

Sfitting

Tilman Plehn

Likelihood

Errors

Toy model

MSSM

GUT

Higgs sector

Higgs hypotheses

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Stockholm, 9/2010

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Motivation: Supersymmetric parameters

New physics in the LHC era [Review: Morrissey, TP, Tait]

- complex models, including dark matter, flavor physics, low-energy physics,...
- honest parameters: weak-scale Lagrangean
- measurements: masses or edges
branching fractions
cross sections
- errors: statistics & systematics & theory, fully correlated
- grid: find local minima?
fit: find global minima?

First go at problem

- ask a friend how SUSY is broken \Rightarrow mSUGRA
- fit $m_0, m_{1/2}, A_0, \tan \beta, y_t, \dots$ minimizing

$$\chi^2 = -2 \log \mathcal{L} = \vec{\chi}_d^T C^{-1} \vec{\chi}_d \quad \text{with} \quad \chi_{d,i} = \frac{|d_i - \bar{d}_i|}{\sigma_i^{(\text{exp})}}$$

- best-fitting point to LHC data

\Rightarrow **use edges, not masses** [more later]

	SPS1a	Δ_{LHC} masses	Δ_{LHC} edges
m_0	100	3.9	1.2
$m_{1/2}$	250	1.7	1.0
$\tan \beta$	10	1.1	0.9
A_0	-100	33	20

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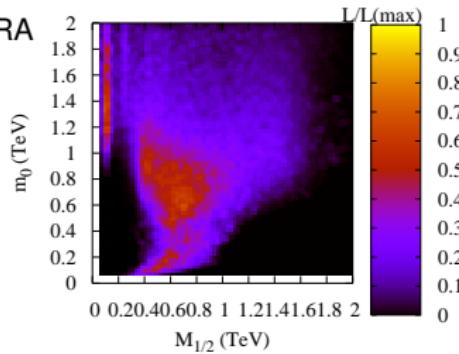
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First go at problem

- ask a friend how SUSY is broken \Rightarrow mSUGRA
 - fit $m_0, m_{1/2}, A_0, \tan \beta, \text{sign}(\mu), y_t, \dots$
 - no problem, include dark matter
 - probability map [Allanach, Lester, Weber]
- \Rightarrow nothing you cannot predict...



Markov chains

Probability maps

- need handle for agreement as function of parameters
 - likelihood: data given a model $p(d|m) \sim |\mathcal{M}|^2$
 - Bayes' theorem: $p(m|d) = p(d|m) p(m)/p(d)$ [$p(d)$ through normalization]
 - theorist's prejudice: model $p(m)$
- ⇒ given measurements:
- (1) compute map $p(d|m)$
 - (2) rank local maxima
 - (3) derive probabilities for parameters

Markov chains

- classical: representative set of spin states
compute average energy on this reduced sample
- Metropolis-Hastings
 - starting probability $p(d|m)$ vs suggestion probability $p(d|m')$
 - (1) accept new point if $p(d|m') > p(d|m)$
 - (2) otherwise accept with $p(d|m')/p(d|m) < 1$
- free proposal probability $q(m \rightarrow m') \neq q(m' \rightarrow m)$
no memory ↔ detailed balance
- 25% success rate to aim for

Improving Markov chains

Weighted Markov chains

- special situation in BSM physics
measure of ‘representative’: probability itself
- example with 2 bins, probability 10%:90%
10 entries needed for good Markov chain
2 entries needed if weight kept
- binning with weight would double count
bin with inverse averaging

$$P_{\text{bin}}(p \neq 0) = \frac{\text{bincount}}{\sum_{i=1}^{\text{bincount}} p^{-1}}$$

- good choice for $\mathcal{O}(6)$ dimensions

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Cooling Markov chains

- need to zoom in on peak structures
- modified condition [inspired by simulated annealing]
Markov chain in 100 partitions, numbered by j

$$\frac{p(m')}{p(m)} > r^{\frac{100}{J_c}} \quad \text{with} \quad c \sim 10, \quad r \in [0, 1] \quad \text{random number}$$

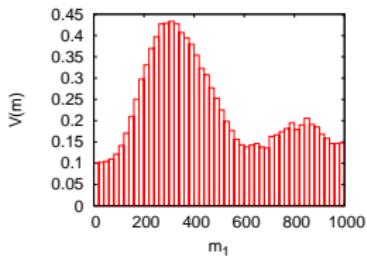
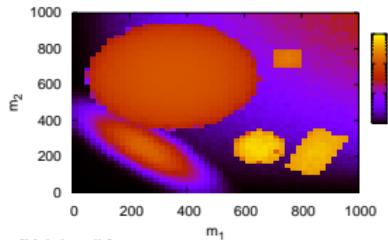
- only reliable for many Markov chains

Frequentist vs Bayesian

Getting rid of model parameters

- poorly constrained directions
interested in special parameter
unphysical parameters [JES part of m_t extraction]
- two ways to marginalize common likelihood map

- (1) integrate over probabilities
normalization etc mathematically correct
integration measure unclear
noise accumulation from irrelevant regions
classical example: convolution of two Gaussians

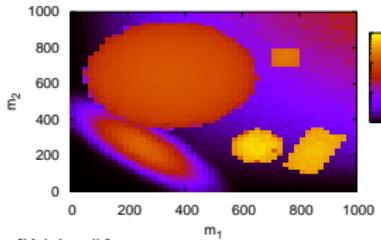


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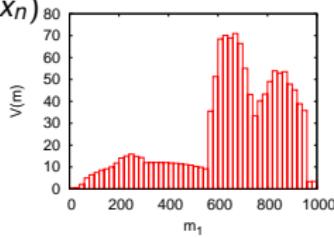
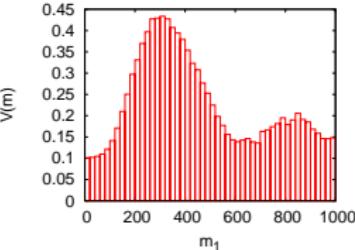
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- (2) profile likelihood $\mathcal{L}(\dots, x_{j-1}, x_{j+1}, \dots) \equiv \max_{x_j} \mathcal{L}(x_1, \dots, x_n)$
not normalized, no comparison of structures
no integration needed
no noise accumulation
classical example: best-fit point

