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Electroweak Sudakov Logarithms in Associated WH-Production at the LHC

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Zusammenfassung

Typischerweise erwartet man, dass die elektroschwachen Korrekturen relativ klein sind. Das ändert sich allerdings bei gewissen hohen Energien. Bei diesen Energien werden die Korrekturen durch so genannte Sudakov-Logarithmen verstärkt. In dieser Arbeit werden wir untersuchen, welche Topologien bzw. welche Strukturen, diese großen Sudakov-Logarithmen hervorrufen. Um das zu beobachten werden wir den Prozess der Higgs-W-Produktion am LHC, $q\bar{q} \rightarrow HW$ analysieren. Anschließend führen wir ein Born-modifiziertes quadriertes Matrixelement ein, das ausschließlich die Sudakov-Beiträge, die von den Dreieckskorrekturen kommen, enthält. Dieses vergleichen wir dann mit dem LO-Prozess und der kompletten elektroschwachen Korrektur. Am Ende wenden wir uns noch den Box Diagrammen und ihren Sudakov Logarithmen zu.

Abstract

Typically, one expects that the electroweak (EW) one-loop corrections are relatively small. However, this changes in certain high energy limits where these corrections are enhanced through the so-called Sudakov logarithms. We will investigate what topologies give rise to these large, Sudakov logarithms. For this study we analyse the process of associated-Higgs production at the LHC, $q\bar{q} \rightarrow HW$. We introduce a Born-improved matrix-element-squared which contains only the Sudakov corrections coming from the triangle corrections. We compare this to both the leading-order and the full NLO Electroweak results. We will also discuss the box diagrams and the Sudakov logarithms appearing there.

Contents

1. Introduction

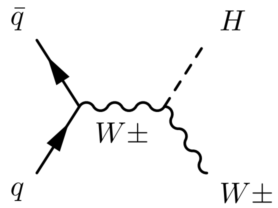


Figure 1.1.: Associated WH-production

With current and future colliders in mind the evaluation of electroweak corrections has become an important part in the search for new physics signals. At the TeV scale the electroweak corrections are enhanced by large, negative, double (or, squared) logarithmic terms. This thesis will search for the origin of those factors inside the EW one-loop corrections of the process of associated-Higgs production at the LHC, $q\bar{q} \to HW$.

In this section we briefly introduce the Standard Model (SM) and then move to the Higgs mechanism, thereby describing the origin of mass in the SM. In Section 2 we give a general overview of the calculation of a full hadronic cross-section within this model. In Section 3 we then outline the general features of the large Sudakov logarithms and how they arise in the large-energy limit of the scalar integrals C_0 and D_0. We then give the standard matrix elements of the triangle and box diagrams arising in associated WH production at the LHC with focus on the Sudakov logarithms only.

1.1. Standard Model

Modern particle physics is the theory of the smallest particles we know so far, the elementary particles and their interactions with the electromagnetic, weak and strong forces. The elementary particles are split up into two different types, the matter particles, fermions - which are further divided into quarks and leptons, and the force-carrier particles, the bosons.

The quarks and leptons consist of six particles each, which are grouped into pairs, or 'generations'. The electromagnetic force has infinite range and therefore is measurable on a macroscopic level. Conversely, the weak and strong forces each have small ranges and dominate only at the subatomic level. It is precisely to study these forces that large, high-energy colliders must be built.

The strong force is mediated by the gluon, of which there are eight types, or 'colors'. The photon is the carrier particle of the electromagnetic force. Both the gluons and photons are massless. The W^+, W^-, and Z bosons carry the weak force and are massive. The mass is acquired through a mechanism known as Electroweak symmetry breaking. This will be explained later in more detail.

The Standard Model is a theory which quantifies, to very high precision, the strength and types of interactions which can occur between the fermions and boson. We note that the gravitaitonal force, the most familar force on our macroscopic level, is not included in the Standard Model. Attempts to describe the gravitational force as being mediated by a particle-like boson (the graviton) are so far not physically consistent. That is, predictions for the rates and strengths of gravitational interactions in the framework of the Standard Model give divergent results. At the energies involved with stuying the subatomic level the effect of the gravity is negligible. [17]

We close this brief overview of the Standard Model with a tabulation of all known fermions and bosons:

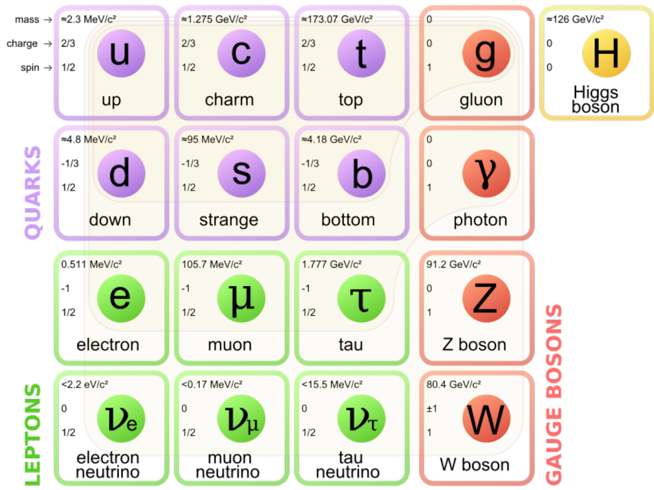


Figure 1.2.: The Standard Model[18]

1.2. Higgs Mechanism

The Higgs mechanism is considered by many to be the most straightforward way to account for masses for the fundamental particles. This mechanism is essentially nothing more than considering that there is another fundamental field, namely the Higgs field, which, upon interactions with another fundamental field convey a mass to it. As the implementation of the Higgs mechanism into the Standard Model is quite lengthy, we consider in the subsection below a toy model which would give rise to a massive photon field. We then discuss how one could extend this to assign masses to the fields with known mass. The discussion of this section is based on [7] and [8].

1.2.1. Abelian Higgs Model

First we apply the Higgs mechanism to an abelian, $U(1)$ gauge theory to get a better understanding of where the mass of the corresponding gauge boson (here the photon) comes from. The kinetic term of the photon is given by,

$$\mathcal{L}_{kin} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{1.1}$$
$$\text{where } F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu, \tag{1.2}$$

with A_ν representing the photon field.
This Lagrangian is invariant under a local gauge transformation of the photon field, $A_\mu(x) \to A_\mu(x) - \partial_\mu \eta(x)$ for any η and x^μ. To impart mass to the photon we can simply add an additional kinetic term to our Lagrangian,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu. \tag{1.3}$$

The problem is that this new Lagrangian clearly breaks local gauge invariance. We can fix this and still obtain a massive photon field by extending the model and introducing a complex scalar field with charge $-e$ that couples to itself and to the photon.

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi), \tag{1.4}$$
$$\text{where} \quad D_\mu = \partial_\mu - ieA_\mu$$
$$\text{and} \quad V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2.$$

This $V(\phi)$ is the most general, renormalizable potential allowed by $U(1)$ gauge invariance. This Lagrangian is now invariant under global $U(1)$ rotations, $\phi \to e^{i\theta}\phi$, and local gauge transformations of both fields,

$$A_\mu \to A_\mu(x) - \partial_\mu\eta(x),$$
$$\phi(x) \to e^{-ie\eta(x)}\phi(x).$$

The next step towards incorporating mass is crucial. We must consider the sign of the parameter μ^2 that we have introduced into the potential. We could choose $\mu^2 > 0$ or $\mu^2 < 0$. If $\mu^2 > 0$ is chosen, the state of lowest energy equals $\phi = 0$, the vacuum state. This therefore describes a simple Quantum Electrodynamics (QED) model with a massless photon and an extra charged scalar field ϕ with mass μ.

However, if $\mu^2 > 0$, the vaccum expectation value (VEV) is nonzero and equals

$$\langle\phi\rangle = \sqrt{\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$$

To make the Higgs boson field manifest, let us now parameterize ϕ as

$$\phi = \frac{v+h}{\sqrt{2}} e^{i\frac{\chi}{v}},$$

where h and χ (the Higgs boson and the Goldsone boson) are real scalar fields which have no VEV. The Lagrangian (1.4) then transforms into

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - evA_\mu\partial^\mu\chi + \frac{e^2v^2}{2}A_\mu A^\mu \\ &+ \frac{1}{2}(\partial_m uh\partial^\mu h - 2\mu^2h^2) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + (h, \chi\text{-interactions}).\end{aligned} \tag{1.5}$$

This is now a theory with a photon of mass $m_A = ev$, a Higgs boson h with mass $m_h = \sqrt{2\lambda}v$, and a massless Goldstone χ. We can get rid of the nonphysical $\chi - A$ mixing by making the following gauge transformation:

$$A_\mu \to A'_\mu = A_\mu - \frac{1}{ev}\partial_\mu\chi$$

This is the so called unitary gauge. The Goldstone χ will then disappear from the theory, [7].

1.2.2. Electroweak Standard Model

We now bring the elements of the toy model of the previous section to life by describing how to embed it into the portion of the Standard Model which describes Electroweak interactions. Formally, the Electroweak Standard Model is a $SU(2)_L \otimes U(1)_Y$ gauge theory with three $SU(2)_L$ gauge bosons fields, $W^i_\mu (i = 1, 2, 3)$, and one $U(1)_Y$ gauge boson field, B_μ. The Lagrangian of this model is neatly divided into four subsets, each dealing with a unique class of interactions involving the fermion and the gauge boson fields,

$$\mathcal{L} = \mathcal{L}_{\text{ferm}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yuk}}. \tag{1.6}$$

We now examine in turn each of the four subsets, moving from left to right in equation (1.6) above, beginning with the fermion portion of the Lagrangian which describes only the interactions between the fermions and the gauge boson fields. It reads,

$$\mathcal{L}_{\text{f}} = i\overline{\Psi}_L \not{D} \Psi_L + i\overline{\Psi}_R \not{D} \Psi_R. \tag{1.7}$$

The L and R refer to the left and right chiral projections onto the fermion fields,

$$\Psi_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\Psi. \tag{1.8}$$

Furthermore, the left-handed fermion fields (Ψ_L) are arranged into separate doublets for the quark and lepton fields,

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}, \qquad l_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix},$$

of which for each there are three generations. The right-handed fermion fields (Ψ_R)

$$u_R, d_R, \nu_{eR}, e_R^-$$

are singlets, of which there are also three, one for each generation of fermions.
These fields are invariant under the gauge transformations

$$\begin{aligned} \Psi_L \to \Psi_L' &= e^{iY_L\theta(x)} U_L \Psi_L, \\ \Psi_R \to \Psi_R' &= e^{iY_R\theta(x)} \Psi_R, \end{aligned}$$

where $U_L = e^{iT^i\beta^i(x)}$ is the $SU(2)_L$ transformation which only acts on the doublet fields. $T^i = \frac{\tau^i}{2}(\tau^i$ are the three Pauli matrices) denote the generators of the fundamental representation of the $SU(2)_L$ Lie algebra with the identity

$$[T^i, T^j] = i\varepsilon^{ijk}T^k.$$

The covariant derivative operates on Ψ_L and Ψ_R as

$$\begin{aligned} D_\mu \Psi_L &= (\partial_\mu + igW_\mu + ig'Y_L B_\mu)\Psi_L, \\ D_\mu \Psi_R &= (\partial_\mu + ig'Y_R B_\mu)\Psi_R. \end{aligned}$$

The transformation properties of B_μ and $W_\mu = W_\mu T^i$ are fixed by the gauge symmetry of the fermion Lagrangian,

$$\begin{aligned} B_\mu \to B_\mu' &= B_\mu - \frac{1}{g'}\partial_\mu\theta, \\ W_\mu \to W_\mu' &= U_L W_\mu U_L^\dagger + \frac{1}{g}(\partial_\mu U_L)U_L^\dagger. \end{aligned}$$

The three $SU(2)_L$ gauge bosons W^i couple to the weak-isospin T and the one $U(1)_Y$ gauge boson B couples to the hypercharge. The electroweak symmetry will turn out to be spontaneously broken. This will generate masses for the physical gauge bosons $W^\pm$ and Z.
Note that the electric charge is defined as the sum of the hypercharge and the third component of the weak-isospin,

$$Q = T^3 + \frac{Y}{2} = \frac{(\tau_3 + Y)}{2}$$

with $T^3 = \frac{\tau}{2}$ for left-handed doublets and $T^3 = 0$ for right-handed singlets. Hence we get the following eigenvalues for the leptons and quarks:

$$\begin{array}{lll} Y(l_L) = -1 & Y(l_R) = -2 & \\ Y(q_L) = \dfrac{1}{3} & Y(u_R) = \dfrac{4}{3} & Y(d_R) = -\dfrac{2}{3} \end{array}$$

When all the dust settles, we can simply interpret $\mathcal{L}_{\text{ferm}}$ as quantifying the strength of interaction between each fermion field with each boson field. This concludes the discussion of $\mathcal{L}_{\text{ferm}}$.

We turn next to the second term in (1.6). This term describes the self-interactions of the gauge boson fields.

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} W^i_{\mu\nu} W^{\mu\nu i} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \tag{1.9}$$

where

$$\begin{aligned} W^i_{\mu\nu} &= \partial_\nu W^i_\mu + g\varepsilon^{ijk} W^j_\mu W^k_\nu, \\ B_{\mu\nu} &= \partial_\nu B_\mu - \partial_\mu B_\nu. \end{aligned} \tag{1.10}$$

The first term of (1.9) gives rise to cubic and quardic self-interactions among the gauge fields. For the fermions it is quite simple to add a mass term such as the Dirac mass term,

$$m\overline{\psi}\psi = m(\overline{\psi}_L + \overline{\psi}_R)(\psi_L + \psi_R) = m(\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L)$$

We can't do that in the case of the gauge bosons. This is the step where the spontaneous symmetry breaking gives rise to the massive physical gauge bosons.

Spontaneous Symmetry Breaking

We now have a complex scalar $SU(2)$ doublet Φ which is coupled to the gauge fields

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

with a scalar potential

$$V(\Phi) = \mu^2 |\Phi^\dagger \Phi| + \lambda \left(|\Phi^\dagger \Phi| \right)^2 \qquad (\lambda > 0). \tag{1.11}$$

Again we have chosen the most general renormalizable potential. Addionally the term is $SU(2)_L$ invariant. Once more we look at the state of minimum energy. For $\mu^2 < 0$ it develops a VEV as before in the Abelian model. We can choose

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix},$$

because the direction of the minimum in $SU(2)$ is not fixed since the potential only depends on terms with $\Phi^\dagger\Phi$. The scalar doublet has now $U(1)_Y$ charge (hypercharge) $Y_\Phi = 1$ and the electromagnetic charge is hence

$$Q\langle\Phi\rangle = 0. \tag{1.12}$$

The electromagnetism is then unbroken by the scalar VEV. This means that the VEV from above provides the wanted symmetry breaking scheme,

$$SU(2)_L \times U(1)_Y \to U(1)_{EM}. \tag{1.13}$$

We will now see how the higgs mechanism generates masses for the gauge bosons W and Z. It works in the same way as in the Abelian model. The contribution of the scalar doublet to the Lagrangian is

$$\mathcal{L}_s = (D^\mu\Phi)^\dagger(D_\mu\Phi) - V(\Phi), \tag{1.14}$$

where

$$D_\mu = \partial_\mu + i\frac{g}{2}\tau W_\mu + i\frac{g'}{2}B_\mu Y.$$

If we choose the unitary gauge again, only the physical Higgs remains in the spectrum after the spontaneous symmetry breaking. All the Goldstone bosons will disappear. As a result the scalar doublet in unitary gauge can be written as

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}. \tag{1.15}$$

This will lead to the following masses for the gauge bosons from the scalar kinetic energy term of eq. 1.14. The physical gauge fields are then two charged fields $W^\pm$ and two neutral gauge bosons, Z and A. It is

$$\begin{aligned} W_\mu^\pm &= \frac{1}{2}(W_\mu^1 \mp iW_\mu^2), \\ Z_\mu &= \frac{-g'B_\mu + gW_\mu^3}{\sqrt{g^2+g'^2}}, \\ A_\mu &= \frac{gB_\mu + g'W_\mu^3}{\sqrt{g^2+g'^2}}. \end{aligned} \tag{1.16}$$

The masses then are

$$\begin{aligned}
M_W^2 &= \frac{1}{4}g^2v^2, \\
M_Z^2 &= \frac{1}{4}(g^2+g'^2)v^2, \\
M_A &= 0.
\end{aligned} \tag{1.17}$$

Notice that the massless photon has to couple with the electromagnetic force e. Therefore the coupling constants define the weak mixing angle θ_W,

$$\begin{aligned}
e &= g\sin\theta_W, \\
e &= g'\cos\theta_W.
\end{aligned} \tag{1.18}$$

Like in the Abelian system if we go into another gauge other than the unitary, there will be Goldstone bosons and the scalar field can be written as

$$\Phi = \frac{1}{\sqrt{2}}e^{i\frac{\omega\tau}{2v}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

with the three Standard Model Goldstone bosons $\vec{\omega} = (\omega^{\pm}, z)$ having the masses M_w and M_Z. We now have to include the last missing piece of the final Lagrangian of the Electroweak Standard Model, the Yukawa Lagrangian

$$\mathcal{L}_{\text{Yuk}} = \sum_{m,n}\Gamma^u_{mn}\overline{q}_{m,L}\tilde{\Phi}u_{n,R} + \Gamma^d_{m,n}\overline{q}_{m,L}\Phi d_{n,R} \tag{1.19}$$

$$+ \Gamma^e_{m,n}\bar{l}_{m,L}\Phi e_{n,R} + \Gamma^\nu_{m,n}\bar{l}_{m,L}\tilde{\Phi}\nu_{n,R} + h.c. \tag{1.20}$$

This also shows where the fermion mass comes from. The matrices Γ_{mn} are the Yukawa couplings between the single Higgs doublet Φ and the fermions. The combinations of $\overline{L}\Phi R$ are $SU(2)_L$ singlets. This means that the Yukawa Lagrangian is gauge invariant. The mass terms should be hyperchargeless. Therefore we introduce two representations of the Higgs fields. Those will give mass to the down quarks and electrons, and to the up quarks and neutrinos. The neutrino has no right-handed partner in the SM, so it can not be acquired through Yukawa coupling. The representations are

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad \text{with} \qquad Y(\Phi) = +1$$

and $\tilde{\Phi}_i = \varepsilon_{ij}\Phi^*_j$, where

$$\tilde{\Phi} = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} \qquad \text{with} \qquad Y(\Phi) = -1.$$

Under $SU(2)$ they transform as

$$\Phi_i \to \Phi'_i = U_{ij}\Phi_j, \qquad \tilde{\Phi}_i \to \tilde{\Phi}'_i = U_{ij}\tilde{\Phi}_j$$

Note that

$$(U^\dagger)_{ln}(U^\dagger)_{kl}\varepsilon_{kj} = det(U^\dagger)\varepsilon_{lk} \qquad \Rightarrow \qquad (U^\dagger)_{kj}\varepsilon_{ij} = U_{il}\varepsilon_{lk}.$$

With this the transformation properties of $\tilde{\Phi}$ is indeed true,

$$\tilde{\Phi}'_i = \varepsilon_{ij}\Phi^*_j = \varepsilon_{ij}U^*_{jk}\Phi^*_k = (U^\dagger)_{kl}\varepsilon_{ij}\Phi^*_k = U_{il}\varepsilon_{lk}\Phi^*_k.$$

We can now generate all of the fermion masses with a single Higgs-doublet by making use of Φ and $\tilde{\Phi}$, as shown in the following for the first family.

Choose

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$
$$\tilde{\Phi} = \frac{1}{\sqrt{2}}\begin{pmatrix} v \\ 0 \end{pmatrix}$$

The Lagrangian then looks like

$$\mathcal{L}_{\text{Yuk}} = \frac{f_e v}{\sqrt{2}}(\overline{e}_L e_R + \overline{e}_R e_L) + \frac{f_u v}{\sqrt{2}}(\overline{u}_L u_R + \overline{u}_R u_L) + \frac{f_d v}{\sqrt{2}}(\overline{d}_L d_R + \overline{d}_R d_L).$$

We can now read off the fermion masses as

$$m_i = -\frac{f_i v}{\sqrt{2}}, \qquad i = e, u, d. \tag{1.21}$$

This finishes the discussion of the Electroweak Standard Model. We can now move on to the general definition of the cross section.

