Tilman Plehn

Counting jet

. .....

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Jet veto

New physic

### From Jet Scaling to Jet Vetos

Tilman Plehn

Heidelberg

CMS week, 2/2012

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Counting jets

Poisson

Stairca

Interpolatin

Jet veto

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### Jet counting

### Why count jets

- complete event reconstruction crucial at LHC  $_{[Higgs\ plus\ 0,1,2\ jets;\ jets\ plus\ \vec{p}_{T}]}$
- utilize tagging and recoil jets in Higgs searches
- identify decay jets in BSM searches
- reduce  $t \bar{t}$  and  $\tilde{g} \tilde{g}$  backgrounds
- $\Rightarrow d\sigma/dn_{\rm jets}$  just another distribution to cut on (?)

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#### Why not [intro: arXiv:0910.4182, Springer Lecture Notes]

- remember  $qg \rightarrow qZ$
- collinear divergence from g o qar q splitting [overlapping with soft divergence]

$$\int_{\rho_T^{\rm min}}^{\rho_T^{\rm max}} d\rho_T^2 \frac{C}{\rho_T^2} = 2 \int_{\rho_T^{\rm min}}^{\rho_T^{\rm max}} d\rho_T \; \rho_T \; \frac{C}{\rho_T^2} \simeq 2C \int_{\rho_T^{\rm min}}^{\rho_T^{\rm max}} d\rho_T \frac{1}{\rho_T} = 2C \; \log \frac{\rho_T^{\rm max}}{\rho_T^{\rm min}}$$

universal form following factorization

$$\sigma_{n+1} = \int \sigma_n \; \frac{dp_a^2}{p_a^2} dz \; \frac{\alpha_s}{2\pi} \; \hat{P}(z)$$

- universally divergent, fixed order poorly defined
- find object to 'renormalize' [i.e. absorbe universal divergence]

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#### DGLAP equation and jet-inclusive rates

re-organize perturbation series [sum collinear logs]

$$\sigma_{n+1}(x,\mu) \sim \frac{1}{n!} \left( \frac{1}{2\pi b_0} \log \frac{\alpha_s(\mu_0^2)}{\alpha_s(\mu^2)} \right)^n \int_{x_0}^1 \frac{dx_n}{x_n} \hat{P}\left(\frac{x}{x_n}\right) \cdots \int_{x_0}^1 \frac{dx_1}{x_1} \hat{P}\left(\frac{x_2}{x_1}\right) \sigma_1(x_1,\mu_0)$$

- DGLAP equivalent to 'infrared RGE'
- factorization scale as inclusive momentum cutoff [vanishing at all orders]

$$\sigma_{\text{tot}}(\mu) = \int_0^1 dx_1 \int_0^1 dx_2 \sum_{ij} f_i(x_1, \mu) f_j(x_2, \mu) \, \hat{\sigma}_{ij}(x_1 x_2 S, \mu)$$

- collinear jets automatically included

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## Poisson scaling

#### Theory: soft gluon radiation [Peskin & Schroeder Ch 6]

- example: photons off hard electron only abelian diagrams, successive radiation
- eikonal approximation

$$\mathcal{M}_{n+1} = g_s T^a \; \epsilon_{\mu}^*(k) \; \bar{u}(q) rac{q^{\mu} + \mathcal{O}(k)}{(qk) + \mathcal{O}(k^2)} \; \mathcal{M}_n$$

- factorization of 'hard process' and soft radiation factors
- Poisson distribution [normalized pdf for n if  $\bar{n}$  expected]

$$\sigma_n = \frac{\bar{n}^n e^{-\bar{n}}}{n!} \iff$$

$$\sigma_n = \frac{\bar{n}^n e^{-\bar{n}}}{n!} \qquad \Longleftrightarrow \qquad \boxed{R_{(n+1)/n} = \frac{\sigma_{n+1}}{\sigma_n} = \frac{\bar{n}}{n+1}}$$

#### Ingredients of Poisson distribution

- 1– radiation matrix element  $\bar{n}^n$ : abelian fine, non-abelian a little tricky [ISR-FSR, see loffe, Fadin, Lipatov]
- 2- phase space factor 1/n!: only combinatorics, matrix element ordered [angular ordering, color suppression]
- 3– normalization factor  $e^{-\bar{n}}$
- similar to parton shower for collinear regime



# Jet Scaling Tilman Plehn Counting jets Staircase scaling Experiment: from United Staircase scaling

Staircase

Jet veto

Experiment: from UA1 to ATLAS/CMS [Steve Ellis, Kleiss, Stirling]

Volume 154B, number 5,6

PHYSICS LETTERS

9 May 1985

#### W's, Z's AND JETS

S.D. ELLIS <sup>1,2</sup>, R. KLEISS and W.J. STIRLING CERN. CH 1211 Geneva 23. Switzerland

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Received 24 January 1985

The process  $p + p \rightarrow W^{\pm}$ ,  $Z^0$  plus 2 jets is discussed in the context of perturbative QCD. The magnitude of the expected rate for this process and the correlations anticipated between the jets are presented.