- frequentist: flavor, Higgs, ...
Bayesian: cosmology, dark matter, new physics, ...
- simply: two questions — two answers, we may not get to pick



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Error analysis

Sources of uncertainty

- statistical error: Gaussian
- systematic error: Gaussian, if measured
- theory error: not Gaussian
- simple argument [Ben's talk]
 - QCD rate $\pm 10\%$ off: no problem
 - QCD rate $\pm 30\%$ off: no problem
 - QCD rate $\pm 300\%$ off: Standard Model wrong
- theory likelihood flat centrally and zero far away
- profile likelihood construction: RFit [CKMFitter]

$$\chi^2 = \vec{\chi}_d^T C^{-1} \vec{\chi}_d$$

$$\chi_{d,i} = \begin{cases} 0 & |d_i - \bar{d}_i| < \sigma_i^{(\text{theo})} \\ \frac{|d_i - \bar{d}_i| - \sigma_i^{(\text{theo})}}{\sigma_i^{(\text{exp})}} & |d_i - \bar{d}_i| > \sigma_i^{(\text{theo})} \end{cases},$$

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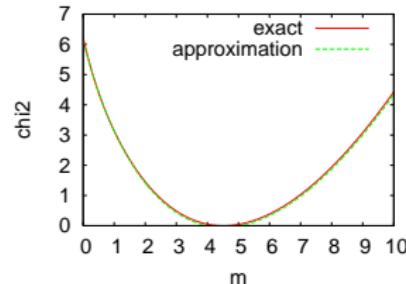
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(Inconsistent) combination of errors

- Gaussian \otimes Gaussians: half width added in quadrature
- Gaussian \otimes flat: RFit scheme
- Gaussian \otimes Poisson
- approximate formula

$$\frac{1}{\log \mathcal{L}_{\text{comb}}} = \frac{1}{\log \mathcal{L}_{\text{Gauss}}} + \frac{1}{\log \mathcal{L}_{\text{Poisson}}}$$

- good to 5% for 5 events with 10% Gaussian



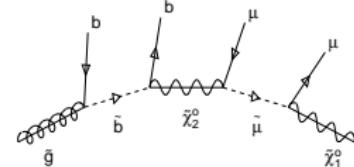
Masses from cascade decays

Cascade decays [Atlas-TDR, Oxbridge]

- if new particles strongly interacting and LSP weakly interacting
- like Tevatron: jets + missing energy
- tough: $(\sigma BR)_1 / (\sigma BR)_2$ [unavoidable: focus point]
- easier: cascade kinematics [$10^7 \dots 10^8$ events]
- thresholds & edges

$$0 < m_{\mu\mu}^2 < \frac{m_{\tilde{\chi}_2^0}^2 - m_{\tilde{\ell}}^2}{m_{\tilde{\ell}}} \frac{m_{\tilde{\ell}}^2 - m_{\tilde{\chi}_1^0}^2}{m_{\tilde{\ell}}}$$

⇒ new-physics mass spectrum from cascade kinematics



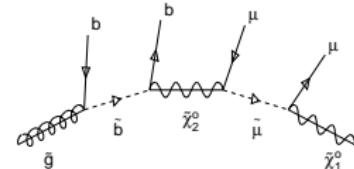
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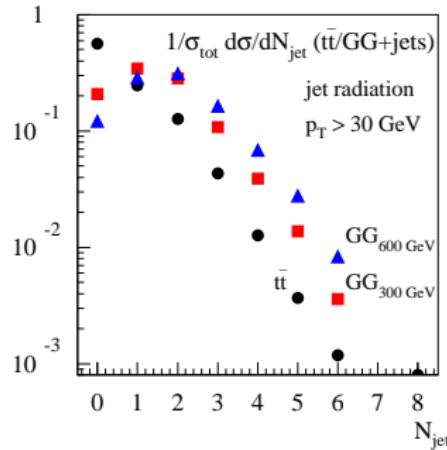
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Likely bad ideas

- decay jets vs QCD radiation
 - collinear initial state radiation [$p_{T,j} < M_{\text{hard}}$]
 - proper description: CKKW/MLM [e.g. MadEvent]
 - $\langle N_{\text{jet}} \rangle$ dependent on hard scale
 - study: scalar gluons [TP & Tait]
- ⇒ QCD basics always useful at LHC



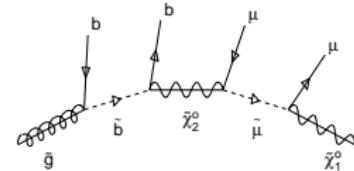
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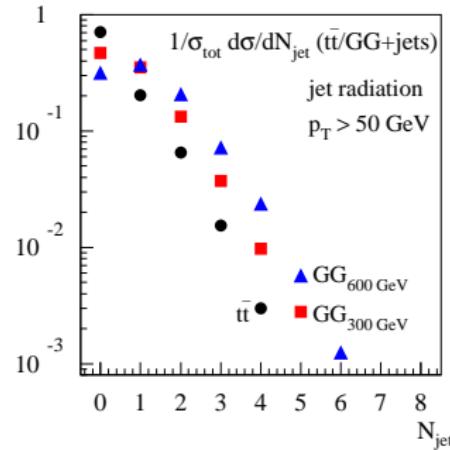
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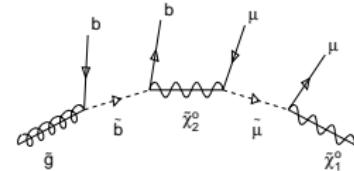
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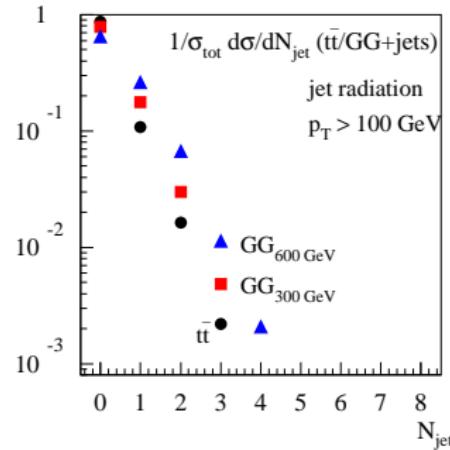
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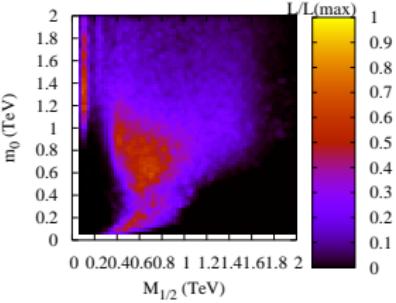
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Toy model: mSUGRA

mSUGRA as of today [Allanach, Cranmer, Lester, Weber]

- ‘Which is the most likely parameter point?’
‘How does dark matter annihilate/couple?’
- trivial model, but still issues...