2. Calculation Framework

In this section we provide an overview of the tools necessary for performing a full numerical calculation of the associated WH production cross section. This includes a description of the hadronic cross section formula as well as how to solve it numerically with a Monte Carlo algorithm. We conclude this section by outlining the general features of the Sudakov logarithms arising in the large-energy limit of the triangle and box diagrams of this process.

2.1. The Hadronic Cross Section Formula

The general cross section for two incoming particles p_a, p_b and n outgoing particles can be written as the product of two terms [2]

$$\hat{\sigma}_n = \frac{1}{\mathcal{F}} \mathcal{I}_n \tag{2.1}$$

$$\tag{2.2}$$

The term in the denominator is the so-called 'flux factor'

$$\mathcal{F} = 2\lambda^{\frac{1}{2}}(s, m_a^2, m_b^2)(2\pi)^{3n-4}, \tag{2.3}$$

$$\tag{2.4}$$

and depends on the triangle function

$$\lambda(s, m_a^2, m_b^2) = \lambda(a, b, c) = s - 2s(m_a^2 + m_b^2) + (m_a^2 - m_b^2)^2 \tag{2.5}$$

The numerator term contains all the information about the probability of the process to occur.

$$\mathcal{I}_n = \int \prod_{i=1}^{n} \frac{\mathrm{d}^3 p_i}{2E_i} \delta^4(p_a + p_b - \sum_i p_i) \left|\mathcal{M}_{a+b\to n}\right|^2 . \tag{2.6}$$

The general definition for the Lorentz invariant phase space with m incoming and n outgoing paticles is

$$\mathrm{dLIPS}_n = d\Phi_n = (2\pi)^4 \delta^4(\sum_{i=1}^{m} k_i - \sum_{j=1}^{n} k'_j) \prod_{i=1}^{n} \frac{\mathrm{d}^3 k'_j}{(2\pi)^3 2E'_j} . \tag{2.7}$$

With our phase space definition (2.7) we can write the cross section as

$$\hat{\sigma} = \int \frac{\mathrm{d}\Phi_{n'}}{2\lambda^{\frac{1}{2}}(s, m_a^2, m_b^2)} \left|\mathcal{M}_{a+b\to n'}\right|^2 . \tag{2.8}$$

At a hadron collider, this equation is not sufficient to quantify the probability for a certain interaction to occur. To acheive this we must account for the probability of finding a certain initial-state particle a or b inside each proton. At or below the so-called factorisation scale ($\sim 1\,\mathrm{GeV}$) the hadronic cross section can be written as products of each parton distribution function (pdf) for each incoming particle of the proton times the above expression for the cross section.

$$\sigma = \int \mathrm{d}x_1 \int \mathrm{d}x_2 \sum_{q_1,\bar{q}_2} \left[f_{q_1}(x_1, Q^2) f_{\bar{q}_2}(x_2, Q^2)\right] \left|\mathcal{M}_{a+b\to n'}\right|^2 \tag{2.9}$$

where $f(x_1, Q^2)$ are the pdf's with the energy scale $Q = M_W$ of the process.

2.1.1. Massive Two-Particle Phase Space

We will now calculate the phase for two massive particles. We define the momentum-sum of two incoming partons as P and calculate the system in the rest frame $P = (\sqrt{\hat{s}}, 0, 0, 0)$

$$\begin{aligned}
\int \mathrm{d}\Phi_2(p_1, p_2) &= \int \frac{\mathrm{d}^3p_1}{(2\pi)^3 2E_1} \frac{\mathrm{d}^3p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(P - p_1 - p_2) \\
&= \int \frac{\mathrm{d}^3p_1}{(2\pi)^3 2E_1} \frac{\mathrm{d}^3p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta(\sqrt{s} - E_1 - E_2)\delta(\vec{p}_1 + \vec{p}_2) \\
&= \int \frac{\mathrm{d}^3p_1}{(2\pi)^3 2E_1} \frac{1}{2E_2} (2\pi)\delta(\sqrt{s} - E_1 E_2) \\
&= \int \frac{\mathrm{d}p\ \mathrm{d}\cos\theta\ \mathrm{d}\phi}{(2\pi)^3 2E_1} \frac{p^2}{2E_2} (2\pi)\delta(\sqrt{s} - E_1 E_2)
\end{aligned} \tag{2.10}$$

where we simplify the notation a bit by writing $|\vec{p_1}| = p$, as we have chosen to solve the $|\vec{p_2}|$ integral above.

The next step is to use the δ-function to solve the integral over p. We know that in the center of momentum system (CMS), $\hat{s} = (p_1 + p_2)^2 = (E_1 + E_2)^2$. We first look at the following two equations:

$$p_1(p_1 + p_2) = p_1^2 + p_1 p_2 = m_1^2 + E_1 E_2 + \vec{p}^2 = E_1(E_1 + E_2) \tag{2.11}$$

$$p_1(p_1 + p_2) = p_1^2 + p_1 p_2 = m_1^2 + \frac{1}{2}(p_1 + p_2)^2 - \frac{1}{2}m_1^2 - \frac{1}{2}m_2^2 \tag{2.12}$$

Now we set these two equations equal and solve for E_1.

$$E_1 \underbrace{(E_1 + E_2)}_{\sqrt{s}} = m_1^2 + \frac{1}{2}\underbrace{(p_1 + p_2)^2}_{s^2} - \frac{1}{2}m_1^2 - \frac{1}{2}m_2^2 \tag{2.13}$$

$$E_1\sqrt{s} = \frac{1}{2}\left(s + m_1^2 - m_2^2\right) \tag{2.14}$$

$$\Rightarrow E_1 = \frac{1}{2\sqrt{s}}(s + m_1^2 - m_2^2) \tag{2.15}$$

Inserting $|\vec{p}\,| = \sqrt{E^2 - m^2}$ into the above expression yields:

$$|\vec{p}| = \frac{\sqrt{s}}{2}\beta \tag{2.16}$$

with

$$\begin{aligned}\beta &= [1 - \frac{2(m_1^2 + m_2^2)}{s} + \frac{(m_1^2 - m_2^2)^2}{s^2}]^{\frac{1}{2}} \\ &= \frac{\lambda^{\frac{1}{2}}(s, m_1^2, m_2^2)}{s},\end{aligned} \tag{2.17}$$

where $\lambda(a, b, c) = a^2 - 2a(b+c) + (b-c)^2$ is the triangle function. The δ-function from before now looks like

$$\delta(\sqrt{s} - E_1 - E_2) = \frac{\delta(p - \beta\frac{\sqrt{s}}{2})}{(\frac{p}{E_1}) + (\frac{p}{E_2})}. \tag{2.18}$$

Inserting this back into the expression for the phase space integral, we finally get

$$\begin{aligned}\int \mathrm{d}\Phi_2(p_1, p_2) &= \int \frac{\mathrm{d}\cos\theta\ \mathrm{d}\phi}{(2\pi)^3} \frac{p^2}{4E_1E_2} \frac{2\pi}{(\frac{p}{E_1}) + (\frac{p}{E_2})}\Bigg|_{p=\beta\frac{\sqrt{s}}{2}} \\ &= \int \frac{\mathrm{d}\cos\theta\ \mathrm{d}\phi}{(2\pi)^2} \frac{p}{\underbrace{E_1 + E_2}_{\sqrt{s}}}\Bigg|_{p=\beta\frac{\sqrt{s}}{2}} \\ &= \frac{\beta}{8\pi}\int \frac{\mathrm{d}\cos\theta}{2}\frac{\mathrm{d}\phi}{2\pi}.\end{aligned} \tag{2.19}$$

The final step is to write down the four-momenta of all particles. To this end we need to calculate the value of the energies E_1, E_2 and the three-momenta $|\vec{p}| = |\vec{p}_1| = |\vec{p}_2|$.

$$
\begin{aligned}
\hat{s} &= (E_1 + E_2)^2 \\
\hat{s} &= E_1^2 + E_2^2 + 2E_1E_2 \\
\hat{s} - E_2^2 &= E_1^2 + 2E_1E_2 \\
|\vec{p}|^2 - |\vec{p}|^2 + \hat{s} - E_2^2 + E_1^2 &= 2E_1^2 + 2E_1E_2 \\
\hat{s} + m_1^2 - m_2^2 &= 2E_1 \underbrace{(E_1 + E_2)}_{\sqrt{\hat{s}}}
\end{aligned}
$$

If we identify particle 1 with the W boson and particle 2 with the Higgs then we can write

$$E_W = \frac{\hat{s} + m_W^2 - m_H^2}{2\sqrt{\hat{s}}}. \tag{2.20}$$

Next, we simply square this and use the triangle function,

$$
\begin{aligned}
E_W^2 &= \frac{(\hat{s} + m_W - m_H)^2}{4\hat{s}} \\
4\hat{s}(|\vec{p}|^2 + m_W^2) &= \hat{s}^2 + (m_W^2 - m_H^2)^2 + 2\hat{s}(m_W^2 - m_H^2) \\
4\hat{s}|\vec{p}|^2 &= \hat{s}^2 + (m_W^2 - m_H^2)^2 + 2\hat{s}(m_W^2 - m_H^2 - 2m_W^2) \\
4\hat{s}|\vec{p}|^2 &= \lambda(\hat{s}, m_W^2, m_H^2)
\end{aligned}
$$

We now have the results we desired

$$|\vec{p}| = \frac{\lambda^{\frac{1}{2}}(\hat{s}, m_W^2, m_H^2)}{2\sqrt{\hat{s}}} \tag{2.21}$$

and

$$E_H = \sqrt{\hat{s}} - E_W = \frac{\hat{s} - m_W^2 + m_H^2}{2\sqrt{\hat{s}}}. \tag{2.22}$$

It is best to use a spherical coordinate system with polar angle ($-\pi \leq \theta \leq \pi$) and azimuthal angle ($0 \leq \phi \leq 2\pi$) to express the four-momenta of the incoming and outgoing particles in the CMS.

$$
\begin{aligned}
p_a &= \left(\frac{\sqrt{\hat{s}}}{2}\,,\,0\,,\,0\,,\,\frac{\sqrt{\hat{s}}}{2}\right) \\
p_b &= \left(\frac{\sqrt{\hat{s}}}{2}\,,\,0\,,\,0\,,\,-\frac{\sqrt{\hat{s}}}{2}\right) \\
p_W &= \left(E_W\,,\,\frac{\sqrt{\hat{s}}}{2}\beta\sin\theta\cos\phi\,,\,\frac{\sqrt{\hat{s}}}{2}\beta\sin\theta\sin\phi\,,\,\frac{\sqrt{\hat{s}}}{2}\beta\cos\theta\right) \\
p_H &= \left(E_H\,,\,-\frac{\sqrt{\hat{s}}}{2}\beta\sin\theta\cos\phi\,,\,-\frac{\sqrt{\hat{s}}}{2}\beta\sin\theta\sin\phi\,,\,-\frac{\sqrt{\hat{s}}}{2}\beta\cos\theta\right)
\end{aligned}
$$

Out of these four-momenta one can construct the so called Mandelstam variables $\hat{s}$, $\hat{t}$ and $\hat{u}$. $\hat{s}$ is the already introduced square of the CMS energy.

$$\hat{s} = (p_1 + p_2)^2 = (p_3 + p_4)^2 \tag{2.23}$$
$$\hat{t} = (p_1 - p_3)^2 = (p_2 - p_4)^2 \tag{2.24}$$
$$\hat{u} = (p_1 - p_4)^2 = (p_2 - p_3)^2 \tag{2.25}$$

One can express all Lorentz invariant combinations of the 4 external momenta in terms of the particles masses and the three Mandelstam varables above. Two out of these variables are independent while their sum has the fixed value

$$\hat{s} + \hat{t} + \hat{u} = \sum_{i=0}^{3} m_i^2. \tag{2.26}$$

There is however a caveat...

2.1.2. Boost procedure

It has not been stated explicitly, but it is perhaps obvious that the above expressions for the four-momenta are not Lorentz invariant. Above, they were calculated specifically in the CMS frame; a frame that is defined such that the initial-state particles have equal energy and equal, yet opposite three-momentum components. Many of the observable quantities constructed from the four-momenta are actually not Lorentz invariant. Note that the products for these momenta and combination of those (Mandelstam variables) are indeed Lorentz invariant. But the Mandelstam variable are not „real “observables we want to study.

For instance some common observables are:

$$\eta = -\ln\left[tan\left(\frac{\theta}{2}\right)\right] \tag{2.27}$$
$$p_T = \sqrt{p_x^2 + p_y^2} \tag{2.28}$$
$$E_T = m^2 + p_T^2 \tag{2.29}$$

where η is the pseudorapidity which decribes the angle of a particle relative to the beam axis. p_T is the momentum transverse to the beam line. E_T is the energy of it.

As these are not Lorentz invariant, they will take different values in so-called ‘boosted’ reference frames. These are reference frames translated in space, and having a non-zero relative velocity with respect to each other. The boost concept is extremely important for hadron collider observables as this is exactly the situation for the initial state partons whose four-momenta can each be some fraction of the incoming protons four-momenta. Hence, the lab frame is not the CMS frame and if we are observing non-Lorentz invariant quantities then we had better transform the four-momenta to this frame. We outline the simple boost procedure below.

Before we can boost our process we have to remind ourself that we don't have single particles with energy S but single quarks with just a fraction of the total energy coming from the protons. We now perform the boost procedure general momenta p_a and p_b and then get the boosted coordinate of 2.23.

Assume you have the following incoming momenta.

$$\begin{aligned} p_a &= (E_a, 0, 0, P_a) \\ p_b &= (E_b, 0, 0, -P_a) \end{aligned}$$

We are in the boosted frame so $p_a = x_1 P_1$ and $p_b = x_2 P_2$. Where x_1 and x_2 are the fraction of the total energy S. The massless on-shell conditions

$$\begin{aligned} (p_a)^2 &= E_a^2 - P_a^2 = 0 \\ &\Rightarrow E_a = P_a \\ (p_b)^2 &= E_b^2 - P_a^2 = 0 \\ &\Rightarrow E_b = P_a = E_a \end{aligned}$$

yields then to the following expression

$$\begin{aligned} p_a^T &= (E_a, 0, 0, E_a^T), \\ p_b^T &= (\frac{x_2}{x_1} E_a^T, 0, 0, -\frac{x_2}{x_1} E_a^T). \end{aligned}$$

We now have to find boost parameters. These parameters contain all information from viewing one frame from the other [9]. First the relative velocity between the frames

$$\vec{v} = \frac{\vec{p}_{tot}}{E_{tot}} = -\frac{\vec{p}_a^T + \vec{p}_b^T}{E_a^T + E_b^T} = \frac{x_2 - x_1}{x_1 + x_2}.$$

Next is the Lorentz factor γ for the frame we want to boost to.

$$\gamma = \frac{E_{tot}}{m} = \frac{E_a^T + E_b^T}{\sqrt{E_{tot}^2 - p_{tot}^2}} = \frac{x_1 + x_2}{2\sqrt{x_1 x_2}}$$

To boost the system along the z-direction we have to perform the LT (Lorenz Transformation) in that direction. For our general four-momentum vector we have then

$$
\begin{aligned}
{p'_a}^0 &= \gamma(a^0 - va^3),\\
{p'_a}^3 &= \gamma(a^3 - va^0).
\end{aligned}
$$

After doing that our momenta look like

$$
\begin{aligned}
p_a &= \left(x_1\frac{\sqrt{S}}{2}, 0, 0, x_1\frac{\sqrt{S}}{2}\right),\\
p_b &= \left(x_2\frac{\sqrt{S}}{2}, 0, 0, -x_2\frac{\sqrt{S}}{2}\right).
\end{aligned}
$$

The coordinates 2.23 then result in the form

$$
\begin{aligned}
p &= \left(x_1\frac{\sqrt{S}}{2}, 0, 0, x_1\frac{\sqrt{S}}{2}\right),\\
p' &= \left(x_2\frac{\sqrt{S}}{2}, 0, 0, -x_2\frac{\sqrt{S}}{2}\right),\\
k &= \left(\gamma\left(E_W - vP_W\right), \frac{\sqrt{\hat{s}}}{2}\beta\sin\theta\cos\phi, \frac{\sqrt{\hat{s}}}{2}\beta\sin\theta\sin\phi, \gamma\left(P_W - vE_W\right)\right),\\
p_H &= \left(\gamma\left(E_H - vP_H\right), -\frac{\sqrt{\hat{s}}}{2}\beta\sin\theta\cos\phi, -\frac{\sqrt{\hat{s}}}{2}\beta\sin\theta\sin\phi, \gamma\left(P_H - vE_H\right)\right)
\end{aligned}
\tag{2.30}
$$

with

$$
\begin{aligned}
P_W &= \frac{\sqrt{s}}{2}\beta\cos\theta,\\
E_W &= \frac{\hat{s} + m_W^2 - m_H^2}{2\sqrt{s}},\\
P_H &= -\frac{\sqrt{s}}{2}\beta\cos\theta,\\
E_H &= \frac{\hat{s} - m_W^2 + m_H^2}{2\sqrt{s}}.
\end{aligned}
$$

We can now move to the main part of the calculation of the cross-section, the calculation of the matrix element squared.

2.2. Calculation of the Tree-level process

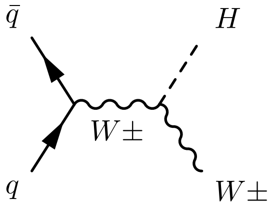


Figure 2.1.: Tree-level process

First we consider the tree-level process

$$q(p,\tau) + \overline{q}'(p',\tau') \to V(k,\lambda_V) + H(p_H). \tag{2.31}$$

where $V = W^{+/-}$. For the W^+H production we include $q = u, c$ and $q' = d, s$. We neglect the fermion masses $m_q, m_{q'}$, so as a consequence, the fermion helicities are conserved in lowest order which means the matrix elements vanish unless $\tau_q = -\tau_{q'} = \tau = \pm 1/2$.
The matrix element $\mathcal{M}_0^\tau$ obtained with the feynman rule in section (A) is the following:

$$\mathcal{M}_0^\tau = \frac{e^2 g^\tau_{qq'V} g_{VVH}}{s - M_V^2} \overline{v}(p') \not{\epsilon}^*_V(\lambda_V) \omega_\tau u(p). \tag{2.32}$$

where $s = (p + p')^2 = (p_H + k)^2$, $\omega_\pm = \frac{1}{2}(1 \pm \gamma_5)$ and $\epsilon^*_V(\lambda_V)$ is the polarization vector of the boson V. The coupling factor are given by

$$g_{qq'W} = \frac{V^*_{qq'}}{\sqrt{2} s_W} \delta_{\tau -} \tag{2.33}$$

$$g_{WWH} = \frac{M_W}{s_W} \tag{2.34}$$

The weak mixing angle s_w is defined by $s_W^2 = \sin^2\theta_W = 1 - c_w^2 = 1 - \frac{M_W^2}{M_Z^2}$. $V_{qq'}$ is the CKM matrix element.
We now go through the calculation of $\left|\overline{\mathcal{M}}\right|^2$.
(Just to shorten the formulas: $\overline{v}' = \overline{v}(p')$ and $u = u(p)$.)

$$\begin{aligned}
\left|\mathcal{M}_0^-\right|^2 &= a^2 \overline{v}' \not{\epsilon}^*_V \omega_- u \overline{u} \not{\epsilon}_V \omega_- v' \\
&= a^2 \epsilon_V^{*\mu} \epsilon_V^\nu \frac{1}{4} tr(\overline{v}' \gamma_\mu (1 - \gamma_5) u \overline{u} (1 + \gamma_5) \gamma_\nu v') \\
&= a^2 \epsilon_V^{*\mu} \epsilon_V^\nu \frac{1}{4} tr(v' \overline{v}' \gamma_\mu (1 - \gamma_5) u \overline{u} \gamma_\nu (1 - \gamma_5)) \\
&= a^2 \epsilon_V^{*\mu} \epsilon_V^\nu \frac{1}{4} tr((\not{p}' - m) \gamma_\mu (1 - \gamma_5)(\not{p} + m) \gamma_\nu (1 - \gamma_5)) \\
&\overset{m=0}{=} a^2 \epsilon_V^{*\mu} \epsilon_V^\nu \frac{1}{2} tr((\not{p}' \gamma_\mu \not{p} \gamma_\nu) - (\not{p}' \gamma_\mu \gamma_5 \not{p} \gamma_\nu))
\end{aligned}$$

with $a = \frac{e^2 g^{\tau}_{qq'V} g_{VVH}}{s - M_V^2}$.