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#### Staircase

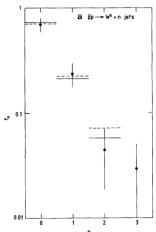
Jet veto

### Staircase scaling

#### Experiment: from UA1 to ATLAS/CMS [Steve Ellis, Kleiss, Stirling]

- W/Z+jets production
- many equivalent descriptions

$$R_{(n+1)/n} = \frac{\sigma_{n+1}}{\sigma_n} = \text{const}$$



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- same for inclusive and exclusive rates [Blackhat-Sherpa]

$$R_{(n+1)/n}^{\text{incl}} = \frac{\sum_{j=n+1}^{\infty} \sigma_j^{(\text{excl})}}{\sigma_n^{(\text{excl})} + \sum_{j=n+1}^{\infty} \sigma_j^{(\text{excl})}} = R_{(n+1)/n}^{\text{excl}}$$

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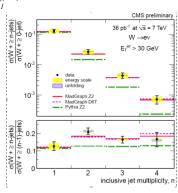
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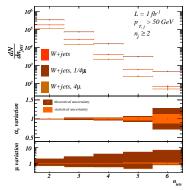
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#### Theoretical studies [Englert, TP, Schichtel, Schumann]

- CKKW/MLM merging [we used Sherpa]
- phase space effects?  $\rightarrow$  moderate
- $\alpha_s$  uncertainties?  $\rightarrow$  small
- scale uncertainties? → tuning parameter?



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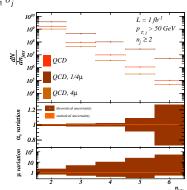
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### Theoretical studies [Englert, TP, Schichtel, Schumann]

- CKKW/MLM merging [we used Sherpa]
- phase space effects? → moderate
- $α_s$  uncertainties? → small
- scale uncertainties? → tuning parameter?
- same for QCD jets
- correctly described by ME-PS merging!?



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Interpolating Jet veto

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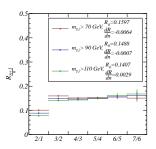
### Interpolating scaling patterns

#### Scaling for photon plus jets [Englert, TP, Schichtel, Schumann]

- naively, no scaling at all [Alex Tapper, private complaint]
- after appropriate cuts, great playground
- 1- staircase

democratic  $\gamma$  and jet acceptance large  $\gamma$ -jet separation [m or  $\Delta R$ , no large logs] no reason for ordered emission [1/n! in Poisson form] dominant: non-abelian splitting of ISR gluon

helping: pdf effect shifting to high-niets



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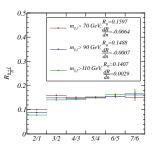
democratic  $\gamma$  and jet acceptance large  $\gamma$ -jet separation  $[m \text{ or } \Delta R, \text{ no large logs}]$  no reason for ordered emission [1/n! in Poisson form] dominant: non-abelian splitting of ISR gluon helping: pdf effect shifting to high- $n_{\text{jets}}$ 

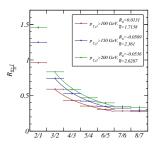
#### 2- Poisson

generate 'hard process'  $_{[m,\, \rho_T,\, \Delta R,\, \ldots]}$  lower general  $\rho_T^{\min}$  for soft-collinear logarithm rely on lot-enhanced radiation

dominant: successive ordered ISR remaining: high-n staircase tail

⇒ staircase–Poisson transition tunable!





