Higgs CP Through Information Geometry

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CP symmetry tests

C and *P* and *T* and $\hat{T}$

– transformations on state with spin/momentum [review: Valencia]

$$C\,|\phi(p,s)\rangle = \left|\phi^*(p,s)\right\rangle \quad P\,|\phi(p,s)\rangle = \eta_\phi\,|\phi(-p,s)\rangle \quad T\,|\phi(p,s)\rangle = \langle\phi(-p,-s)|$$

– transformation of complex scalar

$$C\phi(t,\vec{x})C^{-1} = \eta_C\phi^*(t,\vec{x}) \qquad P\phi(t,\vec{x})P^{-1} = \eta_P\phi(t,-\vec{x}) \qquad T\phi(t,\vec{x})T^{-1} = \phi(-t,\vec{x})$$

– *CPT* symmetry generally assumed, *T* proxy for *CP*

– naive time reversal $\hat{T}$ avoiding inital $\leftrightarrow$ final state

$$\hat{T}\,|\phi(p,s)\rangle = |\phi(-p,-s)\rangle$$

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$$\hat{T}\,|\phi(p,s)\rangle = |\phi(-p,-s)\rangle$$

– genuine *U*-odd is what we want [$U = C, P, \hat{T}$, Atwood, Bar-Shalom, Eilam]

$$\langle O\rangle_{\mathcal{L}=U\mathcal{L}U^{-1}} = 0\ .$$

– *U*-odd is what we usually use

$$O(U\,|i\rangle \to U\,|f\rangle) \overset{\text{odd}}{=} -O\,(|i\rangle \to |f\rangle) \quad \overset{p(|i\rangle)=p(U|i\rangle)}{\Longrightarrow} \quad \langle O\rangle_{\mathcal{L}=U\mathcal{L}U^{-1}} = 0\ .$$

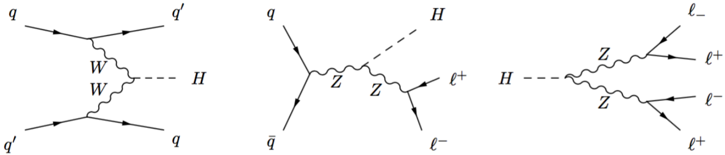
– finite genuine $\hat{T}$-odd measurement means *CP*-violating theory, provided

1- phase space $\hat{T}$-symmetric
2- initial state distribution invariant under $\hat{T}$
3- no re-scattering

$\Rightarrow$ use $\hat{T}$ as proxy to *CP*

Naive time reversal at LHC

Processes with same vertex/information



– four 4-momenta defining 10 observables [Han, Li,...]
  four scalar products giving external masses
  four *C*-even, *P*-even, and $\hat{T}$-even scalar products
  two *C*-odd, *P*-even, and $\hat{T}$-even scalar products
  plus *C*-even, *P*-odd, and $\hat{T}$-odd observable from Levi-Civita-tensor

1– *CP*-odd and $\hat{T}$-odd
  for $q\bar{q} \to ZH$ also genuine *CP*-odd and genuine $\hat{T}$-odd
  non-zero expectation value means *CP* violation

2– *CP*-odd and $\hat{T}$-even
  for $q\bar{q} \to ZH$ also genuine *CP*-odd
  if theory *CP*-violating, *CP*-expectation value should be non-zero
  without re-scattering, $\hat{T}$-expectation value zero
  with re-scattering or complex phase $\langle O \rangle$ matches symmetry

⇒ focus on one *CP*-odd and $\hat{T}$-odd observable

CP in SMEFT Lagrangian

Dimension-6 Lagrangian

– for EFT fit choose symmetries first

– *CP*-conserving couplings [defining Higgs properties]

$$\mathcal{O}_B = i\frac{g}{2}\,(D^\mu\phi^\dagger)(D^\nu\phi)\,B_{\mu\nu} \qquad \mathcal{O}_W = i\frac{g}{2}\,(D^\mu\phi)^\dagger\sigma^k(D^\nu\phi)\,W^k_{\mu\nu}$$

$$\mathcal{O}_{BB} = -\frac{g'^2}{4}\,(\phi^\dagger\phi)\,B_{\mu\nu}\,B^{\mu\nu} \qquad \mathcal{O}_{WW} = -\frac{g^2}{4}\,(\phi^\dagger\phi)\,W^k_{\mu\nu}\,W^{\mu\nu\,k}$$

$$\mathcal{O}_{\phi,2} = \frac{1}{2}\,\partial^\mu(\phi^\dagger\phi)\,\partial_\mu(\phi^\dagger\phi)$$

– *CP*-violating couplings [defining *CP*-violation]

$$\mathcal{O}_{B\tilde{B}} = -\frac{g'^2}{4}\,(\phi^\dagger\phi)\,\widetilde{B}_{\mu\nu}\,B^{\mu\nu} \qquad \mathcal{O}_{W\widetilde{W}} = -\frac{g^2}{4}\,(\phi^\dagger\phi)\,\widetilde{W}^k_{\mu\nu}\,W^{\mu\nu\,k}$$

