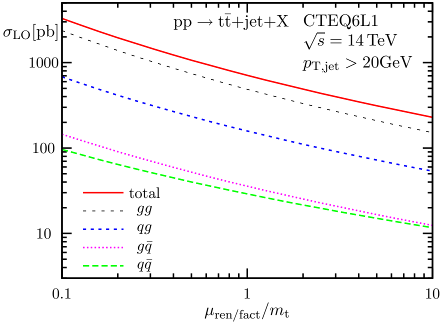
⇒ main background $t\bar{t}$+jets

– background rate after cuts problematic [phase space fragmented]

– NLO corrections to $pp \to t\bar{t}$ available in MC@NLO

– exclusive NLO corrections to $pp \to t\bar{t}j$ almost done

Example: $pp \to ttj$ at LO



Jet radiation properly treated: 1

Hard jets [remember $Z + g$ example]

– described by matrix elements [finite at given order in α_s, e.g. $Z + g$]

– defined only inclusively for soft and collinear jet, distributions wrong [part of pdf]

Soft and collinear jets [no technical details here]

– collinear jet distributions by parton shower [SCET with improved theoretical description?]

– defined in region where matrix elements diverge [finite after log summation]

$\Rightarrow$ holy grail of QCD at LHC: combine the two

Sudakov factors

– Poisson statistics: event with probability p occuring n times

$$f(n;p) = \frac{p^n\, e^{-p}}{n!}$$

– probability of observing no event: $f(0;p) = e^{-p}$

– collinear gluon/photon radiation described by energy-dependent probability

$$dP(x) = \frac{\alpha_s}{2\pi}\, P(x)\, dx \qquad\qquad f(0;x_{\rm min},x_{\rm max}) = \exp\left(- \int_{x_{\rm min}}^{x_{\rm max}} dP(x) \right)$$

$\Rightarrow$ Sudakov giving probability of no radiation/splitting

Jet radiation properly treated: 2

MC@NLO approach

– one hard jet correctly described by exclusive NLO corrections
– soft and collinear jet radiation for all jets from parton shower

Bryan Webber's toy model

– observable for photon radiated with energy $0 < x < 1$

$$\langle O \rangle = \int_0^1 dx \, \frac{O(x)}{x^{2\epsilon}} \left[\frac{d\sigma}{dx}\bigg|_B + \frac{d\sigma}{dx}\bigg|_V + \frac{d\sigma}{dx}\bigg|_R \right]$$

with Born, virtual, real contributions [$\lim_x R(x) = B$, a: coupling]

$$\frac{d\sigma}{dx}\bigg|_B = B\,\delta(x) \qquad \frac{d\sigma}{dx}\bigg|_V = a\left(\frac{B}{2\epsilon} + V\right)\delta(x) \qquad \frac{d\sigma}{dx}\bigg|_R = a\frac{R(x)}{x}$$

– usual subtraction to cancel IR divergence in photon emission

$$\begin{aligned} \langle O \rangle_R =& a\,B\,O(0) \int_0^1 dx \, \frac{1}{x^{1+2\epsilon}} + a \int_0^1 dx \, \frac{R(x)O(x) - BO(0)}{x^{1+2\epsilon}} \\ =& -a\frac{B\,O(0)}{2\epsilon} + a \int_0^1 dx \, \frac{R(x)O(x) - BO(0)}{x} \end{aligned}$$

Jet radiation properly treated: 2

MC@NLO approach

- one hard jet correctly described by exclusive NLO corrections
- soft and collinear jet radiation for all jets from parton shower

Bryan Webber's toy model

- finite NLO Monte-Carlo

$$\langle O \rangle = B\, O(0) + a \left[V\, O(0) + \int_0^1 dx \frac{R(x)\, O(x) - B\, O(0)}{x} \right]$$

- generating functional with additional photon–emission probability $d\sigma/dx|_{PS} = aBQ(x)/x$

$$\int_0^1 dx \left[F \left(B + aV - \frac{aB}{x} \right) + F \frac{a\, R(x)}{x} \right]$$

$$\to \int_0^1 dx \left[F \left(B + aV - \frac{aB}{x} + \frac{aBQ(x)}{x} \right) + F \frac{a\, [R(x) - BQ(x)]}{x} \right]$$

- now: two subtraction terms, new one not at $x = 0$ [harder in practice!]

$\Rightarrow$ MC@NLO subtraction scheme with proper non–collinear subtraction term

Jet radiation properly treated: 2

MC@NLO approach

– one hard jet correctly described by exclusive NLO corrections

– soft and collinear jet radiation for all jets from parton shower

Comments

+ first hard jet correct

+ normalization correct strictly to NLO

+ available from Herwig authors

– additional jets only via parton shower

– subtraction terms not easy or universal

– possible negative weights [why a problem?]

Jet radiation properly treated: 3

CKKW approach [also MLM]

- combine leading–order $X+j, X+jj, X+jjj, X+jjjj$ matrix elements
- avoid double counting between hard matrix elements
- parton shower for soft/collinear radiation

Preparing event sample

(1) create the hard final–state event [k_T algorithm with d_{cut}]

(2) choose jet multiplicty n according to

$$p_n = \frac{\sigma_n}{\sum_{k=0} \sigma_k} \qquad \text{with} \qquad \sigma_k = \sigma_k(d_{\text{cut}})$$

(3) compute event weight and the k_T algorithm's splittings

(4) remember: n jets means n hard jets plus many soft/collinear jets
reweight event to make matrix–element predictions exactly n-jets [Sudakov]

$$w = \prod_{\text{splittings}} \frac{f(0; d_{\text{cut}}, d_j)}{f(0; d_{\text{cut}}, d_k)} \sim \prod_{\text{splittings}} f(0; d_k, d_j) \qquad \text{plus } \alpha_s \text{ terms}$$

(5) normalize to highest known order

$\Rightarrow$ CKKW: combine exclusive n–jet rates with parton shower

Jet radiation properly treated: 3

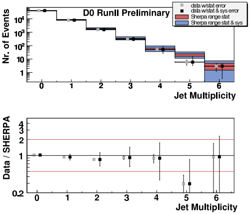
CKKW approach [also MLM]

– combine leading–order $X+j, X+jj, X+jjj, X+jjjj$ matrix elements
– avoid double counting between hard matrix elements
– parton shower for soft/collinear radiation

Comments

+ all additional jets correct
+ available from Sherpa, Madevent, Alpgen authors
– normalization of different processes with error
– over-all normalization unknown

Sherpa–CKKW and Tevatron data



QCD Outlook

LHC phenomenologists need to know QCD [model builders too]

– not all LHC events can be analysed: trigger

– perturbative QCD is not perfect, but the best we have

– some processes need to be known precisely: luminosity, backgrounds

– jets should be properly described: ME-PS matching

⇒ next: what LHC can really do

Apologies...

...to the people I did not cite, because it is a lecture

...to you for not explaining things like parton shower

...to those for whom I went to fast

...to those I bored to death