We use (d is the dimension)

$$
\begin{aligned}
tr(\gamma_5) = tr(\gamma_\mu\gamma_\nu\gamma_5) = 0 \\
tr(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5 &= -di\epsilon^{\mu\nu\rho\sigma} \\
tr(\gamma^\mu\gamma^n u\gamma^\rho\gamma^\sigma) &= d(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}) \\
&= a^2\epsilon_V^{*\mu}\epsilon_V^\nu\frac{1}{2}tr(p'^\rho p^\sigma(\gamma_\rho\gamma_\mu\gamma_\sigma\gamma_\nu) - p'^\rho p^\sigma(\gamma_\rho\gamma_\mu\gamma_\sigma\gamma_\nu\gamma_5)) \\
&= a^2\epsilon_V^{*\mu}\epsilon_V^\nu\frac{d}{2}(p'_\mu p_\nu - p'_\sigma p^\sigma g_{\mu\nu} + p'_\nu p_\mu + ip'^\rho p^\sigma\epsilon_{\rho\mu\sigma\nu})
\end{aligned}
$$

We now form the sum over the polarizations of the boson V. The summation over the color and the spin is included later because it is just an overall factor of $\frac{1}{12}$.

$$
\begin{aligned}
\sum\left|\mathcal{M}_0^-\right|^2 &= a^2\sum\epsilon_V^{*\mu}\epsilon_V^\nu\frac{d}{2}(p'_\mu p_\nu - p'_\sigma p^\sigma g_{\mu\nu} + p'_\nu p_\mu + ip'^\rho p^\sigma\epsilon_{\rho\mu\sigma\nu}) \\
&= \frac{d}{2}a^2(-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2})(p'_\mu p_\nu - p'_\sigma p^\sigma g_{\mu\nu} + p'_\nu p_\mu + ip'^\rho p^\sigma\epsilon_{\rho\mu\sigma\nu}) \\
&= -\frac{d}{2}a^2(p'_\mu p^\mu - g_{\mu\nu}g^{\mu\nu}p'_\sigma p^\sigma + p'_\nu p^\nu + ip'^\rho p^\sigma\epsilon^\mu_{\rho\mu\sigma} \\
&- \frac{1}{k^2}((p'k)(pk) - p'_\sigma p^\sigma k^\mu k^\nu + (p'k)(pk) + ip'^\rho p^\sigma k^\mu k^\nu\epsilon_{\rho\mu\sigma\nu}))
\end{aligned}
$$

Now we will use, that $(-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2})g_{\mu\nu} = 3$ to get rid of the second term in the first and second line.

$$
\begin{aligned}
\sum\left|\mathcal{M}_0^-\right|^2 &= -\frac{d}{2}a^2((p'p) - 3(p'p) + (p'p) + ip'^\rho p^\sigma\epsilon_{\rho\mu\sigma\nu}g^{\mu\nu} - \frac{1}{k^2}(2(p'k)(pk) + ip'^\rho p^\sigma k^\mu k^\nu\epsilon_{\rho\mu\sigma\nu})) \\
&= -\frac{da^2}{2k^2}(-(p'p)k^2 + ip'^\rho p^\sigma k^\mu k_\mu\epsilon_{\rho\mu\sigma\nu}g^{\mu\nu} - 2(p'k)(pk) - ip'^\rho p^\sigma k^\mu k_\mu\epsilon_{\rho\mu\sigma\nu}g^{\mu\nu})
\end{aligned}
$$

The ϵ-Terms cancel each other and in order to get the final result we use

$$
\begin{aligned}
d &= 4 \\
k^2 &= M_V^2 \\
(t - M_V^2) &= -2pk \\
(u - M_V^2) &= -2p'k \\
s &= 2pp'.
\end{aligned}
$$

$$\left|\mathcal{M}_0^-\right|^2 = \frac{a^2}{M_V^2}(sM_V^2 + (t - M_V^2)(u - M_V^2))$$

The result for $|\mathcal{M}^+|^2$ is the same, except for $g^-_{q'qV}$ is replaced by $g^+_{q'qV}$. The entire matrix element is then given by

$$|\mathcal{M}_0|^2 = \left|\mathcal{M}_0^+\right|^2 + \left|\mathcal{M}_0^-\right|^2 .$$

Now we can use the definition of the phase space (2.19) and the definition of the cross section (2.8) to finally get to the differential cross section for the tree level

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2}{48}\frac{g^2_{VVH}}{M_V^2 s^2}((g^+_{q'qV})^2 + (g^-_{q'qV})^2)\lambda^{\frac{1}{2}}(M_V^2, M_H^2, s)\frac{(sM_V^2 + (t - M_V^2)(u - M_V^2))}{(s - M_V^2)^2} \tag{2.35}$$

with $\alpha = \frac{e^2}{4\pi}$. This is the same result as the one published in the paper of Coiccolini [3].

Now we put everything together to get the full matrix element squared.

$$\begin{aligned}\sigma = \int \mathrm{d}x_1 \int \mathrm{d}x_2 \int \mathrm{d}\Phi_2 \sum_{q_1,\overline{q}_2} \left[f_{q_1}(x_1, Q^2) f_{\overline{q}_2}(x_2, Q^2)\right] V^{*2}_{qq'} \lambda^{\frac{1}{2}}(M_V^2, M_H^2, s) \\ \times \frac{\alpha^2}{48}\frac{g^2_{VVH}}{M_V^2 s^2(\sqrt{2}s_W)^2}\frac{(sM_V^2 + (t - M_V^2)(u - M_V^2))}{(s - M_V^2)^2}\end{aligned} \tag{2.36}$$

The written-out sum over the quarks looks like:

$$\begin{aligned}\sum_{q_1,\overline{q}_2} \left[f_{q_1}(x_1, Q^2) f_{\overline{q}_2}(x_2, Q^2)\right] V^{*2}_{qq'} = {} & (f_u(x_1, Q) f_{\overline{d}}(x_2, Q) + f_{\overline{d}}(x_1, Q) f_u(x_2, Q) \\ & + f_d(x_2, Q) f_{\overline{u}}(x_1, Q) + f_{\overline{u}}(x_2, Q) f_d(x_1, Q))V^2_{ud} \\ & + (f_s(x_1, Q) f_{\overline{c}}(x_2, Q) + f_{\overline{s}}(x_1, Q) f_c(x_2, Q) \\ & + f_s(x_2, Q) f_{\overline{c}}(x_1, Q) + f_{\overline{s}}(x_2, Q) f_c(x_1, Q))V^2_{cs} \\ & + (f_c(x_1, Q) f_{\overline{d}}(x_2, Q) + f_{\overline{c}}(x_1, Q) f_d(x_2, Q) \\ & + f_c(x_2, Q) f_{\overline{d}}(x_1, Q) + f_{\overline{c}}(z, Q) f_d(x_1, Q))V^2_{cd} \\ & + (f_u(x_1, Q) f_{\overline{s}}(x_2, Q) + f_{\overline{u}}(x_1, Q) f_s(x_2, Q) \\ & + f_u(x_1, Q) f_{\overline{s}}(x_1, Q) + f_{\overline{u}}(x_2, Q) f_s(x_1, Q))V^2_{us}\end{aligned} \tag{2.37}$$

This sum includes all combination possible for $W^\pm$.

To actually integrate our cross section we will use the Monte Carlo method. We will now give a short introduction of this technique.

2.3. Monte Carlo integration

The basic idea behind the Monte Carlo method is to basically evaluate as much points of the function we wish to integrate and sum all these values and average it by the number of points calculated.
To illustrate the method we demonstrate it for one dimension. We regard the integral

$$\mathcal{I} = \int_a^b f(x)dx \tag{2.38}$$

where $f(x)$ can be any arbitrary function. The average value of $f(x)$ is

$$\overline{f} = \frac{1}{b-a}\mathcal{I}. \tag{2.39}$$

So we can write the integral as

$$\mathcal{I} = (b-a)\overline{f} \tag{2.40}$$

We now choose n random points inside the interval of integral $[a, b]$ to estimate $\overline{f}$

$$\overline{f} \approx \frac{1}{n}\sum_{i=1}^{n} f(x_i). \tag{2.41}$$

With this we can now approximate our wanted integral,

$$\mathcal{I} \approx \frac{(b-a)}{n}\sum_{i=1}^{n} f(x_i). \tag{2.42}$$

That is the fundamental Monte Carlo method.
We now want to estimate the standard error of the Monte Carlo method. The variance can be expressed as,

$$\frac{1}{N^2}\sum_{i=1}^{N} Var(f(x_i)) = \frac{1}{N^2}\sum_{i=1}^{N}(f(x_i) - \langle f\rangle)^2 \tag{2.43}$$

$$= \frac{1}{N}\left(\langle f^2\rangle - \langle f\rangle^2\right), \tag{2.44}$$

Because of the independence of the variables the appearing covariance is zero. The $\langle f^2\rangle$ and $\langle f\rangle$ are given below

$$\langle f\rangle = \frac{1}{N}\sum_{i=1}^{N} f(x_i) \tag{2.45}$$

and

$$\langle f^2 \rangle = \frac{1}{N} \sum_{i=1}^{N} f^2(x_i). \tag{2.46}$$

In our Monte Carlo integration we will use the Monte Carlo algorithm Vegas [13] which uses importance sampling as a variance-reduction technique.
With the cross-section fully computed we now arrive at the discussion of the NLO corrections.

3. Sudakov logarithm

In the explicit one-loop calculation of our process we encounter the so-called Sudakov logarithms. They are normally negligibly small, but in the high energy limit they are large enough to require more careful investigation [19]. We will find the following Sudakov logarithms:

$$\Lambda_{\hat{s}}(M^2) = \frac{1}{2\hat{s}} \ln^2\left(\frac{-\hat{s} - i\epsilon}{M^2}\right) \tag{3.1}$$

$$\Lambda_{\hat{t}}(M^2) = \frac{1}{2\hat{t}} \ln^2\left(\frac{-\hat{t} - i\epsilon}{M^2}\right) \tag{3.2}$$

$$\Lambda_{\hat{u}}(M^2) = \frac{1}{2\hat{u}} \ln^2\left(\frac{-\hat{u} - i\epsilon}{M^2}\right) \tag{3.3}$$

$$\Lambda_{\hat{s}}(M_1^2, M_2^2) = \frac{1}{\hat{s}} \ln\left(\frac{-\hat{s} - i\epsilon}{M_1^2}\right) \ln\left(\frac{-\hat{s} - i\epsilon}{M_2^2}\right) \tag{3.4}$$

where we also include the Sudakov logs with t and u.

In the high-energy limit, $s \gg M_W$, and we effectively have $M_W \sim 0$. Here, the Sudakov logs are at the infrared (IR) limit of EW corrections. We will now try to find out how they appear and what the origin of them is. This search begins with an examination of all scalar integrals appearing in the Passorino-Veltman reduction of the tensor integrals encountered at one-loop (see Appendix C). We will neglect the discussion of the A_0 integral because it is relatively clear that no Sudakov logarithms come from that integral.

Note that we will later use the identities

$$\ln\left(\frac{-\hat{s} - i\epsilon}{M^2}\right) = \ln\left(\frac{s}{M^2}\right) - i\pi, \tag{3.5}$$

$$\ln\left(\frac{-\hat{t} - i\epsilon}{M^2}\right) = \ln\left(\left|\frac{\hat{t}}{M^2}\right|\right), \tag{3.6}$$

$$\ln\left(\frac{-\hat{u} - i\epsilon}{M^2}\right) = \ln\left(\left|\frac{\hat{u}}{M^2}\right|\right). \tag{3.7}$$

3.1. High-Energy Limit of the B_0 Integral

In Appendix C.1.2 we introduced the general solution of the B_0 integral

$$B_0(p^2, M_0, M_1) = \left[\frac{1}{\epsilon} + 2 - \ln(p^2) + \sum_{i=1}^{2}\left[\gamma_i \ln(\frac{\gamma_i - 1}{\gamma_i}) - ln(\gamma_i - 1)\right] + 2\ln\mu\right] \tag{3.8}$$

with

$$\gamma_{1,2} = \frac{p^2 - M_1^2 + M_0^2 \pm \sqrt{(p^2 - M_1^2 + M_0^2)^2 - 4p^2(M_0^2)}}{2p^2}.$$

The interesting case for the B_0 integral is when $p^2 = \hat{s}, \hat{t}, \hat{u}$. If we take that limit the scalar integral reads

$$B_0(\hat{s}, M_0, M_1) = \frac{1}{\epsilon} + 2 - \ln\left(\frac{-\hat{s} - i\epsilon}{\mu^2}\right). \tag{3.9}$$

This shows that no Sudakov logarithms appear so we can safely neglect B_0 for the rest of the calculation.

3.2. High-Energy Limit of the C_0 integral

As shown in Appendix C.1.1 and Section 3.1, the A_0 and B_0 integrals contain no Sudakov logarithms. The next logical step is to take a closer look at the C_0 integral to identify the Sudakov logarithms hidden inside. Therefore, we assume the limit $\hat{s} \gg m_i$ where $i = 0, 1, 2$. We also assume that all the masses are real and that the momenta are on-shell. With these conditions all the η-functions in D.1 vanish. The only terms left are the dilogaritmic functions. The best way to handle the big arguments inside the dilogarithm is to look at the separate parts first.

With our assumption for the C_0 function, D.1 becomes

$$C_0(\hat{p}_{10}, \hat{p}_{20}, m_0, m_1 m_2) = \frac{1}{\hat{\alpha}} \sum_{i=0}^{2} \left[\sum_{\sigma=\pm} \left[\mathrm{Li}_2\left(\frac{\hat{y}_{0i} - 1}{\hat{y}_{i\sigma}}\right) - \mathrm{Li}_2\left(\frac{\hat{y}_{0i}}{\hat{y}_{i\sigma}}\right)\right]\right]. \tag{3.10}$$

Again, we have used the variables

$$\begin{aligned}
\hat{y}_{0i} &= \frac{1}{2\hat{\alpha}\hat{p}_{jk}^2}\left[\hat{p}_{jk}^2(\hat{p}_{jk}^2 - \hat{p}_{ki}^2 - \hat{p}_{ij}^2 + 2m_i^2 - m_j^2 - m_k^2)\right.,\\
&\qquad \left. - (\hat{p}_{ki}^2 - \hat{p}_{ij}^2)(m_j^2 - m_k^2) + \hat{\alpha}(\hat{p}_{jk}^2 - m_j^2 + m_k^2)\right],\\
x_{i\pm} &= [\hat{p}_{jk}^2 - m_j^2 + m_k^2 \pm \hat{\alpha}_i],\\
\hat{y}_{i\pm} &= \hat{y}_{0i} - x_{i\pm},\\
\hat{\alpha} &= \kappa(\hat{p}_{10}^2, \hat{p}_{21}^2, \hat{p}_{20}^2),\\
\hat{\alpha}_i &= \kappa(\hat{p}_{jk}^2, m_j^2, m_k^2)(1 + i\epsilon\hat{p}_{jk}^2).
\end{aligned}$$

We use our limit $\hat{p}_{21}^2 = (\hat{p}_2 - \hat{p}_1)^2 \equiv \hat{s} \gg (m_i^2, |\hat{p}_{10}^2|, |\hat{p}_{20}^2|)$ on the arguments of the dilogarithmic functions.

$$\begin{aligned}
\hat{\alpha} &= \kappa(\hat{p}_{10}^2, \hat{s}, \hat{p}_{20}^2) \approx \hat{s} \\
\hat{\alpha}_0 &= \kappa(\hat{s}, m_1^2, m_2^2) \approx (s - (m_1^2 + m_2^2))(1 + i\epsilon\hat{s}) \\
\hat{\alpha}_1 &= \kappa(\hat{p}_{20}^2, m_2^2, m_0^2)(1 + i\epsilon p_{20}^2) \approx 0 \\
\hat{\alpha}_2 &= \kappa(\hat{p}_{10}^2, m_0^2, m_1^2)(1 + i\epsilon p_{10}^2) \approx 0
\end{aligned}$$

For $i = 0, j = 1$ and $k = 2$ we get

$$\begin{aligned}
y_{00} &\approx \frac{1}{\hat{s}}[\hat{s} - \hat{p}_2^2 - \hat{p}_1^2 + m_0^2 - m_1^2] \\
y_{0\pm} &\approx \frac{1}{2\hat{s}}\left[\hat{s} - \hat{p}_2^2 - \hat{p}_1^2 + 2m_0^2 - m_1^2 - m_2^2 \mp (\hat{s} - (m_1^2 + m_2^2))\right] \\
\Rightarrow &\mathrm{Li}_2\left(\frac{y_{00} - 1}{y_{0\pm}}\right) \approx \mathrm{Li}_2(1) + \mathrm{Li}_2(0) = \frac{\pi^2}{6} \\
\Rightarrow &\mathrm{Li}_2\left(\frac{y_{00}}{y_{0\pm}}\right) \approx \left(\ln\left(\frac{\hat{s}}{m_0^2}\right)\right)^2 + \mathrm{Li}_2(1) = -\frac{1}{2}\left(\ln\left(\frac{\hat{s}}{m_0^2}\right)\right)^2 + \frac{\pi^2}{6}.
\end{aligned}$$

The first term in the final equality is the searched-for Sudakov logarithm. We have used the following identity of the dilogarithmic function for $z \geq 1$:

$$\mathrm{Li}_2(z) = \frac{\pi^2}{3} - \frac{1}{2}\ln^2(z) - \sum_{k=1} \frac{1}{k^2 z^2}.$$

We also also check the other two cases $i = 1, j = 2x, k = 0$ and $i = 2, j = 0, k = 1$.

$$\begin{aligned}
y_{01} &= \frac{1}{2\alpha\hat{p}_{20}^2}\left[\hat{p}_{20}^2(\hat{p}_{20}^2 - \hat{p}_{10}^2 + 2m_1^2 - m_2^2 - m_0^2) - (\hat{p}_{10}^2 - \hat{s})(m_2^2 - m_0^2) + \alpha(-m_2^2 + m_0^2)\right] \\
y_{1\pm} &= \frac{1}{2\alpha\hat{p}_{20}^2}\left[\hat{p}_{20}^2(\hat{p}_{20}^2 - \hat{p}_{10}^2 - \hat{s} + 2m_1^2 - m_2^2 - m_0^2) - (\hat{p}_{10}^2 - \hat{s})(m_2^2 - m_0^2) \mp \alpha\alpha_1\right] \\
\Rightarrow &\mathrm{Li}_2\left(\frac{y_{01} - 1}{y_{1\pm}}\right) \approx \mathrm{Li}_2\left[\frac{2\hat{p}_{20}^2}{\hat{p}_{20}^2 + m_0^2 - m_2^2 \pm \kappa(\hat{p}_{20}^2, m_2^2, m_0^2)}\right] \\
\Rightarrow &\mathrm{Li}_2\left(\frac{y_{01}}{y_{1\pm}}\right) \approx \mathrm{Li}_2(0) = 0
\end{aligned}$$

The result for the last case is the same as for the second under exchange of $\hat{p}_{20}$ with $\hat{p}_{10}$ and $m_0 \to m_1, m_2 \to m_0$. The complicated C_0 functions are now in such a simplified form that we can directly see the Sudakov logarithm.

$$C_0(\hat{p}_{10}, \hat{p}_{20}, m_0, m_1 m_2) = \frac{1}{2\hat{s}}\left(\ln\left(\frac{-\hat{s} - i\epsilon}{m_0^2}\right)\right)^2 + L(\hat{p}_{20}, m_2, m_0) + L(\hat{p}_{10}, m_1, m_0) \tag{3.11}$$

with

$$L(p_a, m_b, m_c) = \mathrm{Li}_2\left[\frac{2\hat{p}_a^2}{\hat{p}_a^2 + m_b^2 - m_c^2 + \kappa(\hat{p}_a^2, m_b^2, m_c^2)}\right] + \mathrm{Li}_2\left[\frac{2\hat{p}_a^2}{\hat{p}_a^2 + m_b^2 - m_c^2 - \kappa(\hat{p}_a^2, m_b^2, m_c^2)}\right]$$

This is the same result as presented in [5]. Now we have the most general result for the high energy limit. However, we still have to consider the case where $m_0^2 = 0$, $m_1^2 \neq p_1^2$ and $m_2^2 \neq p_2^2$. We simply just present the result introduced in [6].

$$C_0(p_1^2, \hat{s}, p_2^2, 0, m_1^2, m_2^2) \approx \frac{1}{\hat{s}}\left(\ln\left(\frac{-\hat{s} - i\epsilon}{m_1^2}\right)\ln\left(\frac{-\hat{s} - i\epsilon}{m_1^2}\right) + \mathrm{Li}_2\left[-\frac{p_1^2}{m_1^2 - p_1^2 - i\epsilon}\right] + \mathrm{Li}_2\left[-\frac{p_2^2}{m_2^2 - p_2^2 - i\epsilon}\right]\right) \tag{3.12}$$

The next step is now to organize the different C_0 functions according to the different arguments appearing. By changing the arguments in 3.12 and using the identities

$$\begin{aligned} C_0(p_b, p_b, p_c, m_a, m_b, m_c) &= C_0(p_b, p_c, p_a, m_b, m_c, m_a), \\ C_0(p_a, p_b, p_c, m_a, m_b, m_c) &= C_0(p_a, p_c, p_b, m_b, m_a, m_c), \end{aligned} \tag{3.13}$$

we can find nearly all Sudakov logarithms. We encounter some C_0 functions where we cannot use equation 3.12 but those will be explained when they appear. The Mandelstam variable $\hat{s}$ in these C_0 functions can also be simply exchanged with other two variables $\hat{t}$ and $\hat{u}$.

