

Signatures for UV-complete Extra Dimensions

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Outline

Large extra dimensions

Effective Kaluza-Klein theory

String theory completion

UV fixed-point completion

Fixed-point black holes

Collider searches

Real particle production

- produce searched-for particle [like $pp \rightarrow$ single jet + E_T]
 - reconstruct decay products [or missing energy]
 - measure mass, spin, branching ratios
- ⇒ high-energy regime

Virtual-particle effects

- produce known final state [like $pp \rightarrow e^+ e^-$]
 - compare to Standard Model predictions
 - trust in quantum theory and error estimates [new physics and old physics]
- ⇒ high-precision regime

Rare effect or rare decays [B physics, EDMs]

- produce something known [like B_S, \dots]
 - find effect forbidden in Standard Model
- ⇒ not multi-purpose
- ⇒ **form complete picture at LHC**

Large extra dimensions

Remember the hierarchy problem

- fundamental scalars bad with high scale present
 - weakness of gravity means large Planck scale $G_N = 1/(16\pi M_{\text{Planck}})^2$
- ⇒ solution: another reason for huge non-fundamental M_{Planck}

Large extra dimensions (ADD) [Antoniadis, Arkani-Hamed, Dimopoulos, Dvali]

- Einstein–Hilbert action for low fundamental Planck scale

$$\begin{aligned} S &= -\frac{1}{2} \int d^4x \sqrt{|g|} M_D^2 R \rightarrow -\frac{1}{2} \int d^{4+n}x \sqrt{|g|} M_D^{2+n} R \\ &= -\frac{1}{2} (2\pi r)^n \int d^4x \sqrt{|g|} M_D^{2+n} R \\ &\equiv -\frac{1}{2} \int d^4x \sqrt{|g|} M_{\text{Planck}}^2 R \end{aligned}$$

- ⇒ express M_{Planck} in terms of fundamental M_D

$$M_{\text{Planck}} = M_D (2\pi r M_D)^{n/2}$$

Numbers to make it work

- free parameter $rM_D \gg 1$
 - constraints from gravity tests above $\mathcal{O}(\text{mm})$
- ⇒ most minimal solution of hierarchy problem

$M_D = 1 \text{ TeV}$	
n	r
1	10^{12} m
2	10^{-3} m
3	10^{-8} m
...	...
6	10^{-11} m

Effective Kaluza-Klein theory

Bottom-up: only gravitons in extra dimensions

- expand the metric [graviton field h]

$$ds^2 = g_{MN}^{(4+n)} dx^M dx^N = \left(\eta_{MN} + \frac{1}{M_D^{n/2+1}} h_{MN} \right) dx^M dx^N$$

- 4D matter in Einstein's equation [no cosmological constant]

$$R_{AB} - \frac{1}{2+n} g_{AB} R = -\frac{1}{M_D^{2+n}} \begin{pmatrix} T_{\mu\nu}(x) \delta^{(n)}(y) & 0 \\ 0 & 0 \end{pmatrix}$$

- Fourier transformation of extra dimensions [periodic boundary conditions]

$$h_{AB}(x; y) = \sum_{m_1=-\infty}^{\infty} \dots \sum_{m_j=-\infty}^{\infty} \frac{h_{AB}^{(m)}(x)}{\sqrt{(2\pi r)^n}} e^{i \frac{m_j y_j}{r}}$$

- universal interactions of KK excitations [$h_{AB} \rightarrow G_{\mu\nu}$, QCD massless]

$$(\square + m_k^2) G_{\mu\nu}^{(k)} = \frac{1}{M_{\text{Planck}}} \left[-T_{\mu\nu} + \left(\frac{\partial_\mu \partial_\nu}{\hat{m}^2} + \eta_{\mu\nu} \right) \frac{T_\lambda^\lambda}{3} \right] = \frac{-T_{\mu\nu}}{M_{\text{Planck}}}$$

- tiny KK mass splitting [$M_D = 1 \text{ TeV}$]

$$\delta m \sim \frac{1}{r} = 2\pi M_D \left(\frac{M_D}{M_{\text{Planck}}} \right)^{2/n} = \begin{cases} 0.003 \text{ eV} & (n=2) \\ 0.1 \text{ MeV} & (n=4) \\ 0.05 \text{ GeV} & (n=6) \end{cases}$$

⇒ continuum of weakly interacting gravitons at the LHC

Gravitons at the LHC

Effective theory predictions [Giudice, Rattazzi, Wells; Han, Lykken, Zhang; Hewett;...]