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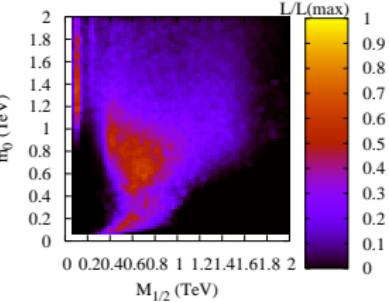
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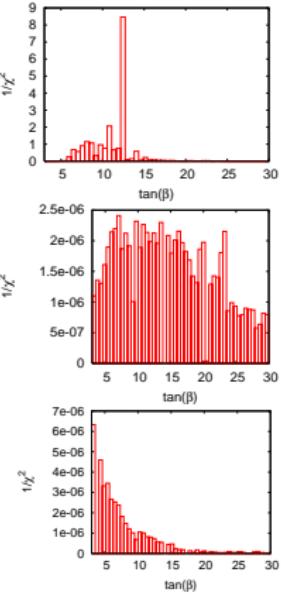
Proper high-scale model [Lafaye, TP, Rauch, D. Zerwas]

- $m_{1/2}$, m_0 , A_0 at high scale
 $\tan \beta$ at low scale
 m_Z assuming e-w symmetry breaking
- replace $\tan \beta$ with high-scale B

$$\mu^2 = \frac{m_{H,2}^2 \sin^2 \beta - m_{H,1}^2 \cos^2 \beta}{\cos 2\beta} - \frac{1}{2} m_Z^2$$

$$2B\mu = \tan 2\beta \left(m_{H,1}^2 - m_{H,2}^2 \right) + m_Z^2 \sin 2\beta$$

- phrase results in $\tan \beta$
no net change for profile likelihood
shift to small $\tan \beta$ for flat B prior



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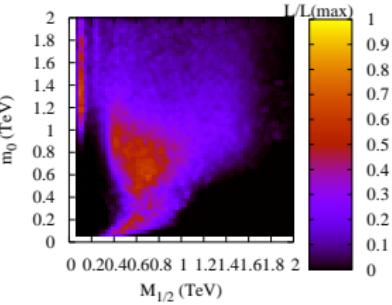
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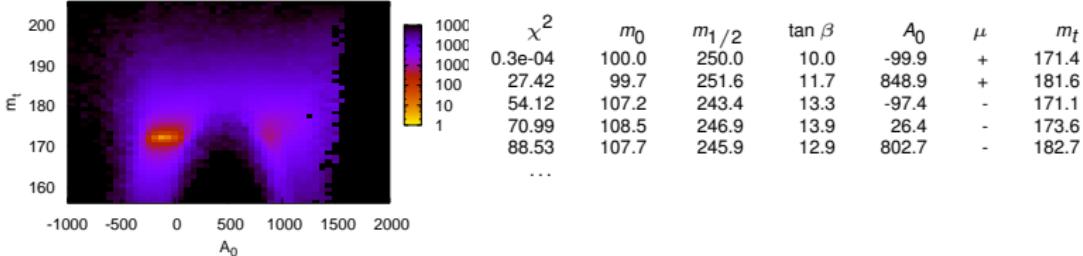
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Correlations and secondary maxima

- SFitter output #1: fully exclusive likelihood map from Markov chain
SFitter output #2: ranked list of local maxima from hill climber
- **strong correlation e.g. of A_0 and y_t**
maxima distinguishable by quality of fit



Real thing: MSSM

19-dimensional MSSM [unless God tells you how she breaks SUSY]

- SFitter approach and outputs still the same [weighted Markov chain plus hill climber]
- but in several steps
 - (1) Markov chains over entire parameter space
 - (2) MC and hill climber over $M_1, m_2, M_3, \mu, \tan\beta$ [flat proposal function, 15 best points]
 - (3) MC and hill-climber over orthogonal coordinates [BW proposal function, 5 best points]
 - (4) error analysis with all parameters [pseudo-measurements]
- degeneracies: 22 measurements from 15 masses
 $m_A, m_{\tilde{\tau}_R}, A_t$ not covered, $\tan\beta$ bad
- assignment of particles to measurements assumed [which neutralino, slepton?]

Alternative solutions

Degenerate best-fit points, not yet discussing errors

- observe 16 parameter points with perfect χ^2 :
 sign of μ
 sign of A_t [now with same m_t]
 lightest neutralino governing M_1 , M_2 or μ [for latter again $M_{1,2}$]

	$\mu < 0$				$\mu > 0$			
					SPS1a			
M_1	96.6	175.1	103.5	365.8	98.3	176.4	105.9	365.3
M_2	181.2	98.4	350.0	130.9	187.5	103.9	348.4	137.8
μ	-354.1	-357.6	-177.7	-159.9	347.8	352.6	178.0	161.5
$\tan \beta$	14.6	14.5	29.1	32.1	15.0	14.8	29.2	32.1
M_3	583.2	583.3	583.3	583.5	583.1	583.1	583.3	583.4
$M_{\tilde{\tau}L}$	114.9	2704.3	128.3	4794.2	128.0	229.9	3269.3	118.6
$M_{\tilde{\tau}R}$	348.8	129.9	1292.7	130.1	2266.5	138.5	129.9	255.1
$M_{\tilde{\mu}L}$	192.7	192.7	192.7	192.9	192.6	192.6	192.7	192.8
$M_{\tilde{\mu}R}$	131.1	131.1	131.1	131.3	131.0	131.0	131.1	131.2
$M_{\tilde{e}L}$	186.3	186.4	186.4	186.5	186.2	186.2	186.4	186.4
$M_{\tilde{e}R}$	131.5	131.5	131.6	131.7	131.4	131.4	131.5	131.6
$M_{\tilde{q}3L}$	497.1	497.2	494.1	494.0	495.6	495.6	495.8	495.0
$M_{\tilde{t}R}$	1073.9	920.3	547.9	950.8	547.9	460.5	978.2	520.0
$M_{\tilde{b}R}$	497.3	497.3	500.4	500.9	498.5	498.5	498.7	499.6
$M_{\tilde{q}L}$	525.1	525.2	525.3	525.5	525.0	525.0	525.2	525.3
$M_{\tilde{q}R}$	511.3	511.3	511.4	511.5	511.2	511.2	511.4	511.5
A_t (-)	-252.3	-348.4	-477.1	-259.0	-470.0	-484.3	-243.4	-465.7
A_t (+)	384.9	481.8	641.5	432.5	739.2	774.7	440.5	656.9
m_A	350.3	725.8	263.1	1020.0	171.6	156.5	897.6	256.1
m_t	171.4	171.4	171.4	171.4	171.4	171.4	171.4	171.4