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Jet veto in Higgs analyses [Barger, Phillips, Zeppenfeld; Rainwater]

particularly useful for WBF signals [remove t̄t and Z+jets backgrounds]

Jet veto in Higgs searches

- veto central (semi-hard) jets estimate P<sub>veto</sub> apply as 'efficiency factor'

> Table C.1: Summary of veto survival probabilities for  $p_T^{veto} = 20$  GeV used in Chapters 3-5.

search	Hjj	$t\bar{t}$	$t\bar{t}j,$	QCD	EW	QCD	QCD	DPS
			$tar{t}jj$	V(V)jj	V(V)jj	Wjjj	$bar{b}jj$	$\gamma\gamma jj$
$\gamma \gamma j j$	0.89	-	-	0.30	0.75	-	-	0.30
$W^{(*)}W^{(*)}jj$	0.89	0.46	0.29	0.29	0.75	-	-	-
au aj j	0.87	-	-	0.28	0.80	0.28	0.28	-

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### Jet veto in Higgs searches

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- problem in Higgs phenomenology

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#### In terms of jet counting [Gerwick, TP, Schumann]

- avoid P<sub>veto</sub> as one number
- study exclusive n<sub>iets</sub> distribution:
  - 1- understand basic features: staircase for inclusive samples Poisson for hard processes
  - 2- predict from theory [including error]
  - 3- validate simulation
  - 4- extrapolate to regimes/processes

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	staircase scaling	Poisson scaling
ση	$\sigma_0~{\rm e}^{-bn}$	$\sigma_{\text{tot}} \frac{e^{-\bar{n}}\bar{n}^n}{n!}$
$R_{(n+1)/n}^{\text{excl}}$	e <sup>-b</sup>	$\frac{\bar{n}}{n+1}$
$R_{(n+1)/n}^{\text{incl}}$	e-b	$\left(\frac{(n+1)e^{-\bar{n}}\bar{n}^{-(n+1)}}{\Gamma(n+1)-n\Gamma(n,\bar{n})}+1\right)^{-1}$
$\langle n_{\rm jets} \rangle$ $P_{\rm veto}$	$\frac{1}{2} \frac{1}{\cosh b - 1}$ $1 - e^{-b}$	īn .
P <sub>veto</sub>	1 - e <sup>-b</sup>	e <sup>- ī</sup>

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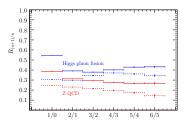
Jet veto

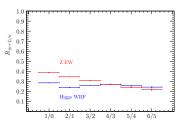
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### Jet veto in Higgs searches

Example: WBF H o au au [Gerwick, TP, Schumann]

- staircase scaling before WBF cuts [QCD and e-w processes]
- first emission sensitive to cuts
- e-w Zjj production with too many structures





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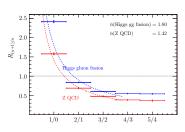
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### WBF cuts: two forward tagging jets

- count add'l jets to reduce backgrounds

$$p_T^{\text{veto}} > 20 \text{ GeV} \qquad \min y_{1,2} < y^{\text{veto}} < \max y_{1,2}$$

- Poisson for QCD processes ['radiation' pattern]



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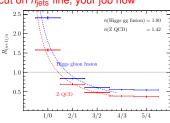
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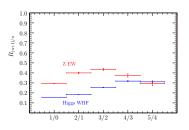
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$$p_T^{\text{veto}} > 20 \text{ GeV} \qquad \min y_{1,2} < y^{\text{veto}} < \max y_{1,2}$$

- Poisson for QCD processes ['radiation' pattern]
- staircase-like for e-w processes [generic]
- features of  $n_{\text{iets}}$  distributions understood

$$\Rightarrow$$
 cut on  $n_{\text{iets}}$  fine, your job now





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### Jet veto in Higgs searches

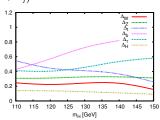
#### Example: WBF H o au au [Gerwick, TP, Schumann]

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#### Why we need the veto probabilities [SFitter: Lafaye, TP, Rauch, Zerwas]

- extraction of Higgs couplings [with error bars]
- including associated Higgs-jet channels [WBF eventually crucial]
- general fit of all couplings [Michael Rauch in Moriond 2012]

$$g_{jjH} = g_{jjH}^{SM} \left(1 + \Delta_j\right)$$



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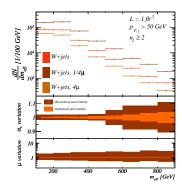
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New physics

### New physics

#### Effective mass [Englert, TP, Schichtel, Schumann]

- obviously sensitive to new physics masses but awful variable, after theory uncertainty
- correlation  $m_{\rm eff} \sim \langle p_T \rangle \times n_{\rm iets}$
- use merged sample for meff estimate scale and  $\alpha_s$  uncertainties



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Effective mass [Englert, TP, Schichtel, Schumann]