3.3. High-Energy Limit of the D_0 Integral

In the calculation of the box diagrams we will encounter the scalar four-point integral, D_0 where we will only use the high-energy limit. The general definition of D_0 is

$$D_0(p_{10}, p_{12}, p_{23}, p_{30}, p_{20}, p_{13}, m_0^2, m_1^2, m_3^2, m_4^2) = \mu^{4-D}\int \frac{\mathrm{d}^D q}{i\pi^{\frac{D}{2}}} \frac{1}{(q^2 - m_0^2)((q+p_{10})^2 - m_1^2)((q+p_{20})^2 - m_2^2)((q+p_{30})^2 - m_3^2)}. \tag{3.14}$$

Again we want a result for the limit

$$\hat{s}, \hat{t}, \hat{u} \gg p_{10}, p_{12}, p_{23}, p_{30}.$$

We won't go through the calculation step by step but use the result presented in the Roth paper [6]:

$$\begin{aligned} D_0(k_1^2, k_2^2, k_2^2, k_4^2, (k_1 - k_2)^2, (k_2 - k_3)^2, m_0^2, m_1^2, m_3^2, m_4^2) = \\ -\frac{1}{ts}\left[\ln^2\left(\frac{-\hat{t} - i\epsilon}{-\hat{s} - i\epsilon}\right) + \pi^2\right] \\ + \frac{1}{s}C_0(k_1^2, (k_1 + k_2)^2, (-k_2)^2, m_1, m_0, m_2) + \frac{1}{s}C_0(k_2^2, (k_2 + k_3)^2, (-k_3)^2, m_2, m_1, m_3) \\ + \frac{1}{s}C_0(k_3^2, (k_3 + k_4)^2, (-k_4)^2, m_3, m_2, m_0) + \frac{1}{s}C_0(k_4^2, (k_4 + k_1)^2, (-k_1)^2, m_0, m_3, m_1) \end{aligned} \tag{3.15}$$

Later we will choose that

$$\mathrm{R}[\hat{t},\hat{s}] = \ln^2\left(\frac{-\hat{t}-i\epsilon}{-\hat{s}-i\epsilon}\right).$$

This expression holds for all non-singular cases, in particular for arbitrary masses. The other cases can be done at the level of 3-point functions.
It is also quite useful to know that we can change the order of the arguments in D_0 again

$$\begin{aligned} D_0(p_1^2,p_2^2,p_3^2,p_4^2,s_{12},s_{23},m_0^2,m_1^2,m_2^2,m_3^2) &= D_0(p_2^2,p_3^2,p_4^2,p_1^2,s_{23},s_{12},m_1^2,m_2^2,m_3^2,m_0^2),\\ D_0(p_1^2,p_2^2,p_3^2,p_4^2,s_{12},s_{23},m_0^2,m_1^2,m_2^2,m_3^2) &= D_0(p_4^2,p_3^2,p_2^2,p_1^2,s_{12},s_{23},m_0^2,m_3^2,m_2^2,m_1^2) \end{aligned} \tag{3.16}$$

with

$$\begin{aligned} s_{12} &= (p_1+p_2)^2,\\ s_{23} &= (p_2+p_3)^2. \end{aligned}$$

4. NLO Triangle Corrections and Their Sudakov Formfactors

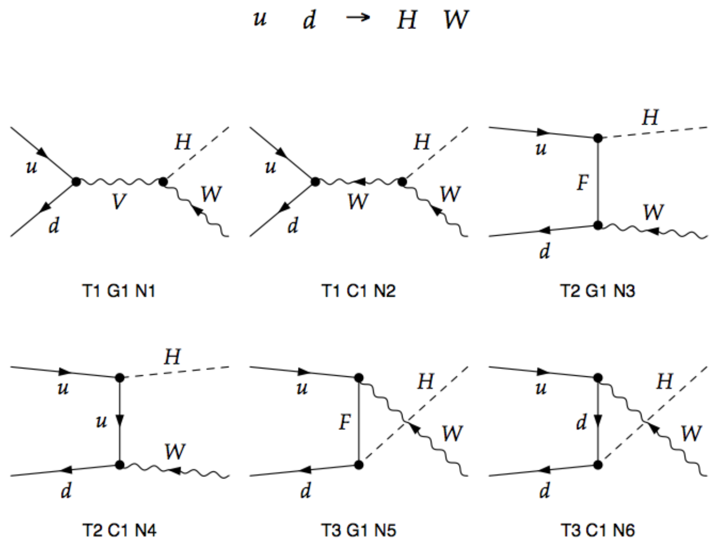


Figure 4.1.: $\hat{s}$-, $\hat{t}$- and $\hat{u}$- channels of the tree level process

In this section we will examine the triangle topology of the one-loop corrections. We have the process $u(p_1) + \bar{d}(p_2) \to H(p_3) + W^{\pm}(p_4)$. As always we can arrange the 2-to-2 process into the standard $\hat{s}$, $\hat{t}$, or $\hat{u}$-channel. We will now go through each of those channels and discuss its NLO triangle corrections. All the results are given after the Passarino reduction and inserting the scalar integrals where we set A_0 and B_0 to zero because they don't give rise to Sudakov logarithms (as shown in C.1.1 and 3.1). Addionally, we set all quark and lepton masses to zero. In the last step all the particles are put on-shell but we leave $p_3^2 = M_3^2$ and $p_4^2 = M_4^2$. This way we can either choose to set the masses of the Higgs and W-boson on-shell or to study the process with the further decay of the Higgs and/or the W-boson. For this study appendix B can be used.

For the loop calculation we used the Mathematica [10] package FeynCalc [11]. The end results are always given in polynomials of the so-called standard matrix elements [1].

Standard Matrix Elements

$$
\begin{aligned}
M_1 &= \overline{v}(p_2).(\gamma \cdot \varepsilon^*(p_4)).\gamma^7.u(p_1) \\
M_2 &= \overline{v}(p_2).(\gamma \cdot p_4).\gamma^7.u(p_1).(p_3 \cdot \varepsilon^*(p_4)) \\
M_3 &= \overline{v}(p_2).(\gamma \cdot p_3).(\gamma \cdot p_4).(\gamma \cdot \varepsilon^*(p_4)).\gamma^7.u(p_1) \\
M_4 &= \overline{v}(p_2).(\gamma \cdot p_3).\gamma^7.u(p_1).(p_2 \cdot \varepsilon^*(p_4)) \\
M_5 &= \overline{v}(p_2).(\gamma \cdot p_4).\gamma^7.u(p_1).(p_2 \cdot \varepsilon^*(p_4)) \\
M_6 &= p_3 \cdot \varepsilon^*(p_4)\overline{v}(p_2).(\gamma \cdot p_3).\gamma^7.u(p_1)
\end{aligned}
$$

In total there are 66 possibilities for triangle diagrams whereas ‘triangle diagrams’ are defined as Feynman diagrams containing a loop with a three particle vertice. We will go through all 8 topologies and point out the emerging Sudakov logarithms coming from the C_0 functions. It is important to point out that most of the diagrams vanish because of the zero quark/lepton masses. This is because the vertex factor of the ffH is proportional to the lepton mass [1],

$$g_{ffH} = -\frac{ie}{2s_W}\frac{m_f}{M_W}. \tag{4.1}$$

4.1. $\hat{s}$-channel

4.1.1. Topology I

In the first topology the correction of the WWH vertex is treated. After setting the quark and lepton masses to zero, five different diagrams remain.

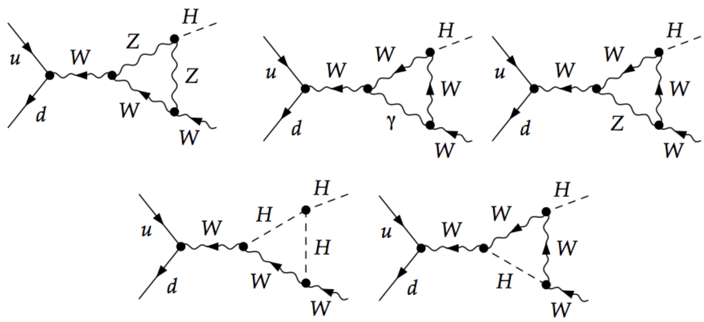


Figure 4.2.: Non-vanishing triangle diagram of the $\hat{s}$-channel first topology

In all of these diagrams we get the following C_0 function:

$$
\begin{aligned}
C_0(p_3^2, p_4^2, \hat{s}, M_1^2, M_2^2, M_3^2) &= C_0(p_4^2, \hat{s}, p_3^2, M_2^2, M_3^2, M_1^2) \\
&= \frac{1}{2\hat{s}}\left[\ln^2\left(\frac{-\hat{s}-i\epsilon}{M_2^2}\right) + 2(L(p_3^2, M_1^2, M_2^2) + L(p_4^2, M_3; M_2^2)\right] \\
&\overset{\text{just Sud log}}{=} \frac{1}{2\hat{s}}\ln^2\left(\frac{-\hat{s}-i\epsilon}{M_2^2}\right) := \Lambda_{\hat{s}}(M_2^2)
\end{aligned} \tag{4.2}
$$

Here we used the high energy assumption of the C_0 function 3.12. The total matrix element for the first topology is then

$$\begin{aligned} M_I^{\hat{s}} = &\left[A_{1,1}\Lambda_{\hat{s}}(M_W^2) + A_{1,2}\Lambda_{\hat{s}}(M_Z^2) + A_{1,3}\Lambda_{\hat{s}}(M_H^2) + A_{1,4}\right] M_1 \\ &+ \left[A_{2,1}\Lambda_{\hat{s}}(M_W^2) + A_{2,2}\Lambda_{\hat{s}}(M_Z^2) + A_{2,3}\Lambda_{\hat{s}}(M_H^2) + A_{2,4}\right] M_2 \\ &+ \left[A_{6,1}\Lambda_{\hat{s}}(M_W^2) + A_{6,2}\Lambda_{\hat{s}}(M_Z^2) + A_{6,3}\Lambda_{\hat{s}}(M_H^2) + A_{6,4}\right] M_6 \end{aligned} \tag{4.3}$$

with the following formfactors:

$$\begin{aligned} A_{1,1} = &\left(\alpha^2 M_W V_{ud}^* \left(c_W^2 \left(-5M_3^6 + 2M_3^4 \left(7M_4^2 + M_W^2 + 6M_Z^2 + 7\hat{s}\right) - M_3^2 \left(13M_4^4\right.\right.\right.\right. \\ &+2M_4^2 \left(7M_W^2 + 7M_Z^2 - 2\hat{s}\right) - 10M_W^4 + 2M_W^2 \left(10M_Z^2 + 7\hat{s}\right) - 10M_Z^4 + 14M_Z^2\hat{s} + 13\hat{s}^2\big) \\ &+2\left(M_4^2 - \hat{s}\right)^2 \left(2M_4^2 + 6M_W^2 + M_Z^2 + 2\hat{s}\right)\Big) - 5M_3^6 s_W^2 + 2M_3^4 \left(7s_W^2 \left(M_4^2 + \hat{s}\right)\right. \\ &+M_W^2 \left(s_W^2 - 1\right)\big) + M_3^2 \left(-13M_4^4 s_W^2 + M_4^2 \left(M_W^2 \left(4 - 14s_W^2\right) + 4\hat{s}s_W^2\right) + 10M_W^4 s_W^2\right. \\ &+2M_W^2\hat{s}\left(2 - 7s_W^2\right) - 13\hat{s}^2 s_W^2\big) + 2\left(M_4^2 - \hat{s}\right)^2 \left(2s_W^2 \left(M_4^2 + \hat{s}\right) + M_W^2 \left(6s_W^2 - 1\right)\right)\Big)\Big) \\ &/\left(2\sqrt{2}s_W^4 \left(M_W^2 - \hat{s}\right)\left(M_3^4 - 2M_3^2\left(M_4^2 + \hat{s}\right) + \left(M_4^2 - \hat{s}\right)^2\right)\right) \end{aligned}$$

$$\begin{aligned} A_{1,2} = &\left(\alpha^2 M_W V_{ud}^* \left(-5M_3^6 + 2M_3^4 \left(7M_4^2 + 6M_W^2 + M_Z^2 + 7\hat{s}\right) - M_3^2 \left(13M_4^4\right.\right.\right. \\ &+2M_4^2 \left(7M_W^2 + 7M_Z^2 - 2\hat{s}\right) - 10M_W^4 + 2M_W^2 \left(10M_Z^2 + 7\hat{s}\right) - 10M_Z^4 + 14M_Z^2\hat{s} + 13\hat{s}^2\big) \\ &+2\left(M_4^2 - \hat{s}\right)^2 \left(2M_4^2 + M_W^2 + 6M_Z^2 + 2\hat{s}\right)\Big)\Big) \\ &/\left(2\sqrt{2}s_W^4 \left(M_W^2 - \hat{s}\right)\left(M_3^4 - 2M_3^2\left(M_4^2 + \hat{s}\right) + \left(M_4^2 - \hat{s}\right)^2\right)\right) \end{aligned}$$

$$A_{1,3} = -\frac{3\alpha^2 \mathrm{MH}^2 M_W V_{ud}^*}{2\sqrt{2}s_W^4\left(M_W^2 - \hat{s}\right)}$$

$$A_{1,4} = \frac{\alpha^2 M_W \left(c_W^2 + s_W^2 + 1\right) V_{ud}^*}{2\sqrt{2}s_W^4\left(M_W^2 - \hat{s}\right)}$$

$$A_{2,1} = -\left(\alpha^2 M_W V_{ud}^* \left(c_W^2 \left(4M_3^8 - M_3^6 \left(22M_4^2 - 7M_W^2 + 17\left(M_Z^2 + \hat{s}\right)\right) + M_3^4 \left(32M_4^4\right.\right.\right.\right.$$
$$+M_4^2\left(31M_W^2 + 3\left(\hat{s} - 7M_Z^2\right)\right) - 30M_W^4 + M_W^2\left(60M_Z^2 + \hat{s}\right) - 30M_Z^4 + 29M_Z^2\hat{s} + 27\hat{s}^2\right)$$
$$-M_3^2\left(14M_4^6 + M_4^4\left(23M_W^2 - 33M_Z^2 + \hat{s}\right) + M_4^2\left(30M_W^4 - 2M_W^2\left(30M_Z^2 + 23\hat{s}\right) + 30M_Z^4\right.\right.$$
$$\left.+26M_Z^2\hat{s} - 34\hat{s}^2\right) + \hat{s}\left(-30M_W^4 + M_W^2\left(60M_Z^2 + 23\hat{s}\right) - 30M_Z^4 + 7M_Z^2\hat{s} + 19\hat{s}^2\right)\Big)$$
$$\left.-5\left(M_4^2 - \hat{s}\right)^3\left(3M_W^2 - M_Z^2 + \hat{s}\right)\right) + s_W^2\left(4M_3^8 + M_3^6\left(-22M_4^2 + 7M_W^2 - 17\hat{s}\right)\right.$$
$$+M_3^4\left(32M_4^4 + M_4^2\left(31M_W^2 + 3\hat{s}\right) - 30M_W^4 + M_W^2\hat{s} + 27\hat{s}^2\right) - M_3^2\left(14M_4^6 + M_4^4\left(23M_W^2 + \hat{s}\right)\right.$$
$$\left.\left.\left.\left.+M_4^2\left(30M_W^4 - 46M_W^2\hat{s} - 34\hat{s}^2\right) + \hat{s}\left(-30M_W^4 + 23M_W^2\hat{s} + 19\hat{s}^2\right)\right) - 5\left(M_4^2 - \hat{s}\right)^3\left(3M_W^2 + \hat{s}\right)\right)\right)\right)$$
$$/\left(\sqrt{2}s_W^4\left(M_W^2 - \hat{s}\right)\left(M_3^4 - 2M_3^2\left(M_4^2 + \hat{s}\right) + \left(M_4^2 - \hat{s}\right)^2\right)^2\right)$$

$$A_{2,2} = \left(\alpha^2 M_W V_{ud}^* \left(-4M_3^8 + M_3^6\left(22M_4^2 + 17M_W^2 - 7M_Z^2 + 17\hat{s}\right) - M_3^4\left(32M_4^4\right.\right.\right.$$
$$\left.+M_4^2\left(-21M_W^2 + 31M_Z^2 + 3\hat{s}\right) - 30M_W^4 + M_W^2\left(60M_Z^2 + 29\hat{s}\right) - 30M_Z^4 + M_Z^2\hat{s} + 27\hat{s}^2\right)$$
$$+M_3^2\left(14M_4^6 + M_4^4\left(-33M_W^2 + 23M_Z^2 + \hat{s}\right) + M_4^2\left(30M_W^4 + M_W^2\left(26\hat{s} - 60M_Z^2\right) + 30M_Z^4\right.\right.$$
$$\left.\left.-46M_Z^2\hat{s} - 34\hat{s}^2\right) + \hat{s}\left(-30M_W^4 + M_W^2\left(60M_Z^2 + 7\hat{s}\right) - 30M_Z^4 + 23M_Z^2\hat{s} + 19\hat{s}^2\right)\right)$$
$$\left.\left.+5\left(M_4^2 - \hat{s}\right)^3\left(-M_W^2 + 3M_Z^2 + \hat{s}\right)\right)\right) / \left(\sqrt{2}s_W^4\left(M_W^2 - \hat{s}\right)\left(M_3^4 - 2M_3^2\left(M_4^2 + \hat{s}\right) + \left(M_4^2 - \hat{s}\right)^2\right)^2\right)$$

$$A_{2,3} = 0$$

$$A_{2,4} = \frac{5\alpha^2 M_W V_{ud}^*\left(M_3^2\left(\hat{s}\left(c_W^2 + s_W^2 + 1\right) + M_W^2 s_W^2\right) + \left(M_4^2 - \hat{s}\right)\left(\hat{s}\left(c_W^2 + s_W^2 + 1\right) - M_W^2 s_W^2\right)\right)}{\sqrt{2}\hat{s}s_W^4\left(M_W^2 - \hat{s}\right)\left(M_3^4 - 2M_3^2\left(M_4^2 + \hat{s}\right) + \left(M_4^2 - \hat{s}\right)^2\right)}$$