- improve by observing more particles or measuring more parameters

Error bars

Locally around SPS1a

- three kinds of parameters
 - well-measured, as expected
 - poorly measured, unexpected
 - poorly measured, as expected
- fixed parameters need check
- poor measurements need explain
- side remark:
fat jets for stops

	no theory error	flat theory error	SPS1a
$\tan \beta$	9.8 ± 2.3	10.0 ± 4.5	10.0
M_1	101.5 ± 4.6	102.1 ± 7.8	103.1
M_2	191.7 ± 4.8	193.3 ± 7.8	192.9
M_3	575.7 ± 7.7	577.2 ± 14.5	577.9
$M_{\tilde{t}_L}$	196.2 ± $\mathcal{O}(10^2)$	227.8 ± $\mathcal{O}(10^3)$	193.6
$M_{\tilde{\tau}_R}$	136.2 ± 36.5	164.1 ± $\mathcal{O}(10^3)$	133.4
$M_{\tilde{\mu}_L}$	192.6 ± 5.3	193.2 ± 8.8	194.4
$M_{\tilde{\mu}_R}$	134.0 ± 4.8	135.0 ± 8.3	135.8
$M_{\tilde{e}_L}$	192.7 ± 5.3	193.3 ± 8.8	194.4
$M_{\tilde{e}_R}$	134.0 ± 4.8	135.0 ± 8.3	135.8
$M_{\tilde{q}_3 L}$	478.2 ± 9.4	481.4 ± 22.0	480.8
$M_{\tilde{q}_3 R}$	429.5 ± $\mathcal{O}(10^2)$	415.8 ± $\mathcal{O}(10^2)$	408.3
$M_{\tilde{b}_R}$	501.2 ± 10.0	501.7 ± 17.9	502.9
$M_{\tilde{q}_L}$	523.6 ± 8.4	524.6 ± 14.5	526.6
$M_{\tilde{q}_R}$	506.2 ± 11.7	507.3 ± 17.5	508.1
A_τ	fixed 0	fixed 0	-249.4
A_t	-500.6 ± 58.4	-509.1 ± 86.7	-490.9
A_b	fixed 0	fixed 0	-763.4
$A_{l1,2}$	fixed 0	fixed 0	-251.1
$A_{u1,2}$	fixed 0	fixed 0	-657.2
$A_{d1,2}$	fixed 0	fixed 0	-821.8
m_A	446.1 ± $\mathcal{O}(10^3)$	406.3 ± $\mathcal{O}(10^3)$	394.9
μ	350.9 ± 7.3	350.5 ± 14.5	353.7
m_t	171.4 ± 1.0	171.4 ± 1.0	171.4

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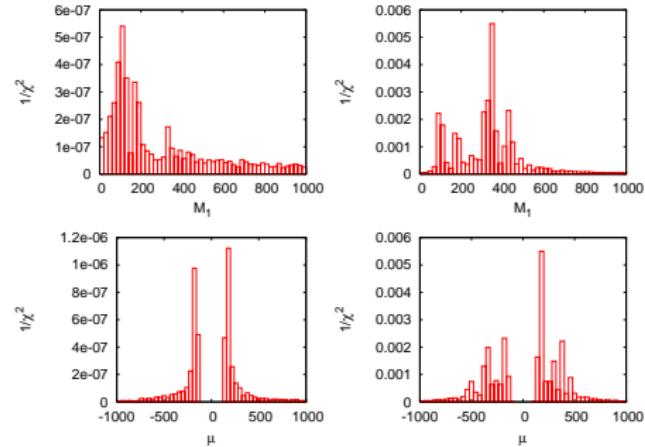
Higgs sector

Higgs hypotheses

Pretty colored plots

Parameters and correlations

- sensitive to marginalization
- test profile likelihood vs marginalized probability
- visible best 1-dimensionally:
clear (dis)advantages of two questions



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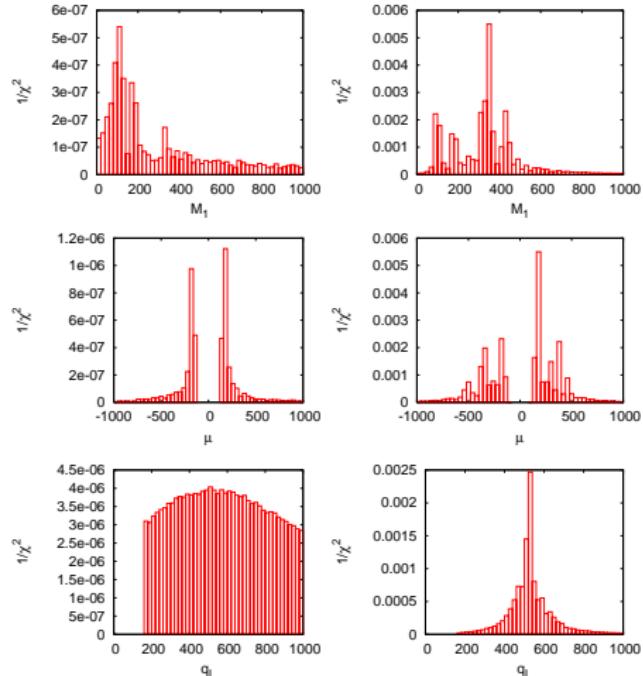
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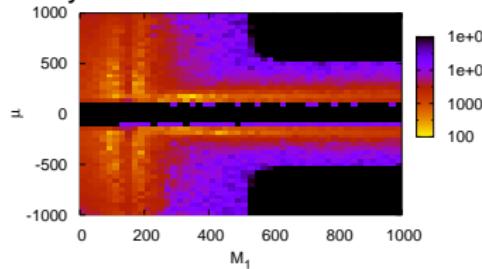
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- visible best 1-dimensionally:
clear (dis)advantages of two questions
- 2-dimensional correlations in color [not crucial for MSSM]
- iso-scalar correlations hard to evaluate, luckily absent



⇒ whatever...