New physics

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Jet veto

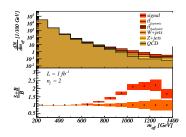
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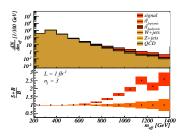
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 use merged sample for m<sub>eff</sub> estimate scale and  $\alpha_s$  uncertainties

estimate SUSY significance as function of m<sub>eff</sub>

but awful variable, after theory uncertainty





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- multijet studies establishing  $m_{\rm eff}$

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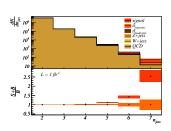
use merged sample for m<sub>eff</sub>

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Mass vs color charge

- now, significance as function of niets



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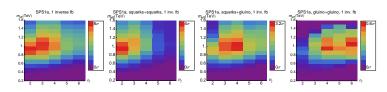
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- multijet studies establishing m<sub>eff</sub>

#### Mass vs color charge

- now, significance as function of n<sub>jets</sub>
- representing new physics color charge [gluino does not decay via gluon]
- exclusive 2D likelihood including all information



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### Exclusive jet counting

#### staircase (non-abelian) vs Poisson (ordered)

- start by measuring  $d\sigma/dn_{\rm jets}$
- test both regimes in photon+jets
- described by ME-PS merging [CKKW/MLM, unchanged by NLO]
- key to jet vetos
- key to SUSY searches

Understanding Jet Scaling and Jet Vetos in Higgs Searches E Gerwick, TP, S Schumann PRL 108 (2012)

Establishing Jet Scaling Patterns with a Photon C Englert, TP, P Schichtel, S Schumann arXiv:1108.5473, JHEP in print

Jets plus Missing Energy with an Autofocus C Englert, TP, P Schichtel, S Schumann PRD83 (2011)

Much of this work was funded by the BMBF Theorie-Verbund which is ideal for hard and relevant LHC work



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Interpolating Jet veto

New physics

### New physics at the LHC

	missing	cascade	mono-	lepton	di-jet	top	WW/ZZ	W'	top	charged	displ.	multi-	spherical
	energy	decays	jets/photon	resnce	resnce	resnce	resnce	resnce	partner	tracks	vertex	photons	events
	(p.89)	(p.91)	(p.15)	(p.109)	(p.109)	(p.120)	(p.15)	(p.93)	(p.116)	(p.123)	(p.123)	(p.29)	(p.47,76)
SUSY (heavy grav.)	11	11							<b>√</b>				
(p.17,26)	• •	· •							•				
SUSY (light grav.)	<b>/</b>	<b>√</b>	<b>√</b>						<b>√</b>	<b>√</b>	<b>/</b>		
(p.17,27)	<b>"</b>	<b>'</b>	· •						•	<b>'</b>	<b>`</b>		
large extra dim	11		<b>//</b>										
(p.39)	V V		V V										<b>√</b>
universal extra dim	11	11		<b>√</b>	1	<b>√</b>	<b>\</b>	<b>√</b>	<b>√</b>				
(p.47)	V V	V V		<b>V</b>	<b>V</b>	<b>'</b>	· •	<b>'</b>	<b>V</b>				
technicolor (vanilla)				/	_	_	_	//					
(p.51)				<b>√</b>	<b>√</b>	<b>√</b>	<b>√</b>	<b>~</b> ~					
topcolor/top seesaw					<b>√</b>	11	1						
(p.53,54)					·	V V	· •						
little Higgs (w/o T)					_	_	_	_					
(p.55,58)				✓	✓	✓	<b>√</b>	✓					
little Higgs (w T)	//	//						_					
(p.55,58)	$ \checkmark \checkmark $	$ \checkmark \checkmark $	✓	✓	<b>√</b>	<b>√</b>	<b>√</b>	<b>√</b>	<b>√</b>				
warped extra dim (IR SM)					/	/							
(p.61,63)				<b>√</b>	<b>√</b>	<b>√</b>	<b>√</b>						
warped extra dim (bulk SM)						//		_					
(p.61,64)				✓	✓	<b>√</b> √	<b>√</b>	<b>√</b>					
Higgsless/comp. Higgs					_	//	//						
(p.69,73)				<b>√</b>	<b>√</b>	<b>√</b> √	<b>V</b>						
hidden valleys													
(p.75)	✓	✓	✓	<b>√</b>	<b>√</b>	<b>√</b>	<b>√</b>	<b>√</b>	<b>√</b>	✓	✓	✓	✓
(p.10)	l						l			l		I	

[arXiv:0912.3259, Morrissey, TP, Tait]