$$A_{6,1} = \left(5\alpha^2 M_W V_{ud}^*\left(c_W^2\left(M_3^6\left(M_4^2 - M_W^2 + M_Z^2\right) - M_3^4\left(M_4^4 + M_4^2\left(3M_W^2 - 7M_Z^2 + \hat{s}\right)\right.\right.\right.\right.$$
$$\left.-\left(M_W^2 - M_Z^2\right)\left(2M_W^2 - 2M_Z^2 + 3\hat{s}\right)\right) - M_3^2\left(M_4^6 + M_4^4\left(3M_W^2 + 5M_Z^2 - 6\hat{s}\right)\right.$$
$$+M_4^2\left(-8M_W^4 + 2M_W^2\left(8M_Z^2 + \hat{s}\right) - 8M_Z^4 + 6M_Z^2\hat{s} + \hat{s}^2\right) + \hat{s}\left(M_W^2 - M_Z^2\right)$$
$$\left.\left(4M_W^2 - 4M_Z^2 + 3\hat{s}\right)\right) + \left(M_4^2 - \hat{s}\right)^2\left(M_4^4 + M_4^2\left(7M_W^2 - 3M_Z^2 + \hat{s}\right) + \left(M_W^2 - M_Z^2\right)\right.$$
$$\left.\left.\left(2M_W^2 - 2M_Z^2 + \hat{s}\right)\right)\right) + s_W^2\left(M_3^6\left(M_4^2 - M_W^2\right) - M_3^4\left(M_4^4 + M_4^2\left(3M_W^2 + \hat{s}\right)\right.\right.$$
$$\left.-2M_W^4 - 3M_W^2\hat{s}\right) - M_3^2\left(M_4^6 + 3M_4^4\left(M_W^2 - 2\hat{s}\right) + M_4^2\left(-8M_W^4 + 2M_W^2\hat{s} + \hat{s}^2\right)\right.$$
$$\left.\left.\left.\left.+M_W^2\hat{s}\left(4M_W^2 + 3\hat{s}\right)\right) + \left(M_4^2 - \hat{s}\right)^2\left(M_4^4 + M_4^2\left(7M_W^2 + \hat{s}\right) + M_W^2\left(2M_W^2 + \hat{s}\right)\right)\right)\right)\right)$$
$$/\left(\sqrt{2}s_W^4\left(M_W^2 - \hat{s}\right)\left(M_3^4 - 2M_3^2\left(M_4^2 + \hat{s}\right) + \left(M_4^2 - \hat{s}\right)^2\right)^2\right)$$

$$\begin{aligned}
A_{6,2} = &\left(5\alpha^2 M_W V_{ud}^* \left(M_3^6\left(M_4^2+M_W^2-M_Z^2\right)-M_3^4\left(M_4^4+M_4^2\left(-7M_W^2+3M_Z^2+\hat{s}\right)\right.\right.\right.\\
&-\left(M_W^2-M_Z^2\right)\left(2M_W^2-2M_Z^2-3\hat{s}\right)\right)-M_3^2\left(M_4^6+M_4^4\left(5M_W^2+3M_Z^2-6\hat{s}\right)\right.\\
&+M_4^2\left(-8M_W^4+2M_W^2\left(8M_Z^2+3\hat{s}\right)-8M_Z^4+2M_Z^2\hat{s}+\hat{s}^2\right)+\hat{s}\left(M_W^2-M_Z^2\right)\\
&\left(4M_W^2-4M_Z^2-3\hat{s}\right)\right)+\left(M_4^2-\hat{s}\right)^2\left(M_4^4+M_4^2\left(-3M_W^2+7M_Z^2+\hat{s}\right)+\left(M_W^2-M_Z^2\right)\right.\\
&\left.\left.\left.\left(2M_W^2-2M_Z^2-\hat{s}\right)\right)\right)\right)/\left(\sqrt{2}s_W^4\left(M_W^2-\hat{s}\right)\left(M_3^4-2M_3^2\left(M_4^2+\hat{s}\right)+\left(M_4^2-\hat{s}\right)^2\right)^2\right)\\
A_{6,3} = &\ 0\\
A_{6,4} = &-\frac{5\alpha^2 M_W V_{ud}^*\left(M_4^2\left(M_W^2 s_W^2-2\hat{s}\left(c_W^2+s_W^2+1\right)\right)+M_W^2 s_W^2\left(\hat{s}-M_3^2\right)\right)}{\sqrt{2}\hat{s}s_W^4\left(M_W^2-\hat{s}\right)\left(M_3^4-2M_3^2\left(M_4^2+\hat{s}\right)+\left(M_4^2-\hat{s}\right)^2\right)}
\end{aligned}$$

4.1.2. Topology II

The second topology of the $\hat{s}$-channel is the NLO correction of the qqW vetex. Again just five diagrams remain.

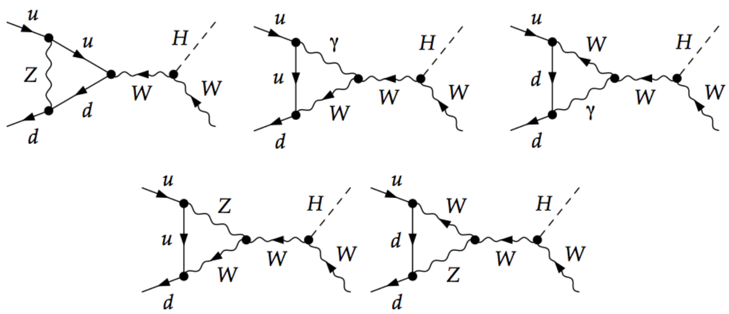


Figure 4.3.: Non-vanishing triangle diagrams of the $\hat{s}$-channel second topology

Unlike before we don't end up with just one type of C_0 function. We will first discuss all types and then present the results and give the matrix element with the relevant formfactors.

The first diagram in figure 4.3 gives

$$C_0(0,\hat{s},0,M^2,0,0) = \frac{1}{2\hat{s}}\ln^2\left(\frac{\hat{s}}{M^2}\right) = \Lambda_{\hat{s}}(M^2). \tag{4.4}$$

Again we used equation 3.12. The next two diagrams give the scalar integrals

$$\begin{aligned}
&C_0(0,\hat{s},0,0,M^2,0),\\
&C_0(0,\hat{s},0,0,0,M^2).
\end{aligned} \tag{4.5}$$

Both of these C_0 functions are divergent. They also don't give rise to Sudakov logarithms so will neglect them.

For the last two diagrams we have used the solution of the C_0 function with $m_0^2 = 0$,

$$\begin{aligned} C_0(0, \hat{s}, 0, 0, M_1^2, M_2^2) &= \frac{1}{\hat{s}} \ln\left(\frac{\hat{s}}{M_1^2}\right) \ln\left(\frac{\hat{s}}{M_2^2}\right) \\ &= \Lambda_{\hat{s}}(M_1^2, M_2^2). \end{aligned} \tag{4.6}$$

Now the total matrix element is

$$\begin{aligned} M_{II}^{\hat{s}} = &\left[B_{1,1}\Lambda_{\hat{s}}(M_Z^2) + B_{1,2}\Lambda_{\hat{s}}(M_W^2, M_Z^2) + B_{1,3}\right] M_1 \\ &+ \left[B_{2,1}\Lambda_{\hat{s}}(M_Z^2) + B_{2,2}\Lambda_{\hat{s}}(M_W^2, M_Z^2) + B_{2,3}\right] M_2 \\ &+ \left[B_{4,1}\Lambda_{\hat{s}}(M_Z^2) + B_{4,2}\Lambda_{\hat{s}}(M_W^2, M_Z^2) + B_{4,3}\right] M_4 \\ &+ \left[B_{5,1}\Lambda_{\hat{s}}(M_Z^2) + B_{5,2}\Lambda_{\hat{s}}(M_W^2, M_Z^2) + B_{5,3}\right] M_5 \\ &+ \left[B_{6,1}\Lambda_{\hat{s}}(M_Z^2) + B_{6,2}\Lambda_{\hat{s}}(M_W^2, M_Z^2) + B_{6,3}\right] M_6 \end{aligned} \tag{4.7}$$

with the following Sudakov formfactors:

$$B_{1,1} = \frac{\alpha^2 M_W \hat{s} \left(8s_W^4 - 18s_W^2 + 9\right) V_{ud}^* \left(-M_3^2 - M_4^2 + M_Z^2 + \hat{t} + \hat{u}\right)^2}{18\sqrt{2} c_W^2 s_W^4 \left(M_W^2 - \hat{s}\right) \left(M_3^2 + M_4^2 - \hat{t} - \hat{u}\right)^2}$$

$$\begin{aligned}
B_{1,2} = {} & \big(\alpha^2 M_W \big(12 \left(s_W^2 - 1\right) M_3^6 + \big(36 \left(s_W^2 - 1\right) M_4^2 + 9M_Z^2 - 10M_Z^2 s_W^2 + 12\hat{s} s_W^2 - 12\hat{s} \\
& +M_W^2 \left(9 - 8s_W^2\right) - 36 s_W^2 \hat{t} + 36\hat{t} - 36 s_W^2 \hat{u} + 36\hat{u}\big) M_3^4 + \big(36 \left(s_W^2 - 1\right) M_4^4 \\
& -2 \left(\left(8s_W^2 - 9\right) M_W^2 + M_Z^2 \left(10s_W^2 - 9\right) - 12 \left(s_W^2 - 1\right) (\hat{s} - 3(\hat{t} + \hat{u}))\right) M_4^2 + 6M_Z^4 \\
& -4M_Z^4 s_W^2 + 2M_Z^2 \hat{s} s_W^2 + 36 s_W^2 \hat{t}^2 - 36\hat{t}^2 + 36 s_W^2 \hat{u}^2 - 36\hat{u}^2 - 3M_Z^2 \hat{s} + M_W^4 \left(6 - 8s_W^2\right) \\
& -18M_Z^2 \hat{t} + 20M_Z^2 s_W^2 \hat{t} - 24\hat{s} s_W^2 \hat{t} + 24\hat{s}\hat{t} - 18M_Z^2 \hat{u} + 20M_Z^2 s_W^2 \hat{u} - 24\hat{s} s_W^2 \hat{u} + 24\hat{s}\hat{u} \\
& +72 s_W^2 \hat{t}\hat{u} - 72\hat{t}\hat{u} + M_W^2 \left(12 \left(s_W^2 - 1\right) M_Z^2 + \hat{s} \left(4s_W^2 - 3\right) + 2 \left(8s_W^2 - 9\right) (\hat{t} + \hat{u})\right)\big) M_3^2 \\
& -12 s_W^2 \hat{t}^3 + 12\hat{t}^3 - 12 s_W^2 \hat{u}^3 + 12\hat{u}^3 - 8M_W^4 \hat{s} s_W^2 - 4M_Z^4 \hat{s} s_W^2 + 9M_W^2 \hat{t}^2 + 9M_Z^2 \hat{t}^2 \\
& -8M_W^2 s_W^2 \hat{t}^2 - 10M_Z^2 s_W^2 \hat{t}^2 + 12\hat{s} s_W^2 \hat{t}^2 - 12\hat{s}\hat{t}^2 + 9M_W^2 \hat{u}^2 + 9M_Z^2 \hat{u}^2 - 8M_W^2 s_W^2 \hat{u}^2 \\
& -10M_Z^2 s_W^2 \hat{u}^2 + 12\hat{s} s_W^2 \hat{u}^2 - 12\hat{s}\hat{u}^2 - 36 s_W^2 \hat{t}\hat{u}^2 + 36\hat{t}\hat{u}^2 + 6M_W^4 \hat{s} + 6M_Z^4 \hat{s} + 12M_4^6 \left(s_W^2 - 1\right) \\
& -6M_W^4 \hat{t} - 6M_Z^4 \hat{t} + 12M_W^2 M_Z^2 \hat{t} + 8M_W^4 s_W^2 \hat{t} + 4M_Z^4 s_W^2 \hat{t} - 12M_W^2 M_Z^2 s_W^2 \hat{t} \\
& -4M_W^2 \hat{s} s_W^2 \hat{t} - 2M_Z^2 \hat{s} s_W^2 \hat{t} + 3M_W^2 \hat{s}\hat{t} + 3M_Z^2 \hat{s}\hat{t} - 6M_W^4 \hat{u} - 6M_Z^4 \hat{u} + 12M_W^2 M_Z^2 \hat{u} \\
& +8M_W^4 s_W^2 \hat{u} + 4M_Z^4 s_W^2 \hat{u} - 12M_W^2 M_Z^2 s_W^2 \hat{u} - 4M_W^2 \hat{s} s_W^2 \hat{u} - 2M_Z^2 \hat{s} s_W^2 \hat{u} - 36 s_W^2 \hat{t}^2 \hat{u} \\
& +36\hat{t}^2 \hat{u} + 3M_W^2 \hat{s}\hat{u} + 3M_Z^2 \hat{s}\hat{u} + 18M_W^2 \hat{t}\hat{u} + 18M_Z^2 \hat{t}\hat{u} - 16M_W^2 s_W^2 \hat{t}\hat{u} - 20M_Z^2 s_W^2 \hat{t}\hat{u} \\
& +24\hat{s} s_W^2 \hat{t}\hat{u} - 24\hat{s}\hat{t}\hat{u} + M_4^4 \big(\left(9 - 8s_W^2\right) M_W^2 + M_Z^2 \left(9 - 10s_W^2\right) \\
& +12 \left(s_W^2 - 1\right) (\hat{s} - 3(\hat{t} + \hat{u}))\big) + M_4^2 \big(\left(6 - 8s_W^2\right) M_W^4 + \big(12 \left(s_W^2 - 1\right) M_Z^2 + \hat{s} \left(4s_W^2 - 3\right) \\
& +2 \left(8s_W^2 - 9\right) (\hat{t} + \hat{u})\big) M_W^2 + M_Z^4 \left(6 - 4s_W^2\right) + 12 \left(s_W^2 - 1\right) (\hat{t} + \hat{u})(3(\hat{t} + \hat{u}) - 2\hat{s}) \\
& +M_Z^2 \left(\hat{s} \left(2s_W^2 - 3\right) + 2 \left(10s_W^2 - 9\right) (\hat{t} + \hat{u})\right)\big)\big) V_{ud}^*\big) \\
& \Big/ \left(6\sqrt{2} \left(M_W^2 - \hat{s}\right) s_W^4 \left(M_3^2 + M_4^2 - \hat{t} - \hat{u}\right)^2\right)
\end{aligned}$$

$$B_{1,3} = \frac{\alpha^2 M_W \left(8s_W^4 - 18s_W^2 + 9\right) V_{ud}^*}{18\sqrt{2} c_W^2 s_W^4 \left(M_W^2 - \hat{s}\right)}$$

$$B_{2,1} = \frac{\alpha^2 M_W M_Z^2 \left(8s_W^4 - 18s_W^2 + 9\right) V_{ud}^* \left(-M_3^2 - M_4^2 + M_Z^2 + \hat{t} + \hat{u}\right)}{9\sqrt{2} c_W^2 s_W^4 \left(M_W^2 - \hat{s}\right) \left(M_3^2 + M_4^2 - \hat{t} - \hat{u}\right)^2}$$

$$\begin{aligned}
B_{2,2} = {} & \big(\alpha^2 M_W V_{ud}^* \big(M_3^2 \left(M_W^2 \left(3 - 4s_W^2\right) + M_Z^2 \left(3 - 2s_W^2\right)\right) + M_4^2 \left(M_W^2 \left(3 - 4s_W^2\right) + M_Z^2 \left(3 - 2s_W^2\right)\right) \\
& -8M_W^4 s_W^2 + 6M_W^4 + 4M_W^2 s_W^2 \hat{t} + 4M_W^2 s_W^2 \hat{u} - 3M_W^2 \hat{t} - 3M_W^2 \hat{u} - 4M_Z^4 s_W^2 + 6M_Z^4 \\
& +2M_Z^2 s_W^2 \hat{t} + 2M_Z^2 s_W^2 \hat{u} - 3M_Z^2 \hat{t} - 3M_Z^2 \hat{u}\big)\big) \Big/ \left(3\sqrt{2} s_W^4 \left(M_W^2 - \hat{s}\right) \left(M_3^2 + M_4^2 - \hat{t} - \hat{u}\right)^2\right)
\end{aligned}$$

$$\begin{aligned}
B_{2,3} = {} & \big(\alpha^2 M_W V_{ud}^* \big(M_Z^2 \hat{s} \left(8s_W^4 - 18s_W^2 + 9\right) - 6c_W^2 \big(M_W^2 \left(2s_W^2 \left(M_3^2 + M_4^2 - \hat{t} - \hat{u}\right) + \hat{s} \left(2s_W^2 + 3\right)\right) \\
& +M_Z^2 \hat{s} \left(3 - 4s_W^2\right)\big)\big)\big) \Big/ \left(18\sqrt{2} c_W^2 \hat{s} s_W^4 \left(M_W^2 - \hat{s}\right) \left(M_3^2 + M_4^2 - \hat{t} - \hat{u}\right) \left(M_3^2 + M_4^2 + \hat{s} - \hat{t} - \hat{u}\right)\right)
\end{aligned}$$

$$
\begin{aligned}
B_{4,1} = &-\big(\alpha^2 M_W \hat{s}\left(8s_W^4-18s_W^2+9\right)V_{ud}^*\big(M_3^4+2M_3^2\left(M_4^2-2M_Z^2-\hat{t}-\hat{u}\right)+M_4^4\\
&-2M_4^2\left(2M_Z^2+\hat{t}+\hat{u}\right)+3M_Z^4+4M_Z^2\hat{t}+4M_Z^2\hat{u}+\hat{t}^2+2\hat{t}\hat{u}+\hat{u}^2\big)\big)\\
&\ \Big/\Big(9\sqrt{2}c_W^2 s_W^4\left(M_W^2-\hat{s}\right)\left(M_3^2+M_4^2-\hat{t}-\hat{u}\right)^3\Big)
\end{aligned}
$$