– dimension six means non-SM momentum dependence

– link to loop-induced coupling: $\text{Tr}(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5) \to \epsilon_{\mu\nu\rho\sigma}$

– complex phases [re-scattering]

typical from absorptive diagrams with light particles
not forseen in EFT approach
mimick with complex *CP*-conserving Wilson coefficients, lacking better idea

$\Rightarrow$ all estabished and known

Information geometry

Quantify what is there to learn

– covariance matrix [measurement error in model space $\mathbf{g}$]

$$C_{ij}(\mathbf{g}) \equiv E\left[(\hat{g}_i - \bar{g}_i)(\hat{g}_j - \bar{g}_j)\middle|\mathbf{g}\right]$$

– Fisher information [sensitivity in model space]

$$I_{ij}(\mathbf{g}) \equiv -E\left[\frac{\partial^2 \log f(\mathbf{x}|\mathbf{g})}{\partial g_i\, \partial g_j}\middle|\mathbf{g}\right]$$

– phase space distribution [phase space $\mathbf{x}$, additive]

$$I_{ij} = \frac{L}{\sigma}\, \partial \sigma g_i\, \partial \sigma g_j - L\, \sigma\, E\left[\frac{\partial^2 \log f^{(1)}(\mathbf{x}|\mathbf{g})}{\partial g_i\, \partial g_j}\right]$$

– Cramèr-Rao bound defining lowest possible covariance

$$C_{ij}(\mathbf{g}) \geq (I^{-1})_{ij}(\mathbf{g})$$

– model-space distance [probability to measure $\mathbf{g}_b$ with true $\mathbf{g}_a$]

$$d(\mathbf{g}_b; \mathbf{g}_a) = \sqrt{(\mathbf{g}_a - \mathbf{g}_b)_i\, I_{ij}(\mathbf{g}_a)\, (\mathbf{g}_a - \mathbf{g}_b)_j}$$

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1– parametrization-invariant elipses of constant distance/reach in model space

2– diagonalize I_{ij}, define model-space eigenvectors

3– compute information in distributions or phase space regions

$\Rightarrow$ best tool to understand modern analyses

WBF production

Testing CP in WBF

– four external 4-momenta $\to$ 10 scalar products

four external masses [zero]
four C-even, P-even, $\hat{T}$-even [not interesting]
two C-odd, P-even, $\hat{T}$-even [not possible]
one C-even, P-odd, $\hat{T}$-odd [yeah]

$$\epsilon_{\mu\nu\rho\sigma}\; k_1^\mu\; k_2^\nu\; q_1^\rho\; q_2^\sigma \quad \to \quad O \equiv \epsilon_{\mu\nu\rho\sigma}\; k_1^\mu\; k_2^\nu\; q_1^\rho\; q_2^\sigma\; \text{sign}\left[(k_1 - k_2)\cdot(q_1 - q_2)\right]$$

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- azimuthal angle difference [lab frame]

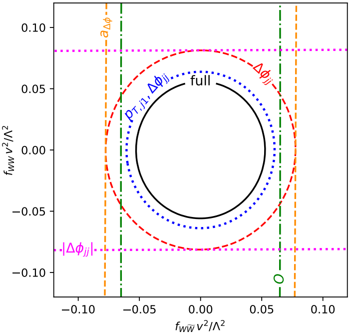
$$O = 2E_-(\vec{q}_- \times \vec{q}_+)\cdot \vec{k}_+ \quad \to \quad \sin\Delta\phi_{jj}$$

- CP asymmetry

$$a_{\Delta\phi_{jj}} \equiv \frac{\mathrm{d}\sigma(\Delta\phi_{jj}) - \mathrm{d}\sigma(-\Delta\phi_{jj})}{\mathrm{d}\sigma(\Delta\phi_{jj}) + \mathrm{d}\sigma(-\Delta\phi_{jj})}$$

- separating dimension-6 effects
- check with imaginary Wilson coefficients

$\Rightarrow$ testing CP, but assuming no re-scattering



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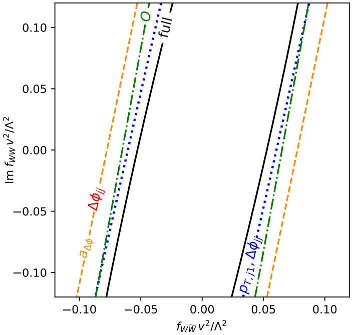

ZH production

Testing *CP* in *ZH* production

– same 10 scalar products as for WBF

– *CP*-odd and $\hat{T}$-odd angle as for WBF

$$O_1 = \epsilon_{\mu\nu\rho\sigma}\; k_1^\mu k_2^\nu q_{\ell+}^\rho q_{\ell-}^\sigma \;\text{sign}((k_1 - k_2)\cdot(q_1 - q_2)) \quad \to \quad \sin\Delta\phi_{\ell\ell}$$