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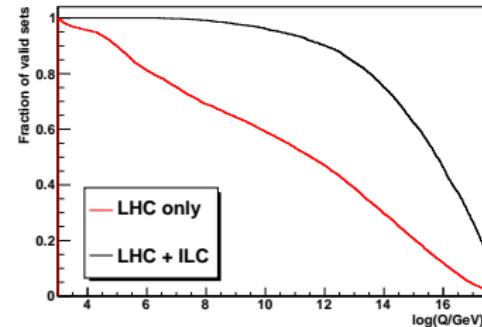
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Testing a SUSY GUT

Renormalization group analysis [Adams, Kneur, Lafaye, TP, Rauch, Zerwas; SFitter+SuSpect]

- are all mass parameters defined at high scales? [tachyonic solutions]
do they unify?
where is the GUT scale?
what are the unified values?



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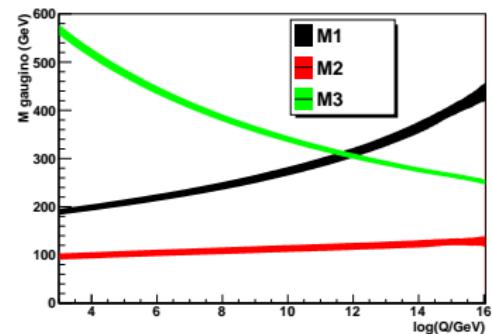
Testing a SUSY GUT

Renormalization group analysis [Adams, Kneur, Lafaye, TP, Rauch, Zerwas; SFitter+SuSpect]

- are all mass parameters defined at high scales? [tachyonic solutions]
do they unify?
where is the GUT scale?
what are the unified values?

Interpretation of LHC results

- measuring $m_{1/2}$: 8-fold degeneracy solved [modulo sign of μ]



Likelihood

Errors

Toy model

MSSM

GUT

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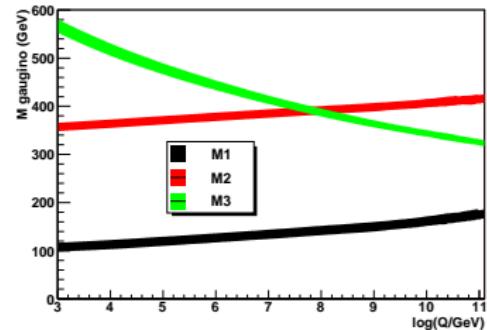
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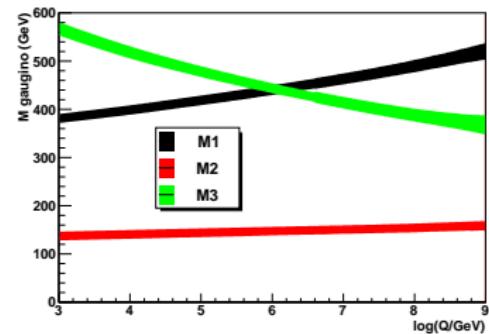
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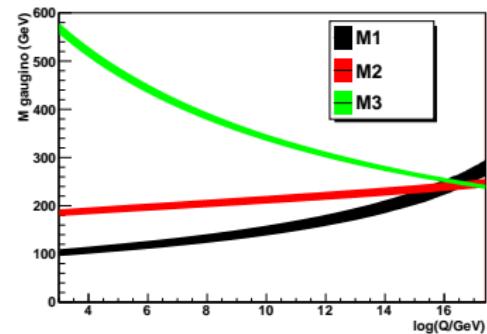
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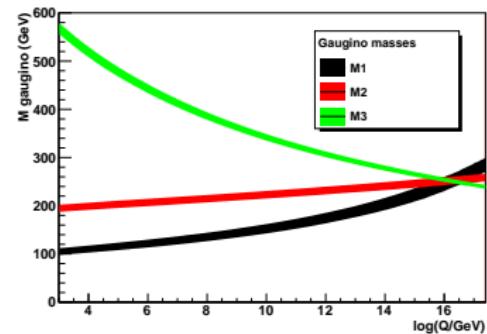
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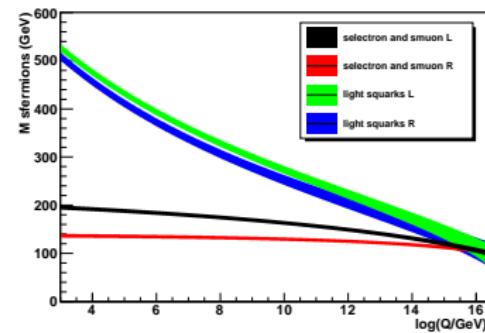
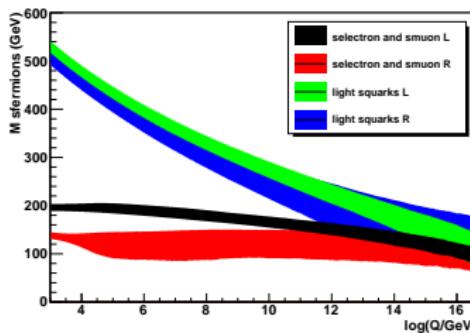
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Interpretation of LHC results

- measuring $m_{1/2}$: 8-fold degeneracy solved [modulo sign of μ]
- measuring m_0 : bottom-up vs top-down
- measured high-scale masses

	$\Delta m/m_{\text{top-down}}$	m_{unified}	$\log M_{\text{GUT}}/\text{GeV}$	$m(1.7 \cdot 10^{16} \text{ GeV})$
$m_{1/2}$	1%	251.9 ± 5.9	16.23 ± 0.29	252.3 ± 3.2
m_0	2%	98.5 ± 10.5	16.5 ± 0.6	100.8 ± 4.9

⇒ one more aspects to better do right

Likelihood

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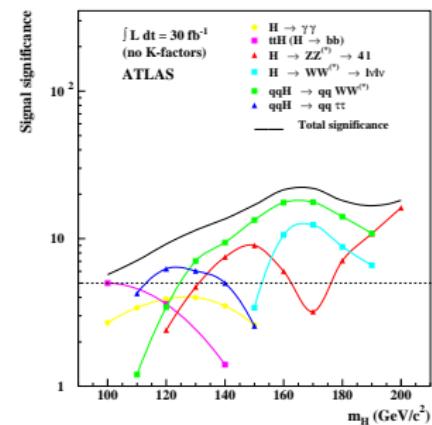
Higgs sector

Higgs hypotheses

Higgs sector

Higgs-sector analysis at the LHC [Zeppenfeld, Kinnunen, Nikitenko, Richter-Was; Dührssen et al.]