$$
\begin{aligned}
B_{4,2} = &\big(\alpha^2 M_W \big(-6\left(s_W^2-1\right)M_3^6+\big(-18\left(s_W^2-1\right)M_4^2-9M_Z^2+8M_Z^2 s_W^2-6\hat{s}s_W^2+6\hat{s}\\
&+M_W^2\left(10s_W^2-9\right)+18s_W^2\hat{t}-18\hat{t}+18s_W^2\hat{u}-18\hat{u}\big)M_3^4-2\big(9\left(s_W^2-1\right)M_4^4\\
&+\left(\left(9-10s_W^2\right)M_W^2+M_Z^2\left(9-8s_W^2\right)+6\left(s_W^2-1\right)(\hat{s}-3(\hat{t}+\hat{u}))\right)M_4^2+6M_Z^4\\
&-4M_Z^4 s_W^2-2M_Z^2\hat{s}s_W^2+9s_W^2\hat{t}^2-9\hat{t}^2+9s_W^2\hat{u}^2-9\hat{u}^2+3M_Z^2\hat{s}+M_W^4\left(6-8s_W^2\right)\\
&-9M_Z^2\hat{t}+8M_Z^2 s_W^2\hat{t}-6\hat{s}s_W^2\hat{t}+6\hat{s}\hat{t}-9M_Z^2\hat{u}+8M_Z^2 s_W^2\hat{u}-6\hat{s}s_W^2\hat{u}+6\hat{s}\hat{u}+18s_W^2\hat{t}\hat{u}\\
&-18\hat{t}\hat{u}+M_W^2\left(12\left(s_W^2-1\right)M_Z^2+\hat{s}\left(3-4s_W^2\right)+\left(10s_W^2-9\right)(\hat{t}+\hat{u})\right)\big)M_3^2+6s_W^2\hat{t}^3\\
&-6\hat{t}^3+6s_W^2\hat{u}^3-6\hat{u}^3+24M_W^4\hat{s}s_W^2+12M_Z^4\hat{s}s_W^2-9M_W^2\hat{t}^2-9M_Z^2\hat{t}^2+10M_W^2 s_W^2\hat{t}^2\\
&+8M_Z^2 s_W^2\hat{t}^2-6\hat{s}s_W^2\hat{t}^2+6\hat{s}\hat{t}^2-9M_W^2\hat{u}^2-9M_Z^2\hat{u}^2+10M_W^2 s_W^2\hat{u}^2+8M_Z^2 s_W^2\hat{u}^2\\
&-6\hat{s}s_W^2\hat{u}^2+6\hat{s}\hat{u}^2+18s_W^2\hat{t}\hat{u}^2-18\hat{t}\hat{u}^2-18M_W^4\hat{s}-18M_Z^4\hat{s}-6M_4^6\left(s_W^2-1\right)+12M_W^4\hat{t}\\
&+12M_Z^4\hat{t}-24M_W^2M_Z^2\hat{t}-16M_W^4 s_W^2\hat{t}-8M_Z^4 s_W^2\hat{t}+24M_W^2M_Z^2 s_W^2\hat{t}-8M_W^2\hat{s}s_W^2\hat{t}\\
&-4M_Z^2\hat{s}s_W^2\hat{t}+6M_W^2\hat{s}\hat{t}+6M_Z^2\hat{s}\hat{t}+12M_W^4\hat{u}+12M_Z^4\hat{u}-24M_W^2M_Z^2\hat{u}-16M_W^4 s_W^2\hat{u}\\
&-8M_Z^4 s_W^2\hat{u}+24M_W^2M_Z^2 s_W^2\hat{u}-8M_W^2\hat{s}s_W^2\hat{u}-4M_Z^2\hat{s}s_W^2\hat{u}+18s_W^2\hat{t}^2\hat{u}-18\hat{t}^2\hat{u}+6M_W^2\hat{s}\hat{u}\\
&+6M_Z^2\hat{s}\hat{u}-18M_W^2\hat{t}\hat{u}-18M_Z^2\hat{t}\hat{u}+20M_W^2 s_W^2\hat{t}\hat{u}+16M_Z^2 s_W^2\hat{t}\hat{u}-12\hat{s}s_W^2\hat{t}\hat{u}+12\hat{s}\hat{t}\hat{u}\\
&+M_4^4\left(\left(10s_W^2-9\right)M_W^2+M_Z^2\left(8s_W^2-9\right)-6\left(s_W^2-1\right)(\hat{s}-3(\hat{t}+\hat{u}))\right)\\
&+2M_4^2\big(\left(8s_W^2-6\right)M_W^4+\left(-12\left(s_W^2-1\right)M_Z^2+\hat{s}\left(4s_W^2-3\right)-\left(10s_W^2-9\right)(\hat{t}+\hat{u})\right)M_W^2\\
&+M_Z^4\left(4s_W^2-6\right)+3\left(s_W^2-1\right)(\hat{t}+\hat{u})(2\hat{s}-3(\hat{t}+\hat{u}))\\
&+M_Z^2\left(\hat{s}\left(2s_W^2-3\right)-\left(8s_W^2-9\right)(\hat{t}+\hat{u})\right)\big)\big)\big)V_{ud}^*\big)\Big/\Big(3\sqrt{2}\left(M_W^2-\hat{s}\right)s_W^4\left(M_3^2+M_4^2-\hat{t}-\hat{u}\right)^3\Big)
\end{aligned}
$$

$$
\begin{aligned}
B_{4,3} = &\big(\alpha^2 M_W V_{ud}^*\big(M_3^4\left(-\left(36c_W^2+8s_W^4-18s_W^2+9\right)\right)+M_3^2\big(-2M_4^2\left(36c_W^2+8s_W^4-18s_W^2+9\right)\\
&+6c_W^2\left(M_W^2\left(4s_W^2+3\right)+M_Z^2\left(3-4s_W^2\right)-6(\hat{s}-2(\hat{t}+\hat{u}))\right)\\
&-\left(8s_W^4-18s_W^2+9\right)\left(M_Z^2+\hat{s}-2(\hat{t}+\hat{u})\right)\big)-M_4^4\left(36c_W^2+8s_W^4-18s_W^2+9\right)\\
&+M_4^2\big(6c_W^2\left(M_W^2\left(4s_W^2+3\right)+M_Z^2\left(3-4s_W^2\right)-6(\hat{s}-2(\hat{t}+\hat{u}))\right)\\
&-\left(8s_W^4-18s_W^2+9\right)\left(M_Z^2+\hat{s}-2(\hat{t}+\hat{u})\right)\big)+24c_W^2M_W^2\hat{s}s_W^2+36c_W^2M_W^2\hat{s}\\
&-24c_W^2M_W^2 s_W^2\hat{t}-24c_W^2M_W^2 s_W^2\hat{u}-18c_W^2M_W^2\hat{t}-18c_W^2M_W^2\hat{u}-48c_W^2M_Z^2\hat{s}s_W^2\\
&+36c_W^2M_Z^2\hat{s}+24c_W^2M_Z^2 s_W^2\hat{t}+24c_W^2M_Z^2 s_W^2\hat{u}-18c_W^2M_Z^2\hat{t}-18c_W^2M_Z^2\hat{u}+36c_W^2\hat{s}\hat{t}\\
&+36c_W^2\hat{s}\hat{u}-36c_W^2\hat{t}^2-72c_W^2\hat{t}\hat{u}-36c_W^2\hat{u}^2-16M_Z^2\hat{s}s_W^4+36M_Z^2\hat{s}s_W^2-18M_Z^2\hat{s}\\
&+8M_Z^2 s_W^4\hat{t}+8M_Z^2 s_W^4\hat{u}-18M_Z^2 s_W^2\hat{t}-18M_Z^2 s_W^2\hat{u}+9M_Z^2\hat{t}+9M_Z^2\hat{u}+8\hat{s}s_W^4\hat{t}\\
&+8\hat{s}s_W^4\hat{u}-18\hat{s}s_W^2\hat{t}-18\hat{s}s_W^2\hat{u}+9\hat{s}\hat{t}+9\hat{s}\hat{u}-8s_W^4\hat{t}^2-16s_W^4\hat{t}\hat{u}-8s_W^4\hat{u}^2+18s_W^2\hat{t}^2\\
&+36s_W^2\hat{t}\hat{u}+18s_W^2\hat{u}^2-9\hat{t}^2-18\hat{t}\hat{u}-9\hat{u}^2\big)\big)\\
&\ \Big/\Big(18\sqrt{2}c_W^2 s_W^4\left(M_W^2-\hat{s}\right)\left(M_3^2+M_4^2-\hat{t}-\hat{u}\right)^2\left(M_3^2+M_4^2+\hat{s}-\hat{t}-\hat{u}\right)\Big)
\end{aligned}
$$

$$
\begin{aligned}
B_{5,1} = &-\left(\alpha^2 M_W \hat{s}\left(8s_W^4 - 18s_W^2 + 9\right) V_{ud}^* \left(M_3^4 + 2M_3^2\left(M_4^2 - 2M_Z^2 - \hat{t} - \hat{u}\right) + M_4^4\right.\right. \\
&\left.\left.-2M_4^2\left(2M_Z^2 + \hat{t} + \hat{u}\right) + 3M_Z^4 + 4M_Z^2\hat{t} + 4M_Z^2\hat{u} + \hat{t}^2 + 2\hat{t}\hat{u} + \hat{u}^2\right)\right) \\
&\quad / \left(9\sqrt{2}c_W^2 s_W^4\left(M_W^2 - \hat{s}\right)\left(M_3^2 + M_4^2 - \hat{t} - \hat{u}\right)^3\right)
\end{aligned}
$$

$$
\begin{aligned}
B_{5,2} = &\left(\alpha^2 M_W\left(-6\left(s_W^2 - 1\right)M_3^6 + \left(-18\left(s_W^2 - 1\right)M_4^2 - 9M_Z^2 + 8M_Z^2 s_W^2 - 6\hat{s}s_W^2 + 6\hat{s}\right.\right.\right. \\
&\left.+M_W^2\left(10s_W^2 - 9\right) + 18s_W^2\hat{t} - 18\hat{t} + 18s_W^2\hat{u} - 18\hat{u}\right)M_3^4 - 2\left(9\left(s_W^2 - 1\right)M_4^4\right. \\
&+\left(\left(9 - 10s_W^2\right)M_W^2 + M_Z^2\left(9 - 8s_W^2\right) + 6\left(s_W^2 - 1\right)\left(\hat{s} - 3(\hat{t} + \hat{u})\right)\right)M_4^2 + 6M_Z^4 \\
&-4M_Z^4 s_W^2 - 2M_Z^2\hat{s}s_W^2 + 9s_W^2\hat{t}^2 - 9\hat{t}^2 + 9s_W^2\hat{u}^2 - 9\hat{u}^2 + 3M_Z^2\hat{s} + M_W^4\left(6 - 8s_W^2\right) \\
&-9M_Z^2\hat{t} + 8M_Z^2 s_W^2\hat{t} - 6\hat{s}s_W^2\hat{t} + 6\hat{s}\hat{t} - 9M_Z^2\hat{u} + 8M_Z^2 s_W^2\hat{u} - 6\hat{s}s_W^2\hat{u} + 6\hat{s}\hat{u} \\
&\left.+18s_W^2\hat{t}\hat{u} - 18\hat{t}\hat{u} + M_W^2\left(12\left(s_W^2 - 1\right)M_Z^2 + \hat{s}\left(3 - 4s_W^2\right) + \left(10s_W^2 - 9\right)(\hat{t} + \hat{u})\right)\right)M_3^2 \\
&+6s_W^2\hat{t}^3 - 6\hat{t}^3 + 6s_W^2\hat{u}^3 - 6\hat{u}^3 + 24M_W^4\hat{s}s_W^2 + 12M_Z^4\hat{s}s_W^2 - 9M_W^2\hat{t}^2 - 9M_Z^2\hat{t}^2 \\
&+10M_W^2 s_W^2\hat{t}^2 + 8M_Z^2 s_W^2\hat{t}^2 - 6\hat{s}s_W^2\hat{t}^2 + 6\hat{s}\hat{t}^2 - 9M_W^2\hat{u}^2 - 9M_Z^2\hat{u}^2 + 10M_W^2 s_W^2\hat{u}^2 \\
&+8M_Z^2 s_W^2\hat{u}^2 - 6\hat{s}s_W^2\hat{u}^2 + 6\hat{s}\hat{u}^2 + 18s_W^2\hat{t}\hat{u}^2 - 18\hat{t}\hat{u}^2 - 18M_W^4\hat{s} - 18M_Z^4\hat{s} \\
&-6M_4^6\left(s_W^2 - 1\right) + 12M_W^4\hat{t} + 12M_Z^4\hat{t} - 24M_W^2 M_Z^2\hat{t} - 16M_W^4 s_W^2\hat{t} - 8M_Z^4 s_W^2\hat{t} \\
&+24M_W^2 M_Z^2 s_W^2\hat{t} - 8M_W^2\hat{s}s_W^2\hat{t} - 4M_Z^2\hat{s}s_W^2\hat{t} + 6M_W^2\hat{s}\hat{t} + 6M_Z^2\hat{s}\hat{t} + 12M_W^4\hat{u} + 12M_Z^4\hat{u} \\
&-24M_W^2 M_Z^2\hat{u} - 16M_W^4 s_W^2\hat{u} - 8M_Z^4 s_W^2\hat{u} + 24M_W^2 M_Z^2 s_W^2\hat{u} - 8M_W^2\hat{s}s_W^2\hat{u} - 4M_Z^2\hat{s}s_W^2\hat{u} \\
&+18s_W^2\hat{t}^2\hat{u} - 18\hat{t}^2\hat{u} + 6M_W^2\hat{s}\hat{u} + 6M_Z^2\hat{s}\hat{u} - 18M_W^2\hat{t}\hat{u} - 18M_Z^2\hat{t}\hat{u} + 20M_W^2 s_W^2\hat{t}\hat{u} \\
&+16M_Z^2 s_W^2\hat{t}\hat{u} - 12\hat{s}s_W^2\hat{t}\hat{u} + 12\hat{s}\hat{t}\hat{u} + M_4^4\left(\left(10s_W^2 - 9\right)M_W^2 + M_Z^2\left(8s_W^2 - 9\right)\right. \\
&\left.-6\left(s_W^2 - 1\right)\left(\hat{s} - 3(\hat{t} + \hat{u})\right)\right) + 2M_4^2\left(\left(8s_W^2 - 6\right)M_W^4 + \left(-12\left(s_W^2 - 1\right)M_Z^2 + \hat{s}\left(4s_W^2 - 3\right)\right.\right. \\
&\left.-\left(10s_W^2 - 9\right)(\hat{t} + \hat{u})\right)M_W^2 + M_Z^4\left(4s_W^2 - 6\right) + 3\left(s_W^2 - 1\right)(\hat{t} + \hat{u})(2\hat{s} - 3(\hat{t} + \hat{u})) \\
&\left.\left.\left.+M_Z^2\left(\hat{s}\left(2s_W^2 - 3\right) - \left(8s_W^2 - 9\right)(\hat{t} + \hat{u})\right)\right)\right)V_{ud}^*\right) / \left(3\sqrt{2}\left(M_W^2 - \hat{s}\right)s_W^4\left(M_3^2 + M_4^2 - \hat{t} - \hat{u}\right)^3\right)
\end{aligned}
$$

$$
\begin{aligned}
B_{5,3} = &\left(\alpha^2 M_W V_{ud}^*\left(M_3^4\left(-\left(36c_W^2 + 8s_W^4 - 18s_W^2 + 9\right)\right) + M_3^2\left(-2M_4^2\left(36c_W^2 + 8s_W^4 - 18s_W^2 + 9\right)\right.\right.\right. \\
&+6c_W^2\left(M_W^2\left(4s_W^2 + 3\right) + M_Z^2\left(3 - 4s_W^2\right) - 6(\hat{s} - 2(\hat{t} + \hat{u}))\right) \\
&\left.-\left(8s_W^4 - 18s_W^2 + 9\right)\left(M_Z^2 + \hat{s} - 2(\hat{t} + \hat{u})\right)\right) - M_4^4\left(36c_W^2 + 8s_W^4 - 18s_W^2 + 9\right) \\
&+M_4^2\left(6c_W^2\left(M_W^2\left(4s_W^2 + 3\right) + M_Z^2\left(3 - 4s_W^2\right) - 6(\hat{s} - 2(\hat{t} + \hat{u}))\right)\right. \\
&\left.-\left(8s_W^4 - 18s_W^2 + 9\right)\left(M_Z^2 + \hat{s} - 2(\hat{t} + \hat{u})\right)\right) + 24c_W^2 M_W^2\hat{s}s_W^2 + 36c_W^2 M_W^2\hat{s} \\
&-24c_W^2 M_W^2 s_W^2\hat{t} - 24c_W^2 M_W^2 s_W^2\hat{u} - 18c_W^2 M_W^2\hat{t} - 18c_W^2 M_W^2\hat{u} - 48c_W^2 M_Z^2\hat{s}s_W^2 \\
&+36c_W^2 M_Z^2\hat{s} + 24c_W^2 M_Z^2 s_W^2\hat{t} + 24c_W^2 M_Z^2 s_W^2\hat{u} - 18c_W^2 M_Z^2\hat{t} - 18c_W^2 M_Z^2\hat{u} + 36c_W^2\hat{s}\hat{t} \\
&+36c_W^2\hat{s}\hat{u} - 36c_W^2\hat{t}^2 - 72c_W^2\hat{t}\hat{u} - 36c_W^2\hat{u}^2 - 16M_Z^2\hat{s}s_W^4 + 36M_Z^2\hat{s}s_W^2 - 18M_Z^2\hat{s} \\
&+8M_Z^2 s_W^4\hat{t} + 8M_Z^2 s_W^4\hat{u} - 18M_Z^2 s_W^2\hat{t} - 18M_Z^2 s_W^2\hat{u} + 9M_Z^2\hat{t} + 9M_Z^2\hat{u} + 8\hat{s}s_W^4\hat{t} \\
&+8\hat{s}s_W^4\hat{u} - 18\hat{s}s_W^2\hat{t} - 18\hat{s}s_W^2\hat{u} + 9\hat{s}\hat{t} + 9\hat{s}\hat{u} - 8s_W^4\hat{t}^2 - 16s_W^4\hat{t}\hat{u} - 8s_W^4\hat{u}^2 \\
&\left.\left.+18s_W^2\hat{t}^2 + 36s_W^2\hat{t}\hat{u} + 18s_W^2\hat{u}^2 - 9\hat{t}^2 - 18\hat{t}\hat{u} - 9\hat{u}^2\right)\right) \\
&\quad / \left(18\sqrt{2}c_W^2 s_W^4\left(M_W^2 - \hat{s}\right)\left(M_3^2 + M_4^2 - \hat{t} - \hat{u}\right)^2\left(M_3^2 + M_4^2 + \hat{s} - \hat{t} - \hat{u}\right)\right)
\end{aligned}
$$

$$B_{6,1} = \frac{\alpha^2 M_W M_Z^2 \left(8s_W^4 - 18s_W^2 + 9\right) V_{ud}^* \left(-M_3^2 - M_4^2 + M_Z^2 + \hat{t} + \hat{u}\right)}{9\sqrt{2} c_W^2 s_W^4 \left(M_W^2 - \hat{s}\right) \left(M_3^2 + M_4^2 - \hat{t} - \hat{u}\right)^2}$$

$$\begin{aligned} B_{6,2} = {} & \big(\alpha^2 M_W V_{ud}^* \left(M_3^2 \left(M_W^2 \left(3 - 4s_W^2\right) + M_Z^2 \left(3 - 2s_W^2\right)\right) + M_4^2 \left(M_W^2 \left(3 - 4s_W^2\right) + M_Z^2 \left(3 - 2s_W^2\right)\right)\right. \\ & -8M_W^4 s_W^2 + 6M_W^4 + 4M_W^2 s_W^2 \hat{t} + 4M_W^2 s_W^2 \hat{u} - 3M_W^2 \hat{t} - 3M_W^2 \hat{u} - 4M_Z^4 s_W^2 + 6M_Z^4 \\ & \left.+2M_Z^2 s_W^2 \hat{t} + 2M_Z^2 s_W^2 \hat{u} - 3M_Z^2 \hat{t} - 3M_Z^2 \hat{u}\right)\big) / \left(3\sqrt{2} s_W^4 \left(M_W^2 - \hat{s}\right) \left(M_3^2 + M_4^2 - \hat{t} - \hat{u}\right)^2\right) \end{aligned}$$

$$\begin{aligned} B_{6,3} = {} & \big(\alpha^2 M_W V_{ud}^* \left(M_Z^2 \hat{s} \left(8s_W^4 - 18s_W^2 + 9\right) - 6c_W^2 \left(M_W^2 \left(2s_W^2 \left(M_3^2 + M_4^2 - \hat{t} - \hat{u}\right) + \hat{s}\left(2s_W^2 + 3\right)\right)\right.\right. \\ & \left.\left.+M_Z^2 \hat{s} \left(3 - 4s_W^2\right)\right)\right)\big) / \left(18\sqrt{2} c_W^2 \hat{s} s_W^4 \left(M_W^2 - \hat{s}\right) \left(M_3^2 + M_4^2 - \hat{t} - \hat{u}\right) \left(M_3^2 + M_4^2 + \hat{s} - \hat{t} - \hat{u}\right)\right) \end{aligned}$$

4.1.3. Topology III

In the third topology only three non-vanishing diagrams appear. The first one will be ignored because only the A_0 and B_0 functions occur in its loops. The other two give constant factors. We will add these terms because of the Sudakov terms appearing inside the mixed terms within the matrix element squared.