- real radiation of KK tower $[dm/d|k| = 1/r; d\sigma \propto 1/M_{\text{Planck}}^2]$

$$d\sigma^{\text{tower}} \sim S_{n-1} r^n \int dm m^{n-1} d\sigma^{\text{graviton}} = \frac{S_{n-1}}{(2\pi M_D)^n} \int dm m^{n-1} \frac{d\sigma^{\text{graviton}} M_{\text{Planck}}^2}{M_D^2}$$

- higher-dimensional operator from virtual KK tower [s-channel in DY]

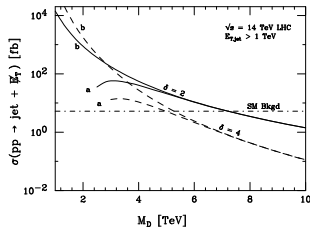
$$S(s) = \frac{S_{n-1}}{M_D^{2+n}} \int^\Lambda dm \frac{m^{n-1}}{s-m^2} \sim \frac{S_{\delta-1}}{2} \frac{\Lambda^{n-2}}{M_D^{n+2}} \neq \frac{S_{\delta-1}}{2} \frac{1}{M_D^4}$$

- saving grace: $1/M_D^2$ interactions of KK tower

⇒ **effective theory valid for $E < M_D$**

(1) Real emission $pp \rightarrow G_{\text{KK}} + \text{jets}$ [Giudice, Rattazzi, Wells; Vacavant, Hichliffe...]

- background: radiation of $Z \rightarrow \nu\bar{\nu}$
 - $\mathcal{M} = 0$ for $E_{\text{parton}} > \Lambda$
 $\mathcal{M} = 0$ automatically for $m_{\text{KK}} > E_{\text{parton}}$
 - effective cutoff: steep gluon density
- ⇒ **explicit cutoff only for $M_D \lesssim 3 \text{ GeV}$**



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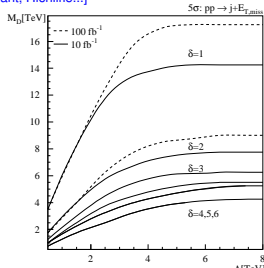
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Virtual gravitons at LHC

(2) Virtual exchange $pp \rightarrow \mu^+ \mu^-$ [Giudice & Strumia; Giudice, Strumia, TP;...]

- Drell-Yan as standard candle [$q\bar{q}$ and gg initial states]
- reconstructed $m_{\mu\mu}$ for photon, Z, graviton
- loop-induced D6 operator [axial vector-axial vector]

$$\frac{1}{16} \frac{1}{M_D^2} \frac{\pi^{n-2}}{\Gamma^2(n/2)} \left(\frac{\Lambda}{M_D} \right)^{2n+2}$$

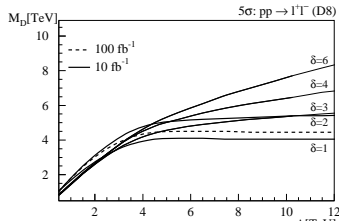
- tree-induced D8 operator [leading constant in \sqrt{s}/Λ]

$$S(s) = \frac{S_{n-1}}{M_D^{2+n}} \int^\Lambda dm \frac{m^{n-1}}{s+m^2} = \begin{cases} \frac{4\pi}{M_D^4} + \dots & \text{effective scale} \\ \frac{S_{n-1}^{\text{eff}}}{M_D^4} \frac{1}{2} \left(\frac{\Lambda}{M_D} \right)^{n-2} + \dots & \text{NDA} \\ \frac{S_{n-1}}{M_D^4} \frac{1}{n-2} \left(\frac{\Lambda}{M_D} \right)^{n-2} + \dots & \text{cutoff } \Theta(\Lambda - m) \end{cases}$$

- two-dimensional integration over (m, s)
cutoff requiring $m \lesssim M_D$
additional cutoff for $\sqrt{s} \lesssim M_D$
- constant rate $M_D^{\text{max}} \sim \Lambda^{(n-2)/(n+2)}$

⇒ **breakdown of effective theory**

[UHECR: Plümacher & Kachelrieß]



String theory completion

String theory as UV completion [e.g. Cullen, Perelstein, Peskin; Antoniadis, Benakli, Laugier...]