- optimistic LHC scenario: everything working and good data
- light Higgs around 120 GeV: 10 main channels ($\sigma \times BR$) [bb channel new]
- measurements: $GF : H \rightarrow ZZ, WW, \gamma\gamma$
 $WBF : H \rightarrow ZZ, WW, \gamma\gamma, \tau\tau$
 $VH : H \rightarrow b\bar{b}$ [Butterworth, Davison, Rubin, Salam]
 $t\bar{t}H : H \rightarrow \gamma\gamma, WW, (b\bar{b})\dots$
- parameters: couplings $W, Z, t, b, \tau, g, \gamma$ [plus Higgs mass]
- hope: cancel uncertainties
 $(WBF : H \rightarrow WW)/(WBF : H \rightarrow \tau\tau)$
 $(WBF : H \rightarrow WW)/(GF : H \rightarrow WW)\dots$



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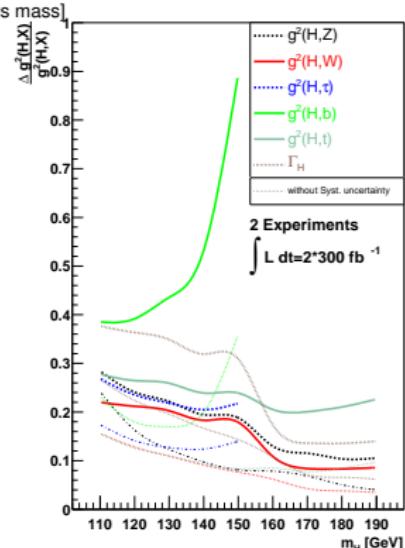
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Total width

- degeneracy: $\sigma BR \propto (g_p^2/\sqrt{\Gamma_H})(g_d^2/\sqrt{\Gamma_H})$
- additional constraint: $\sum \Gamma_i(g^2) < \Gamma_H \rightarrow \Gamma_H|_{\min}$
- $WW \rightarrow WW$ unitarity: $g_{WWH} \lesssim g_{WWH}^{\text{SM}} \rightarrow \Gamma_H|_{\max}$
- width extraction hard
- ⇒ **this analysis: $\Gamma_H = \sum_{\text{obs}} \Gamma_j$**



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Higgs couplings

SFitter analysis [Dührssen, Lafaye, TP, Rauch, Zerwas]

- all couplings varied around SM values $g_{HXX} = g_{HXX}^{\text{SM}} (1 + \delta_{HXX})$
 $\delta_{HXX} \sim -2$ means sign flip [$g_{HWW} > 0$ fixed]
- need assumption about loop-induced couplings $g_{ggH}, g_{\gamma\gamma H}$
- likelihood map and local errors from SFitter
- experimental/theory errors on signal and backgrounds [do not ask theorists!]

luminosity measurement	5 %
detector efficiency	2 %
lepton reconstruction efficiency	2 %
photon reconstruction efficiency	2 %
WBF tag-jets / jet-veto efficiency	5 %
b -tagging efficiency	3 %
τ -tagging efficiency (hadronic decay)	3 %
lepton isolation efficiency ($H \rightarrow 4\ell$)	3 %

σ (gluon fusion)	13 %
σ (weak boson fusion)	7 %
σ (VH -associated)	7 %
σ ($t\bar{t}$ -associated)	13 %

Higgs couplings

SFitter analysis [Dührssen, Lafaye, TP, Rauch, Zerwas]

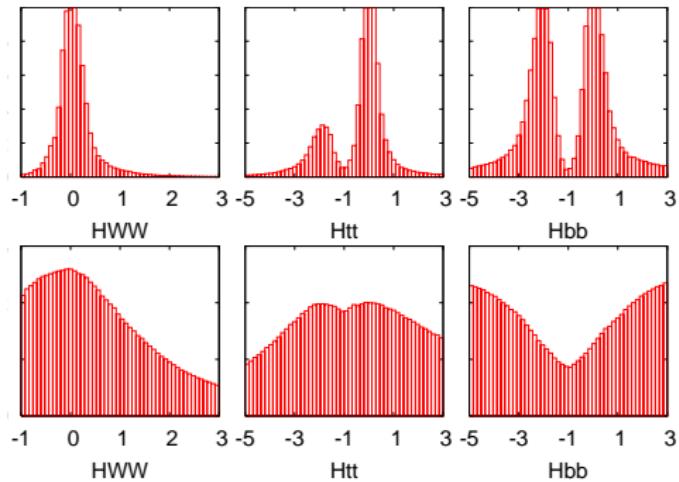
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- likelihood map and local errors from SFitter
- experimental/theory errors on signal and backgrounds [do not ask theorists!]
- error bars for Standard Model hypothesis [smeared data point, 30fb^{-1}]

coupling	without eff. couplings			including eff. couplings		
	σ_{symm}	σ_{neg}	σ_{pos}	σ_{symm}	σ_{neg}	σ_{pos}
δ_{WWH}	± 0.23	-0.21	+0.26	± 0.24	-0.21	+0.27
δ_{ZZH}	± 0.50	-0.74	+0.30	± 0.44	-0.65	+0.24
$\delta_{t\bar{t}H}$	± 0.41	-0.37	+0.45	± 0.53	-0.65	+0.43
$\delta_{b\bar{b}H}$	± 0.45	-0.33	+0.56	± 0.44	-0.30	+0.59
$\delta_{\tau\bar{\tau}H}$	± 0.33	-0.21	+0.46	± 0.31	-0.19	+0.46
$\delta_{\gamma\gamma H}$	—	—	—	± 0.31	-0.30	+0.33
δ_{ggH}	—	—	—	± 0.61	-0.59	+0.62
m_H	± 0.26	-0.26	+0.26	± 0.25	-0.26	+0.25
m_b	± 0.071	-0.071	+0.071	± 0.071	-0.071	+0.072
m_t	± 1.00	-1.03	+0.98	± 0.99	-1.00	+0.98

Higgs couplings

One-dimensional distributions to check....