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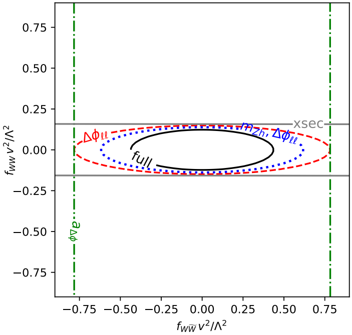
⇒ testing *CP* without assumtions [to leading order]



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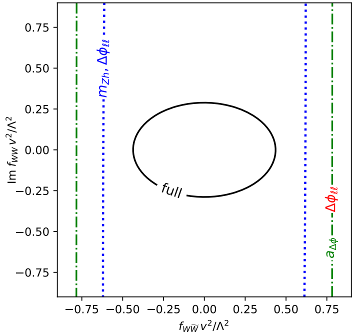

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- separating dimension-6 effects

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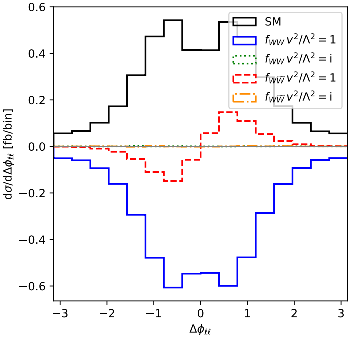
- *CP*-odd and $\hat{T}$-even, requiring second phase

$$O_2 \to \Delta E_{\ell\ell} \qquad\qquad O_3 \to \Delta p_{T,\ell\ell}$$

$\Rightarrow$ interesting subsequent test



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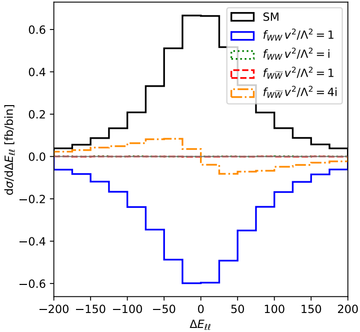


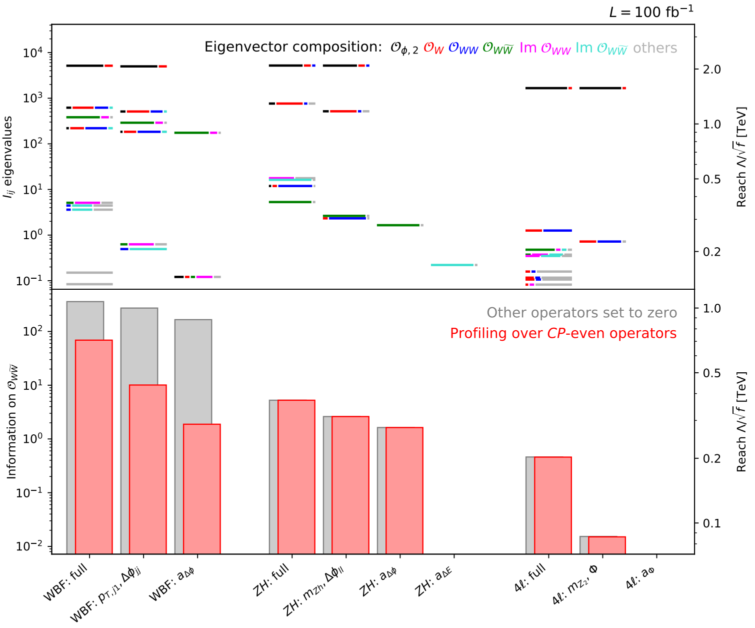
Higgs decays

Testing *CP* in $H \to 4\ell$ decays

– again same 10 scalar products
– momentum flow limited by m_H
– reach for *CP*-even operators shit [1612.05261]
– even making slide is waste of time

⇒ what's the point...

Comparison


$L = 100$ fb^{-1}
Eigenvector composition: $\mathcal{O}_{\phi,2}$ $\mathcal{O}_W$ $\mathcal{O}_{WW}$ $\mathcal{O}_{W\widetilde{W}}$ Im $\mathcal{O}_{WW}$ Im $\mathcal{O}_{W\widetilde{W}}$ others
I_{ij} eigenvalues
Reach $\Lambda/\sqrt{f}$ [TeV]
Other operators set to zero
Profiling over CP-even operators
Information on $\mathcal{O}_{W\widetilde{W}}$
WBF: full
WBF: $p_{T,j1}$, $\Delta\phi_{jj}$
WBF: $a_{\Delta\phi}$
ZH: full
ZH: m_{Zh}, $\Delta\phi_{ll}$
ZH: $a_{\Delta\phi}$
ZH: $a_{\Delta E}$
4ℓ: full
4ℓ: m_{Z_2}, Φ
4ℓ: a_Φ