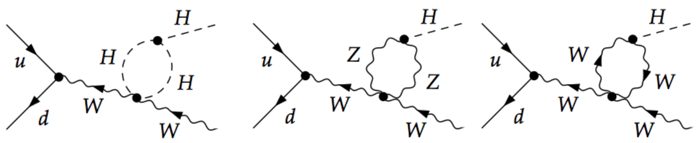


Figure 4.4.: Non-vanishing triangle diagram of the s-channel third topology

$$\text{2nd diagram} \Rightarrow \frac{2\alpha^2 M_W V_{ud}^*}{\sqrt{2} s_W^4 \left(-\hat{s} + M_W^2\right)} M_1$$
$$\text{3rd diagram} \Rightarrow \frac{2\alpha^2 M_W V_{ud}^*}{\sqrt{2} s_W^4 \left(-\hat{s} + M_W^2\right)} M_1$$

Then the matrix element for the third topology looks like

$$M_{III}^{\hat{s}} = \frac{4\alpha^2 M_W V_{ud}^*}{\sqrt{2} s_W^4 \left(-\hat{s} + M_W^2\right)} M_1 \tag{4.8}$$

4.1.4. Topologies IV and V

For completeness we present the fourth and fifth topology of the $\hat{s}$-channel. They vanish for the same reason as in the third topology.

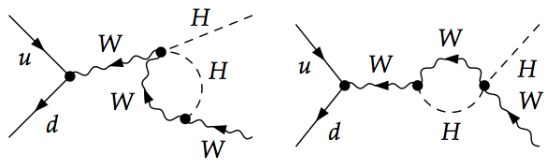


Figure 4.5.: Triangle diagram of the $\hat{s}$-channel fourth and fifth topology

4.2. $\hat{t}$-channel

4.2.1. Topology I

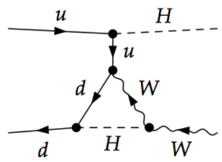


Figure 4.6.: Triangle diagram of the t-channel first topology

This diagram is just an example for the first topology of the t-channel because the vertex qqH 4.1 always appears inside these diagrams. We can immediately move to the second topology.

4.2.2. Topology II

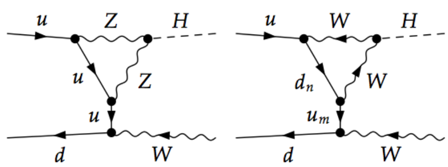


Figure 4.7.: Non-vanishing triangle diagram of the t-channel second topology

These two diagrams are the ones left after setting our masses to zero. Those two give rise to a new C_0 type,

$$\begin{aligned}C_0(p_2^2, p_3^2, \hat{t}, 0, m^2, m^2) &= C_0(p_3^2, \hat{t}, p_2^2, m^2, m^2, 0)\\ &= C_0(p_3^2, \hat{t}, 0, m^2, m^2, 0)\\ &= \frac{1}{2\hat{t}} \ln^2\left(\frac{-\hat{t} - i\epsilon}{m^2}\right) + \frac{1}{\hat{t}} \mathrm{Li}_2\left[\frac{2p_3^2}{p_3^2 \pm \kappa(p_3^2, m^2, m^2)}\right]\\ &\overset{\text{just Sud log}}{=} \Lambda_{\hat{t}}(M^2).\end{aligned} \tag{4.9}$$

Now the total matrix element is

$$\begin{aligned} M_{II}^{\hat{t}} = &\left[C_{1,1}\Lambda_{\hat{t}}(M_W^2) + C_{1,2}\Lambda_{\hat{t}}(M_Z^2) + C_{1,3}\right] M_1 \\ &+ \left[C_{2,1}\Lambda_{\hat{t}}(M_W^2) + C_{2,2}\Lambda_{\hat{t}}(M_Z^2) + C_{2,3}\right] M_2 \\ &+ \left[C_{3,1}\Lambda_{\hat{t}}(M_W^2) + C_{3,2}\Lambda_{\hat{t}}(M_Z^2) + C_{3,3}\right] M_3 \\ &+ \left[C_{4,1}\Lambda_{\hat{t}}(M_W^2) + C_{4,2}\Lambda_{\hat{t}}(M_Z^2) + C_{4,3}\right] M_4 \end{aligned} \tag{4.10}$$

with the following formfactors:

$$\begin{aligned} C_{1,1} = -&\left(\alpha^2 V_{\mathrm{CKM}} M_W \left(M_3^4\left(2\hat{t} - 4M_4^2\right) + M_3^2\left(8M_4^4 + M_4^2\left(-3M_W^2 - 8\hat{t} + 4\hat{u}\right)\right.\right.\right.\\ &\left.\left.\left. + M_W^2(-\hat{s} + 2\hat{t} + \hat{u}) + 2\hat{t}(\hat{t} - \hat{u})\right) + \left(M_4^2 - \hat{s} + \hat{u}\right)\left(M_4^2\left(M_W^2 - 2\hat{u}\right) - M_W^2\hat{s} + \hat{t}\hat{u}\right)\right)\right) \\ &/\left(\sqrt{2}s_W^4\left(2M_4^2 - \hat{t}\right)\left(M_3^2\left(4\hat{t} - 8M_4^2\right) + \left(M_4^2 - \hat{s} + \hat{u}\right)^2\right)\right) \end{aligned}$$

$$\begin{aligned} C_{1,2} = -&\left(\alpha^2 M_W\left(3 - 4s_W^2\right)^2 V_{ud}^*\left(M_3^4\left(2\hat{t} - 4M_4^2\right) + M_3^2\left(8M_4^4 + M_4^2\left(-3M_Z^2 - 8\hat{t} + 4\hat{u}\right)\right.\right.\right.\\ &\left.\left.\left. + M_Z^2(-\hat{s} + 2\hat{t} + \hat{u}) + 2\hat{t}(\hat{t} - \hat{u})\right) + \left(M_4^2 - \hat{s} + \hat{u}\right)\left(M_4^2\left(M_Z^2 - 2\hat{u}\right) - M_Z^2\hat{s} + \hat{t}\hat{u}\right)\right)\right) \\ &/\left(18\sqrt{2}c_W^4 s_W^4\left(2M_4^2 - \hat{t}\right)\left(M_3^2\left(4\hat{t} - 8M_4^2\right) + \left(M_4^2 - \hat{s} + \hat{u}\right)^2\right)\right) \end{aligned}$$

$$C_{1,3} = 0$$

$$C_{2,1} = \frac{\sqrt{2}\alpha^2 V_{\mathrm{CKM}} M_W\left(M_3^2\left(4M_4^2 - 2\hat{t}\right) - \left(2M_4^2 + M_W^2 - \hat{t}\right)\left(M_4^2 - \hat{s} + \hat{u}\right)\right)}{s_W^4\left(2M_4^2 - \hat{t}\right)\left(M_3^2\left(4\hat{t} - 8M_4^2\right) + \left(M_4^2 - \hat{s} + \hat{u}\right)^2\right)}$$

$$C_{2,2} = \frac{\alpha^2 M_W\left(3 - 4s_W^2\right)^2 V_{ud}^*\left(M_3^2\left(4M_4^2 - 2\hat{t}\right) - \left(2M_4^2 + M_Z^2 - \hat{t}\right)\left(M_4^2 - \hat{s} + \hat{u}\right)\right)}{9\sqrt{2}c_W^4 s_W^4\left(2M_4^2 - \hat{t}\right)\left(M_3^2\left(4\hat{t} - 8M_4^2\right) + \left(M_4^2 - \hat{s} + \hat{u}\right)^2\right)}$$

$$C_{2,3} = 0$$

$$C_{3,1} = \frac{\alpha^2 V_{\mathrm{CKM}} M_W\left(M_3^2\left(4M_4^2 - 2\hat{t}\right) - \left(2M_4^2 + M_W^2 - \hat{t}\right)\left(M_4^2 - \hat{s} + \hat{u}\right)\right)}{\sqrt{2}s_W^4\left(2M_4^2 - \hat{t}\right)\left(M_3^2\left(4\hat{t} - 8M_4^2\right) + \left(M_4^2 - \hat{s} + \hat{u}\right)^2\right)}$$

$$C_{3,2} = \frac{\alpha^2 M_W\left(3 - 4s_W^2\right)^2 V_{ud}^*\left(M_3^2\left(4M_4^2 - 2\hat{t}\right) - \left(2M_4^2 + M_Z^2 - \hat{t}\right)\left(M_4^2 - \hat{s} + \hat{u}\right)\right)}{18\sqrt{2}c_W^4 s_W^4\left(2M_4^2 - \hat{t}\right)\left(M_3^2\left(4\hat{t} - 8M_4^2\right) + \left(M_4^2 - \hat{s} + \hat{u}\right)^2\right)}$$

$$C_{3,3} = 0$$

$$C_{4,1} = \frac{\sqrt{2}\alpha^2 V_{\mathrm{CKM}} M_W \left(M_3^2\left(2\hat{t}-4M_4^2\right)+\left(2M_4^2+M_W^2-\hat{t}\right)\left(M_4^2-\hat{s}+\hat{u}\right)\right)}{s_W^4\left(2M_4^2-\hat{t}\right)\left(M_3^2\left(4\hat{t}-8M_4^2\right)+\left(M_4^2-\hat{s}+\hat{u}\right)^2\right)}$$

$$C_{4,2} = \frac{\alpha^2 M_W\left(3-4s_W^2\right)^2 V_{ud}^*\left(M_3^2\left(2\hat{t}-4M_4^2\right)+\left(2M_4^2+M_Z^2-\hat{t}\right)\left(M_4^2-\hat{s}+\hat{u}\right)\right)}{9\sqrt{2}c_W^4 s_W^4\left(2M_4^2-\hat{t}\right)\left(M_3^2\left(4\hat{t}-8M_4^2\right)+\left(M_4^2-\hat{s}+\hat{u}\right)^2\right)}$$

$$C_{4,3} = 0$$

with

$$CKMSum = V_{u,d}^*(V_{d,u}V_{u,d}^* + V_{d,c}V_{c,d}^*) + V_{u,s}^*(V_{s,u}V_{u,d}^* + V_{s,c}V_{c,d}^*).$$

4.3. $\hat{u}$-channel

4.3.1. Topology I

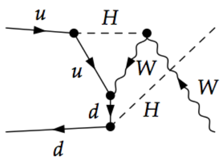


Figure 4.8.: Triangle diagram of the u-channel first topology

Again this kind of diagram vanishes because of the qqH vertex 4.1. We can once more move directly to the second topology.

4.3.2. Topology II

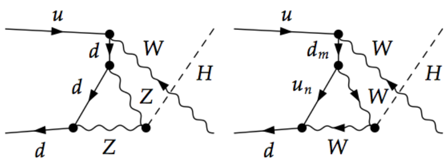


Figure 4.9.: Non-vanishing triangle diagram of the $\hat{u}$-channel second topology

The two non-vanishing diagrams of the $\hat{u}$ channel give rise to a Sudakov logarithm with the Mandelstam variable $\hat{u}$ inside,

$$\begin{aligned} C_0(p_2^2, p_3^2, \hat{u}, 0, m^2, m^2) &= C_0(p_3^2, \hat{u}, p_2^2, m^2, m^2, 0) \\ &= C_0(p_3^2, \hat{u}, 0, m^2, m^2, 0) \\ &= \frac{1}{2\hat{u}} \ln^2\left(\frac{-\hat{u} - i\epsilon}{m^2}\right) + \frac{1}{\hat{u}} \mathrm{Li}_2\left[\frac{2p_3^2}{p_3^2 \pm \kappa(p_3^2, m^2, m^2)}\right] \\ &\overset{\text{just Sud log}}{=} \Lambda_{\hat{u}}(M^2). \end{aligned} \tag{4.11}$$

The matrix element looks like:

$$M_{II}^{\hat{u}} = \left[D_{1,1}\Lambda_{\hat{u}}(M_W^2) + D_{1,2}\Lambda_{\hat{u}}(M_Z^2)\right] M_1 \tag{4.12}$$

with

$$D_{1,1} = -\frac{\alpha^2 V_{\mathrm{CKM}} M_W^3}{\sqrt{2} s_W^4 \left(M_3^2 - \hat{u}\right)} \tag{4.13}$$

$$D_{1,2} = -\frac{\alpha^2 M_W M_Z^2 \left(3 - 2s_W^2\right)^2 V_{ud}^*}{18\sqrt{2} c_W^4 s_W^4 \left(M_3^2 - \hat{u}\right)} \tag{4.14}$$

again with

$$V_{\mathrm{CKM}} = V_{u,d}^*(V_{d,u}V_{u,d}^* + V_{d,c}V_{c,d}^*) + V_{u,s}^*(V_{s,u}V_{u,d}^* + V_{s,c}V_{c,d}^*).$$

4.4. Modified Born Process

Finally we can introduce a modified Born process which only includes the triangle diagrams discussed in this chapter.

$$M_{\mathrm{ModBorn}} = M_0 + M_I^{\hat{s}} + M_{II}^{\hat{s}} + M_{III}^{\hat{s}} + M_{II}^{\hat{t}} + M_{II}^{\hat{u}} \tag{4.15}$$

Later on we will present the result of this matrix element squared.

4.5. Overview of the C_0 Functions

In this section we will give a schedular summary of all different types of the C_0 integral used. Everything in the high energy limit and with constant factors but we will also ppoint out the Sudakov structure.

Diagram	C_0	p_1^2	p_{21}^2	p_2^2	m_0^2	m_1^2	m_2^2	Sudakov Log Structure	constant term
s-channel 1		p_3^2	p_4^2	$\hat{s}$	M_1^2	M_2^2	M_3^2	$\Lambda_{\hat{s}}(M^2)$	$\frac{1}{\hat{s}}\left[L(p_3^2, M_1^2, M_2^2) + L(p_4, M_3^2, M_2^2)\right]$
s-channel 2		0	$\hat{s}$	0	M^2	0	0	$\Lambda_{\hat{s}}(M^2)$	0
s-channel 3								**no Sudakov terms**	
s-channel 4		0	$\hat{s}$	0	0	M^2	0		**no Sudakov terms**
s-channel 5		0	$\hat{s}$	0	0	0	M^2		**no Sudakov terms**
s-channel 6		0	$\hat{s}$	0	0	M_1^2	M_2^2	$\Lambda_{\hat{s}}(M_1^2, M_2^2)$	0
t-channel 1								**no Sudakov terms**	
t-channel 2		p_3^2	$\hat{t}$	0	M^2	M^2	0	$\Lambda_{\hat{t}}(M^2)$	$\frac{1}{\hat{t}}L(p_3^2, M^2, M^2)$
u-channel 1								**no Sudakov terms**	
u-channel 2		0	p_3^2	$\hat{u}$	0	M^2	M^2	$\Lambda_{\hat{u}}(M^2)$	$\frac{1}{\hat{u}}L(p_3^2, M^2, M^2)$

5. NLO Box Corrections and Their Sudakov Formfactors

As before we will go through the different topologies and introduce all non-vanishing diagrams and sort them by their topologies. In our calculation we will use the high energy limit of the D_0 scalar integral shown in section 3.3. We will list all different types of C_0 and D_0 and give the chosen result, but not the analytic expression of the formfactors. In total there are around 144 formfactors and 18 constant terms which are each at least half a page long. Since this would definitely go beyond the scope of this thesis, we just give the general structure of the matrix elements.

5.1. Topology I

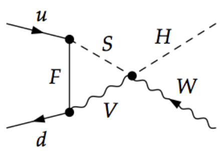


Figure 5.1.: Diagram of the first topology type

As before we can neglect this kind of topology because we set the masses of the quarks to zero. We move directly to the second topology.

5.2. Topology II

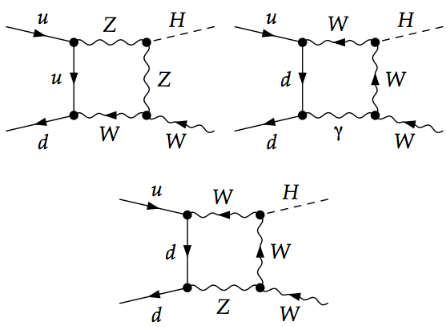


Figure 5.2.: Non-vanishing box diagrams of the second topology

In the second topology we encounter the following C_0 integrals:

$$
\begin{aligned}
C_0(p^2,\hat{s},p'^2,0,m_1^2,m_2^2) &= \Lambda_{\hat{s}}(m_1^2,m_2^2),\\
C_0(p^2,\hat{t},p'^2,m_0^2,m_1^2,0) &= \Lambda_{\hat{t}}(m_0^2),\\
C_0(p^2,p'^2,\hat{s},m_0^2,m_1^2,m_2^2) &= \Lambda_{\hat{s}}(m_1^2),\\
C_0(p^2,p'^2,\hat{t},0,m_1^2,m_2^2) &= \Lambda_{\hat{t}}(m_1^2),
\end{aligned}
$$

where p and p' are a combination of our momenta p_1, p_2, p_3 and p_4. The same holds true for the masses m_i.

The D_0 integral are

$$
\begin{aligned}
&D_0(p_2^2,p_4^2,p_3^2,p_1^2,\hat{t},\hat{s},0,m_1^2,m_2^2,m_3^2) =\\
&\qquad -\frac{1}{\hat{s}\hat{t}}\left[\mathrm{R}(\hat{t},\hat{s})+\pi^2-\frac{1}{2}\left[\Lambda_{\hat{t}}(m_1^2)+\Lambda_{\hat{s}}(m_2^2)+\Lambda_{\hat{t}}(m_3)+\Lambda_{\hat{s}}(m_3^2,m_1^2)\right]\right], \qquad (5.1)\\
&D_0(\hat{t},p_3^2,\hat{s},p_2^2,p_1^2,p_4^2,0,m_1^2,m_2^2,m_3^2) = D_x^{\hat{s},\hat{t}}(0,m_1^2,m_2^2,m_3^2). \qquad (5.2)
\end{aligned}
$$

With these functions we can solve the scalar integrals from the first and the third diagram of the second topology. In the second diagram however, we encounter the fact that there are two zero masses. We therefore have to solve the integrals inside them seperately:

$$
\begin{aligned}
C_0(p^2,p'^2,t,0,0,m) &= div.\\
C_0(p^2,\hat{s},p'^2,0,0,m) &= div.
\end{aligned}
$$

As for those kind of C_0 cases before (see 4.5), these functions are divergent and won't give Sudakov logarithms so we will neglect them again.
The D_0 function appearing in the second diagram then becomes

$$
\begin{aligned}
&D_0(p_2^2,p_4^2,p_3^2,p_1^2,\hat{t},\hat{s},0,0,m_2^2,m_3^2) =\\
&\qquad -\frac{1}{\hat{s}\hat{t}}\left[\mathrm{R}(\hat{t},\hat{s})+\pi^2-\frac{1}{2}\left[\Lambda_{\hat{s}}(m_2^2)+\Lambda_{\hat{t}}(m_3)\right]\right]. \qquad (5.3)
\end{aligned}
$$

The results are found again by using the equations of sections 3.2 and 3.3. Note that there are some D_0 integrals we haven't calculated yet,

$$
D_0(\hat{t},p_3^2,\hat{s},p_2^2,p_1^2,p_4^2,0,m_1^2,m_2^2,m_3^2) = D_x^{\hat{s},\hat{t}}(0,m_1^2,m_2^2,m_3^2). \qquad (5.4)
$$

With these arguments of the D_0 integral we can't use the high energy limit

$$\hat{s}, \hat{t}, \hat{u} \gg p_{10}, p_{12}, p_{23}, p_{30}.$$

To solve this integral we have to start from scratch and calculate the D_0 integral with that combination of momenta and then of course perform the high energy limit. We will leave that calculation for later discussion outside of this thesis and move on with the general structure of matrix element appearing in the box diagrams,

$$M_{II}^{\text{Box}} = \sum_{i=1}^{6} \left[k_{i,1}\Lambda_{\hat{s}}(M_W^2, M_Z^2) + k_{i,2}\Lambda_{\hat{s}}(M_W^2) + k_{i,3}\Lambda_{\hat{s}}(M_Z^2) \right.$$
$$\left. + k_{i,4}\Lambda_{\hat{t}}(M_W^2) + k_{i,5}\Lambda_{\hat{t}}(M_Z^2) + k_{i,6}\mathrm{R}[\hat{t}, \hat{s}] + k_{i,7}D_x^{\hat{s},\hat{t}} + c_i \right]. \tag{5.5}$$