1– noisy environment preferring profile likelihoods [no effective couplings, 30 fb^{-1}]



Likelihood

Errors

Toy model

MSSM

GUT

Higgs sector

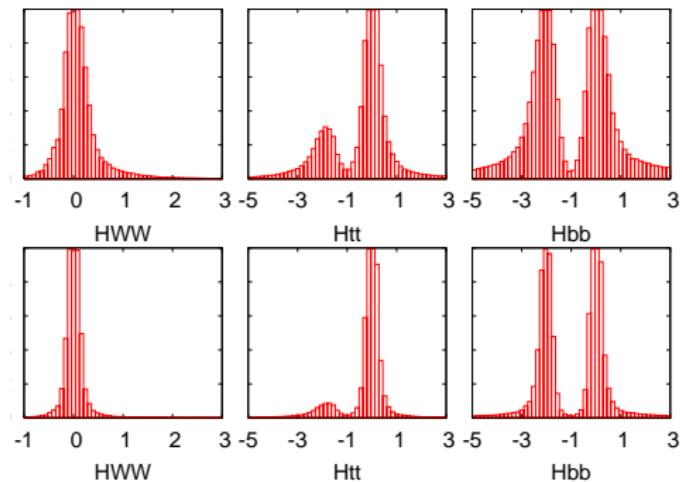
Higgs hypotheses

Higgs couplings

One-dimensional distributions to check....

1– noisy environment preferring profile likelihoods [no effective couplings, 30 fb^{-1}]

2– higher luminosity quantitatively different [no effective couplings, 30 vs 300 fb^{-1}]



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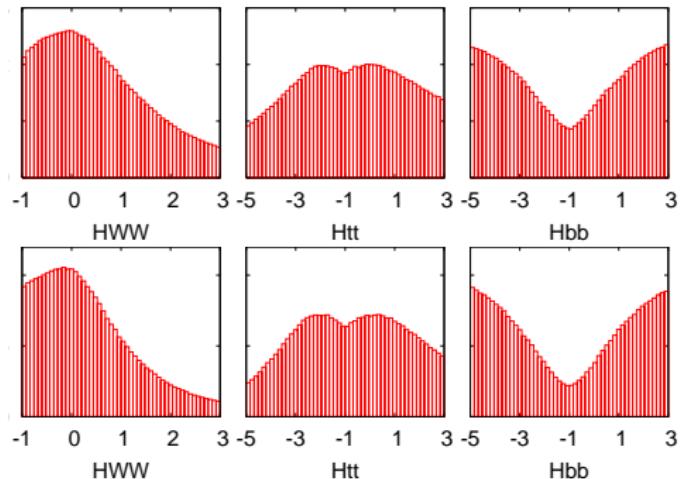
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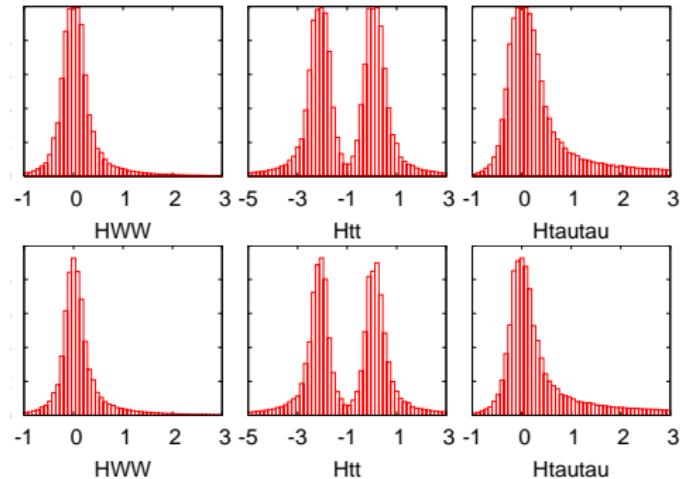
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One-dimensional distributions to check....

- 1– noisy environment preferring profile likelihoods [no effective couplings, 30 fb^{-1}]
- 2– higher luminosity quantitatively different [no effective couplings, 30 vs 300 fb^{-1}]
- 3– but not saving Bayesian statistics [no effective couplings, 300 fb^{-1}]
- 4– theory errors not dominant for 30 fb^{-1} [with effective couplings, 30 fb^{-1}]



⇒ profile likelihood promising for 30 fb^{-1} , errors a mess

Likelihood

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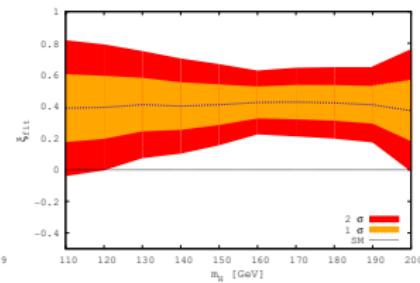
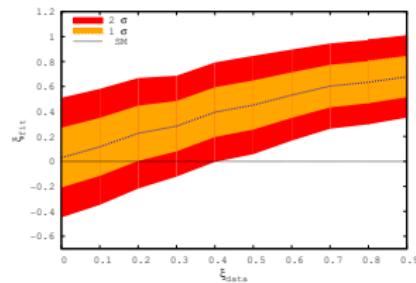
Higgs sector

Higgs hypotheses

Refining Higgs hypotheses

Strongly interacting Higgs at LHC [Espinosa, Grojean, Mühlleitner; SFitter + Block, Zerwas]

- looking like fundamental Higgs
- 1– all couplings scaled $g \rightarrow g\sqrt{1 - \xi}$
- one-parameter fit in SFitter [SFitter + Bock, P Zerwas]
- 30 fb^{-1} and 120 GeV Higgs: $\Delta g/g \sim 10\%$
best around $m_H \sim 160 \text{ GeV}$: $\Delta g/g \sim 5\%$