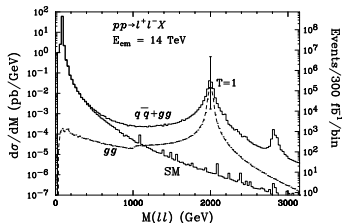
- see Antoniadis' talk, let me skip due to my complete ignorance...
- Regge excitations of Standard Model [Burikham, Fygi, Han]
- compute different helicity contributions, like

$$\begin{aligned} A(e_L^+ e_R^- \rightarrow \gamma_L \gamma_R) &= -2e^2 \sqrt{\frac{u}{t}} \left(\frac{u}{s} + \frac{t}{s} - 1 \right) = 2e^2 \sqrt{\frac{u}{t}} \\ &= -2e^2 \sqrt{\frac{u}{t}} \left(\frac{u}{s} S(s, t) + \frac{t}{s} S(s, u) - S(t, u) \right) \end{aligned}$$

- with Veneziano form factor

$$\begin{aligned} S(s, t) &= \frac{\Gamma(1 - \alpha' s) \Gamma(1 - \alpha' t)}{\Gamma(1 - \alpha'(s+t))} = \frac{\Gamma(1 - s/M_S^2) \Gamma(1 - t/M_S^2)}{\Gamma(1 - (s+t)/M_S^2)} \\ &= 1 - \frac{\pi^2}{6} \frac{st}{M_S^4} + \mathcal{O}(M_S^{-6}) \end{aligned}$$

- dominant over KK because $M_{\text{eff}} \gg M_S$
closed-string gravitons suppressed
- effective operator below string scale
- string resonances: $\sqrt{n} M_S$



UV fixed-point completion

Gravity as its own UV completion [Reuter; Percacci; Fischer & Litim;...]

- effective action: $\Gamma_k = 1/(16\pi G_k) \int d^{4+n}x \sqrt{g} [-R(g) + \dots]$ [Wetterich]
- dimensionless running coupling $g(\mu) = G(\mu)\mu^{2+n} = G_0 Z_G^{-1}(\mu) \mu^{2+n}$
- UV fixed point [anomalous dimension: $\eta = -\mu \partial_\mu \log Z_G \propto g$]

$$\mu \frac{\partial}{\partial \mu} g(\mu) = (2 + n + \eta(g)) g(\mu) = 0 \quad \text{for} \quad g \neq 0 \quad \eta(g) = -2 - n$$

- $G(\mu) < G_N$ weak enough at high energies? [Weinberg]
- IR — no running; M_D regime — strong effects; UV — fixed point

- iterative approach: start with anomalous dimension of graviton

$$P(s, m) = \begin{cases} \frac{1}{s + m^2} & \sqrt{s}, m < k_{\text{trans}} \\ \frac{M_D^{n+2}}{(s + m^2)^{n/2+2}} & \sqrt{s}, m > k_{\text{trans}} \end{cases}$$

- IR and UV contributions to D8 operator [leading in \sqrt{s}/m , matched to $1/s^2$]

$$S^{(\text{FP})} = \frac{S_{n-1}}{M_D^4} \left(\frac{k_{\text{trans}}}{M_D} \right)^{n-2} \frac{n-1}{n-2} = (1 + (n-2)) S^{(\Theta)}$$

⇒ consistent and truly minimal SM extension

UV fixed-point completion

Predicted graviton kernel

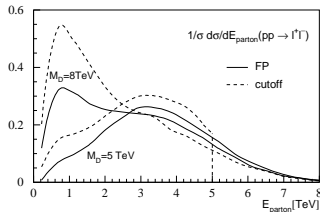
- sizeable rate enhancement $\mathcal{S}^{(FP)} = \mathcal{S}^{(\Theta)} + \mathcal{S}^{(UV)}$:

σ [fb]	$n = 3$			$n = 6$		
	2 TeV	5 TeV	8 TeV	2 TeV	5 TeV	8 TeV
$\mathcal{S}^{(NDA)}$	43.6	0.18	0.0053	263	1.11	0.031
$\mathcal{S}^{(\Theta)}$	173	0.72	0.0204	66	0.28	0.008
$\mathcal{S}^{(FP)}$	408	1.24	0.0317	398	1.21	0.031

- visible effect on $m_{\ell\ell}$
- shift to larger $m_{\ell\ell}$ [shown $n = 3$]
small $m_{\ell\ell}$: factor $\mathcal{S} \propto (n-1)$
large $m_{\ell\ell}$: factor $\mathcal{S}^2 \propto (n-1)^2$
- mild effects from $k_{\text{trans}} = M_D \pm 10\%$
- no resonance structure

⇒ **clearly testable model**

- needed and not needed:
 - (1) transition scale $k_{\text{trans}} \sim M_D$ [anomalous dimension modeled]
 - (2) no artificial Λ for m integration
 - (3) matching/cutoff for s integration [Minkowski picture?]



UV fixed point completion

Alternative: coupling form factor [Hewett & Rizzo: RS gravitons]

- dress s-channel coupling

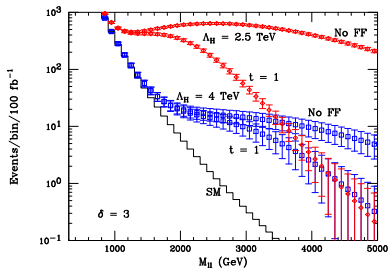
$$\frac{1}{M_D^{2+n}} \longrightarrow \frac{1}{M_D^{2+n}} F(s) = \frac{1}{M_D^{2+n}} \left(1 + \left(\frac{\sqrt{s}}{M_D} \right)^{2+n} \right)^{-1}$$

- no modified s integration (unitarity)

$$\frac{1}{M_D^{2+n}} F(s) \sim \frac{1}{M_D^{2+n}} \left(\frac{M_D}{\sqrt{s}} \right)^{2+n} \sim \frac{1}{s^{1+n/2}}$$

- still cutoff in m integration
- also applicable to graviton emission [less impressive effect]

⇒ **KK and energy integrations understood for fixed point**



Fixed-point black holes

Application to black holes [Falls, Litim, Raghuraman; to appear]

- classical Schwarzschild black holes $r_{\text{class}} = (G_N M)^{1/(n+1)}$ from

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2 \quad \text{with } f = 1 - \frac{G_N M}{r^{n+1}}$$

- improved radius $r_{\text{FP}} < r_{\text{class}}$ from

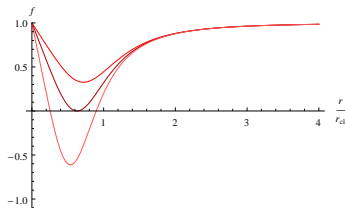
$$f(r_{\text{FP}}) \equiv 0 \quad \text{with } f(r) = 1 - \frac{G(r)M}{r^{n+1}}$$

- critical (minimal) black hole mass $M_{\text{crit}} \sim M_D$

$$(n+1) M_{\text{crit}} = r \left. \frac{d}{dr} G(r) \right|_{r(M_{\text{crit}})}$$

- (1) no arbitrary small black holes
- (2) stable remnant from large black holes

⇒ distinctive UV feature



Outlook

TeV-scale gravity at LHC

- most minimal extension of Standard Model
 - effective field theory:
 - real emission accidentally well defined
 - virtual-graviton predictions poor
 - string theory:
 - resonances and exponentiation suppression
 - gravitons sub-leading
 - fixed-point gravity:
 - gravity asymptotically safe (weak at high energies)
 - effect on KK mass: graviton anomalous dimension
 - effect on energy: coupling form factor
 - application to black holes
 - lots more work to do [Gerwick, Litim, Plehn]
- ⇒ UV-complete extra dimensions needed and available

**Signatures for
UV-complete Extra
Dimensions**

Tilman Plehn

Large dimensions

Effective theory

String theory

UV fixed point

Black holes