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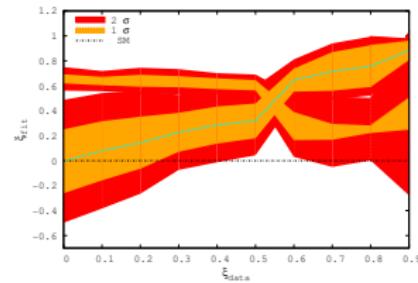
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- 2– gauge couplings $g \rightarrow g\sqrt{1 - \xi}$
Yukawas $g \rightarrow g(1 - 2\xi)/\sqrt{1 - \xi}$
 - sign change of Yukawas, $g_{\gamma\gamma H}$ correlated



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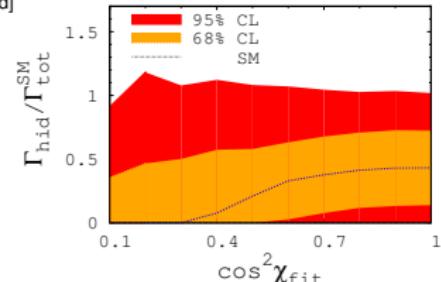
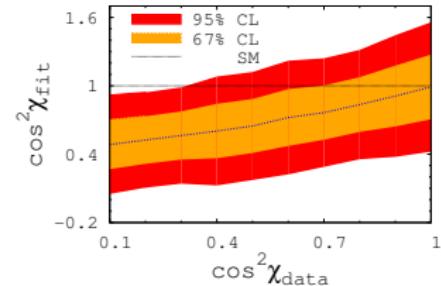
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Higgs portal

- universal scaling $\sqrt{1 - \xi} \equiv \cos \chi$

– invisible Higgs decay measurable [Eboli & Zeppenfeld]
two-parameter fit, project out Γ_{hid} or $\cos \chi$

⇒ hypotheses testable with 30 fb^{-1}



Sfitting

Tilman Plehn

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Outlook

High-dimensional parameter/measurement space everywhere

- MSSM [weak-scale new physics Lagrangian]
- running up to text unification
- Higgs operator analysis
- biased Higgs operator analysis

SFitter technicalities

- profile likelihood and Bayesian probability for SUSY
- unification test from likelihood map in GUT
- profile likelihood for Higgs sector
- Roberto's conclusions: there will never be 'enough statistics'

Likelihood

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SPSa1

Our favorite parameter point: SPS1a

- low masses to help ILC
- many decay chains to help LHC
- studied to death in LHC-ILC report
- SPS1a' with correct dark matter density
- mass spectrum

	m_{SPS1a}	LHC	ILC	LHC+ILC		m_{SPS1a}	LHC	ILC	LHC+ILC
h	108.99	0.25	0.05	0.05	H	393.69		1.5	1.5
A	393.26		1.5	1.5	H^+	401.88		1.5	1.5
χ_1^0	97.21	4.8	0.05	0.05	χ_2^0	180.50	4.7	1.2	0.08
χ_3^0	356.01		4.0	4.0	χ_4^0	375.59	5.1	4.0	2.3
χ_1^\pm	179.85		0.55	0.55	χ_2^\pm	375.72		3.0	3.0
\tilde{g}	607.81	8.0		6.5					
\tilde{t}_1	399.10		2.0	2.0					
\tilde{b}_1	518.87	7.5		5.7	\tilde{b}_2	544.85	7.9		6.2
\tilde{q}_L	562.98	8.7		4.9	\tilde{q}_R	543.82	9.5		8.0
\tilde{e}_L	199.66	5.0	0.2	0.2	\tilde{e}_R	142.65	4.8	0.05	0.05
$\tilde{\mu}_L$	199.66	5.0	0.5	0.5	$\tilde{\mu}_R$	142.65	4.8	0.2	0.2
$\tilde{\tau}_1$	133.35	6.5	0.3	0.3	$\tilde{\tau}_2$	203.69		1.1	1.1
$\tilde{\nu}_e$	183.79		1.2	1.2					

SPSa1

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many decay chains to help LHC
studied to death in LHC-ILC report
- SPS1a' with correct dark matter density
- endpoint measurements

type of measurement	nominal value	stat.	LES error	JES	theo.
m_h	108.99	0.01	0.25		2.0
m_t	171.40	0.01		1.0	
$m_{\tilde{t}_L} - m_{\chi_1^0}$	102.45	2.3	0.1		2.2
$m_{\tilde{g}} - m_{\chi_1^0}$	511.57	2.3		6.0	18.3
$m_{\tilde{q}_R} - m_{\chi_1^0}$	446.62	10.0		4.3	16.3
$m_{\tilde{g}} - m_{\tilde{b}_1}$	88.94	1.5		1.0	24.0
$m_{\tilde{g}} - m_{\tilde{b}_2}$	62.96	2.5		0.7	24.5
$m_{ll}^{\max}: \text{three-particle edge}(\chi_2^0, \tilde{t}_R, \chi_1^0)$	80.94	0.042	0.08		2.4
$m_{llq}^{\max}: \text{three-particle edge}(\tilde{q}_L, \chi_2^0, \chi_1^0)$	449.32	1.4		4.3	15.2
$m_{lq}^{\text{low}}: \text{three-particle edge}(\tilde{q}_L, \chi_2^0, \tilde{t}_R)$	326.72	1.3		3.0	13.2
$m_{ll}^{\max}(\chi_4^0): \text{three-particle edge}(\chi_4^0, \tilde{t}_R, \chi_1^0)$	254.29	3.3	0.3		4.1
$m_{\tau\tau}^{\max}: \text{three-particle edge}(\chi_2^0, \tilde{\tau}_1, \chi_1^0)$	83.27	5.0		0.8	2.1
$m_{lq}^{\text{high}}: \text{four-particle edge}(\tilde{q}_L, \chi_2^0, \tilde{t}_R, \chi_1^0)$	390.28	1.4		3.8	13.9
$m_{llq}^{\text{thres}}: \text{threshold}(\tilde{q}_L, \chi_2^0, \tilde{t}_R, \chi_1^0)$	216.22	2.3		2.0	8.7
$m_{llb}^{\text{thres}}: \text{threshold}(\tilde{b}_1, \chi_2^0, \tilde{t}_R, \chi_1^0)$	198.63	5.1		1.8	8.0

- challenge: find more LHC measurements
add flavor, $(g - 2)_\mu$, dark matter

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