5.3. Topology III

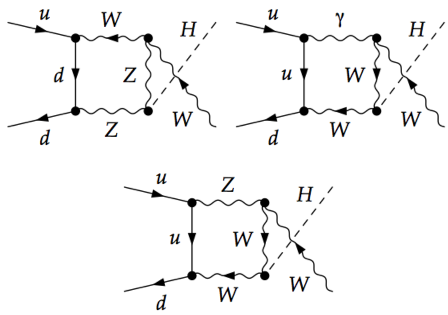


Figure 5.3.: Non-vanishing box diagrams of the third topology

For the third topology we have for the first and third diagram the following C_0 functions:

$$\begin{aligned}
C_0(p^2, \hat{s}, p'^2, 0, m_1^2, m_2^2) &= \Lambda_{\hat{s}}(m_1^2, m_2^2) \\
C_0(\hat{u}, p^2, p'^2, 0, m_1^2, m_2^2) &= \Lambda_{\hat{u}}(m_2^2) \\
C_0(p^2, p'^2, \hat{s}, m_0^2, m_1^2, m_2^2) &= \Lambda_{\hat{s}}(m_1^2) \\
C_0(p^2, p'^2, \hat{u}, 0, m_1^2, m_2^2) &= \Lambda_{\hat{u}}(m_1^2)
\end{aligned}$$

The D_0 integrals are

$$D_0(p_2^2, p_4^2, p_3^2, p_1^2, \hat{u}, \hat{s}, 0, m_1^2, m_2^2, m_3^2) =$$
$$-\frac{1}{\hat{s}\hat{u}} \left[\mathrm{R}(\hat{u}, \hat{s}) + \pi^2 - \frac{1}{2}\left[\Lambda_{\hat{u}}(m_1^2) + \Lambda_{\hat{s}}(m_2^2) + \Lambda_{\hat{u}}(m_3) + \Lambda_{\hat{s}}(m_3^2, m_1^2) \right] \right], \tag{5.6}$$
$$D_0(\hat{u}, p_3^2, \hat{s}, p_2^2, p_1^2, p_4^2, 0, m_1^2, m_2^2, m_3^2) = D_x^{\hat{s},\hat{u}}(0, m_1^2, m_2^2, m_3^2). \tag{5.7}$$

The second D_0 integral is of the same type as equation 5.4 in the second topology. Once again we will leave that for later discussions.
As before, the second diagram has two zero masses. Hence, we encounter divergent functions once again and we will neglect them.

$$C_0(\hat{u}, p^2, p'^2, 0, m, 0) = div.$$
$$C_0(p^2, \hat{s}, p'^2, 0, m, 0) = div.$$

The result of the D_0 integral also changes to

$$D_0(p_2^2, p_4^2, p_3^2, p_1^2, \hat{u}, \hat{s}, 0, m_1^2, m_2^2, 0) = \\ -\frac{1}{\hat{s}\hat{u}}\left[\mathrm{R}(\hat{u}, \hat{s}) + \pi^2 - \frac{1}{2}\left[\Lambda_{\hat{u}}(m_1^2) + \Lambda_{\hat{s}}(m_2^2)\right]\right]. \tag{5.8}$$

Finally we can write our matrix element as

$$M_{III}^{\mathrm{Box}} = \sum_{i=1}^{6}\left[o_{i,1}\Lambda_{\hat{s}}(M_W^2, M_Z^2) + o_{i,2}\Lambda_{\hat{s}}(M_W^2) + o_{i,3}\Lambda_{\hat{s}}(M_Z^2) \right. \\ \left. + o_{i,4}\Lambda_{\hat{u}}(M_W^2) + o_{i,5}\Lambda_{\hat{u}}(M_Z^2) + o_{i,6}\mathrm{R}(\hat{u}, \hat{s}) + o_{i,7}D_x^{\hat{s},\hat{u}} + \tilde{c}_i\right]. \tag{5.9}$$

5.4. Topology IV

The fourth topology has just one non-vanishing diagram.

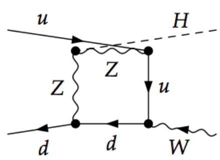


Figure 5.4.: Non-vanishing box diagram of the fourth topology

Again we encounter divergent C_0 functions.

$$C_0(\hat{u}, p^2, p'^2, 0, 0, m) = div.$$
$$C_0(p^2, p'^2, \hat{t}, m, 0, 0) = div.$$

We also find C_0 and D_0 functions of the type

$$C_0(p^2, \hat{t}, p'^2, m_0^2, m_1^2, 0) = \Lambda_{\hat{t}}(m_0^2) \tag{5.10}$$
$$C_0(p^2, p'^2, \hat{u}, 0, m_1^2, m_2^2) = \Lambda_{\hat{u}}(m_1^2) \tag{5.11}$$

$$D_0(p_2^2, p_3^2, p_1^2, p_4^2, \hat{u}, \hat{t}, 0, m_1^2, m_2^2, 0) = \\ -\frac{1}{\hat{t}\hat{u}}\left[\mathrm{R}(\hat{u}, \hat{t}) + \pi^2 - \frac{1}{2}\left[\Lambda_{\hat{u}}(m_1^2) + \Lambda_{\hat{t}}(m_2^2)\right]\right] \tag{5.12}$$
$$D_0(\hat{u}, p_2^2, \hat{t}, p_1^2, p_3^2, p_4^2, m_0^2, 0, m_2^2, 0) = D_x^{\hat{t},\hat{u}}(m_0^2, 0, m_2^2, 0) \tag{5.13}$$

The matrix element then looks like

$$M_{IV}^{\mathrm{Box}} = \sum_{i=1}^{6}\left[r_{i,1}\Lambda_{\hat{u}}(M_W^2) + r_{i,2}\Lambda_{\hat{u}}(M_Z^2)\right. \\ \left. + r_{i,3}\Lambda_{\hat{t}}(M_W^2) + r_{i,4}\Lambda_{\hat{t}}(M_Z^2) + r_{i,5}\mathrm{R}(\hat{t}, \hat{s}) + r_{i,6}D_x^{\hat{t},\hat{u}} + \check{c}_i\right]. \tag{5.14}$$

5.5. Overview over the D_0 functions

In this section we give a short recap over the used D_0 integrals.

$$D_0(p_2^2, p_4^2, p_3^2, p_1^2, \hat{t}, \hat{s}, 0, m_1^2, m_2^2, m_3^2) = \\ -\frac{1}{\hat{s}\hat{t}}\left[\mathrm{R}(\hat{t}, \hat{s}) + \pi^2 - \frac{1}{2}\left[\Lambda_{\hat{t}}(m_1^2) + \Lambda_{\hat{s}}(m_2^2) + \Lambda_{\hat{t}}(m_3) + \Lambda_{\hat{s}}(m_3^2, m_1^2)\right]\right], \tag{5.15}$$
$$D_0(p_2^2, p_4^2, p_3^2, p_1^2, \hat{u}, \hat{s}, 0, m_1^2, m_2^2, m_3^2) = \\ -\frac{1}{\hat{s}\hat{u}}\left[\mathrm{R}(\hat{u}, \hat{s}) + \pi^2 - \frac{1}{2}\left[\Lambda_{\hat{u}}(m_1^2) + \Lambda_{\hat{s}}(m_2^2) + \Lambda_{\hat{u}}(m_3) + \Lambda_{\hat{s}}(m_3^2, m_1^2)\right]\right], \tag{5.16}$$
$$D_0(p_2^2, p_3^2, p_1^2, p_4^2, \hat{u}, \hat{t}, 0, m_1^2, m_2^2, 0) = \\ -\frac{1}{\hat{t}\hat{u}}\left[\mathrm{R}(\hat{u}, \hat{t}) + \pi^2 - \frac{1}{2}\left[\Lambda_{\hat{u}}(m_1^2) + \Lambda_{\hat{t}}(m_2^2)\right]\right] \tag{5.17}$$

All D_0 integral of the type

$$D_0(\hat{r}_1, \tilde{p}_1^2, \hat{r}_2, \tilde{p}_2^2, \tilde{p}_3^2, \tilde{p}_4^2, m_0^2, m_1^2, m_2^2, m_3^2) \tag{5.18}$$

with $r_1, r_2 = \hat{s}, \hat{t}, \hat{u}$ and $\tilde{p}_i \ll r_1, r_2$, still have to be calculated.

6. Results

In this thesis we have explored the origin of the Sudakov logarithms appearing in the high-energy limit of the loop corrections of associated WH production at the LHC. To this end, we studied in detail the scalar integrals arising from the Passorino Veltman reduction. It became clear that only certain subsets of the three-point function, C_0, and the four-point function give rise to Sudakov logarithms. With that in mind we continued with the discussion of the triangle and box corrections.

We introduced a modified Born matrix element 4.15 where we only included the Sudakov terms of the triangle diagrams. The modified born process essentially consists of the sum of all matrix elements determined in Chapter 4 overall squared. When we calculated the total cross sections with the procedure introduced in Chapter 2, we get a result of $\sigma_{\text{Mod}} = 1.293 \pm 0.242$ pb whereas for the LO we get the result $\sigma_o = 1.195 \pm 0.223$ pb. This gives a k-factor of $k = 1.082$.

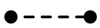
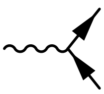
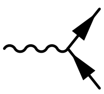
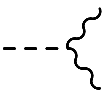
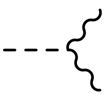
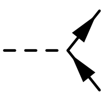
This indicates that the Sudakov corrections have an overall positive effect at the level of the total cross-section. On the other hand, the full NLO electroweak corrections have an overall negative effect on the total cross-section [3] on the order of 1-13% depending on the choice of the scheme for α, $\sigma_0|_{\alpha(0)} = 0.8114(2)$ pb, $\sigma_0|_{\alpha(M_Z^2)} = 0.9166(2)$ pb and $\sigma_0|_{G_\mu} = 0.8673(2)$ pb.

In the end we also gave a rough overview of the structures of the boxes. In order to advance further this point has to be finished. Overall, we have completed most of the first steps involved with a resummation of the electroweak Sudakov logarithms arising in this process. An interesting next step would be to finish extracting the Sudakov logarithms from the box diagrams and explore the possiblity of resumming them.

This will be very relevant to future measurements of the Higgs boson at the LHC, mainly the measurements of the WWH vertex. Also, it should be noted, that this entire procedure can be performed also for associated ZH production, following the same plan outlined in this thesis.

A. Feynman rules

Here is a table of all Feynman rules used in the Treelevel Calculation.

	incoming fermion	$u(\vec{p})$
	outgoing fermion	$\bar{u}(\vec{p})$
	incoming antifermion	$\bar{v}(\vec{p})$
	outgoing antifermion	$u(\vec{p})$
	incoming, outgoing vector boson	$\epsilon^*(p)$, $\epsilon(p)$
	fermion propagator	$ie\gamma_\mu \frac{V^*_{ud}}{\sqrt{2}s_W}\delta_{r-}$
	Higgs boson propagator	$ie\gamma_\mu \left(-\frac{s_W}{c_W}Q_q + \frac{I^3_q}{c_W s_W}\delta_{r-}\right)$
	W boson propagator	$ieg_{\mu\nu}\frac{M_W}{s_W}$
	Wff vertex	$ie\gamma_\mu \frac{V^*_{f_1,f_2}}{\sqrt{2}s_W}\delta_{r-}$
	Zff vertex	$ie\gamma_\mu \left(-\frac{s_W}{c_W}Q_q + \frac{I^3_q}{c_W s_W}\delta_{r-}\right)$
	HWW vertex	$ieg_{\mu\nu}\frac{M_W}{s_W}$
	HZZ vertex	$ieg_{\mu\nu}\frac{M_W}{s_W c_W}$
	Hff vertex	$-\frac{ie}{2s_W}\frac{m_f}{M_W}$

B. Calculations

B.1. Next-to-leading order process (2 to 3)

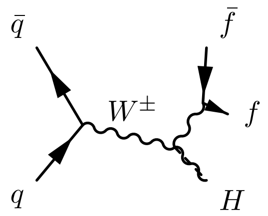


Figure B.1.: 2 to 3 process

Now we consider the Next-to-leading order

$$q_1(p_1) + \overline{q}_2(p2) \to V + H \to l_1(p_3) + \overline{l}_2(p_4) + H(p_H)$$

The matrix element for the 2 to 3 process is:

$$\mathcal{M}^{\tau\tau'} = b\overline{v}_2\gamma^{\nu}\omega_{\tau}u_1\overline{u}_3\gamma_{\nu}\omega_{\tau'}v_4 \tag{B.1}$$

with $u_j = u(p_j)$ (same vor v) and $b = e^3 g^{\tau}_{q_1q_2V} g_{VVH} g^{\tau'}_{l_1l_2V} D(s_{12})D(s_{34})$, where $D(s) = \frac{1}{s-M^2}$. The coupling factors $g^{\tau}_{q_1q_2V}$ and g_{VVH} are the same as in equation (2.33) to (**??**). The definition for the coupling factor for the l_1l_1V vertex ($g^{\tau'}_{l_1l_2V}$) is the same as for $g^{\tau}_{q_1q_2V}$ just with adjusted T_3^f and Q_f.

$$\begin{aligned}
(\mathcal{M}^{dd'})^{\dagger} &= b^{\dagger}v_4^{\dagger}\omega_{d'}^{\dagger}\gamma^{\mu\dagger}\gamma^{0\dagger}u_3u_1^{\dagger}\omega_d^{\dagger}\gamma_{\mu}^{\dagger}\gamma^{0\dagger}v_2 \\
&= b^{\dagger}v_4^{\dagger}\gamma^0\gamma^0\omega_{d'}^{\dagger}\gamma^0\gamma^0\gamma^{\mu\dagger}\gamma^{0\dagger}u_3u_1^{\dagger}\gamma^0\gamma^0\omega_d^{\dagger}\gamma^0\gamma^0\gamma_{\mu}^{\dagger}\gamma^{0\dagger}v_2 \\
&= b^{\dagger}\overline{v}_4\omega_{-d'}\gamma^{\mu}u_3\overline{u}_1\omega_{-d}\gamma_{\mu}v_2
\end{aligned}$$

$$\begin{aligned}
\left|\mathcal{M}^{\tau\tau'dd'}\right| &= b^2(\overline{v}_2\gamma^{\nu}\omega_{\tau}u_1)(\overline{u}_3\gamma_{\nu}\omega_{\tau'}v_4)(\overline{v}_4\omega_{-d'}\gamma^{\mu}u_3)(\overline{u}_1\omega_{-d}\gamma_{\mu}v_2) \\
&= \sum_{a\to l} b^2(\overline{v}_{2a}\gamma^{\nu}_{ab}(\omega_{\tau})_{bc}u_{1c})(\overline{u}_{3d}\gamma_{\nu de}(\omega_{\tau'})_{ef}v_{4f})(\overline{v}_{4g}(\omega_{-d'})_{gh}\gamma^{\mu}_{hi}u_{3i})(\overline{u}_{1j}(\omega_{-d})_{jk}\gamma_{\mu kl}v_{2l}) \\
&= \sum_{a\to l} b^2[v_{2l}\overline{v}_{2a}\gamma^{\nu}_{ab}(\omega_{\tau})_{bc}u_{1c}\overline{u}_{1j}(\omega_{-d})_{jk}\gamma_{\mu kl}][u_{3i}\overline{u}_{3d}\gamma_{\nu de}(\omega_{\tau'})_{ef}v_{4f})(\overline{v}_{4g}(\omega_{-d'})_{gh}\gamma^{\mu}_{hi}] \\
&= b^2 tr[\underbrace{v_2\overline{v}_2\gamma^{\nu}\omega_{\tau}u_1\overline{u}_1\omega_{-d}\gamma_{\mu}}_{A}]tr[\underbrace{u_3\overline{u}_3\gamma_{\nu}\omega_{\tau'}v_4\overline{v}_4\omega_{-d'}\gamma^{\mu}}_{B}]
\end{aligned}$$

$d = \tau$ and $d' = \tau'$

$$\begin{aligned}
A^{--} &= tr[v_2\overline{v}_2\gamma^\nu\omega_- u_1\overline{u}_1\omega_+\gamma_\mu] \\
&= tr[\not{p}_2\gamma^\nu\omega_-\not{p}_1\omega_+\gamma_\mu] \\
&= tr[\not{p}_2\gamma^\nu\omega_-\omega_-\not{p}_1\gamma_\mu] \\
&= \frac{2}{4}tr[\not{p}_2\gamma^\nu(1-\gamma_5)\not{p}_1\gamma_\mu] \\
&= \frac{2}{4}[tr[\not{p}_2\gamma^\nu\not{p}_1\gamma_\mu] - tr[\not{p}_2\gamma^\nu\gamma_5\not{p}_1\gamma_\mu]] \\
&= \frac{2}{4}p_2^\sigma p_1^\rho[tr[\gamma_\sigma\gamma^\nu\gamma_\rho\gamma_\mu] - tr[\gamma_\sigma\gamma^\nu\gamma_\rho\gamma_\mu\gamma_5]] \\
&= \frac{2}{4}p_2^\sigma p_1^\rho g^{\tau\nu}[g_{\sigma\tau}g_{\rho\mu} - g_{\sigma\rho}g_{\tau\mu} + g_{\sigma\mu}g_{\tau\rho} + i\epsilon_{\sigma\tau\rho\mu}] \\
&= \frac{8}{4}g^{\tau\nu}[p_{2\tau}p_{1\mu} - (p_2p_1)g_{\tau\mu} + p_{2\mu}p_{1\tau} + ip_2^\sigma p_1^\rho\epsilon_{\sigma\tau\rho\mu}]
\end{aligned}$$

$$\begin{aligned}
B^{--} &= tr[u_3\overline{u}_3\gamma_\nu\omega_{\tau'}v_4\overline{v}_4\omega_{-\tau'}\gamma^\mu] \\
&= tr[\not{p}_3\gamma_\nu\omega_-\not{p}_4\omega_+\gamma^\mu] \\
&= \frac{1}{4}tr[\not{p}_3\gamma_\nu(1-\gamma_5)\not{p}_4\gamma^\mu] \\
&= \frac{2}{4}p_{3l}p_{4m}g_{\alpha\nu}[g^{l\alpha}g^{m\mu} - g^{lm}g^{\alpha\mu} + g^{l\mu}g^{\alpha m} + i\epsilon^{l\alpha m\mu}] \\
&= \frac{8}{4}g_{\alpha\nu}[p_3^\alpha p_4^\mu - (p_3p_4)g^{\alpha\mu} + p_3^\mu p_4^\alpha + ip_{3l}p_{4m}\epsilon^{l\alpha m\mu}]
\end{aligned}$$

If we change the helicity from - to + term with the $\epsilon_{..}$ get an - instead a + infront!
We now calculate all possible of helicity combinations.

$$\begin{aligned}
A^{--}B^{--} &= 4g^{\tau\nu}g_{\alpha\nu}[p_{2\tau}p_{1\mu} - (p_2p_1)g_{\tau\mu} + p_{2\mu}p_{1\tau} + ip_2^\sigma p_1^\rho\epsilon_{\sigma\tau\rho\mu}] \\
&\times [p_3^\alpha p_4^\mu - (p_3p_4)g^{\alpha\mu} + p_3^\mu p_4^\alpha + ip_{3l}p_{4m}\epsilon^{l\alpha m\mu}] \\
&= 4\delta_\alpha^\tau[(p_1p_4)p_{2\tau}p_3^\alpha - p_{2\tau}p_1^\alpha(p_3p_4) + p_{2\tau}p_4^\alpha(p_1p_3) + ip_{2\tau}p_{1\mu}p_{3l}p_{4m}\epsilon^{l\alpha m\mu} \\
&- [(p_2p_1)p_{4\tau}p_3^\alpha - (p_2p_1)(p_3p_4)\delta_\tau^\alpha + p_{3\tau}p_4^\alpha(p_2p_1) + i(p_2p_1)p_{3l}p_{4m}g_{\tau\mu}\epsilon^{l\alpha m\mu}] \\
&+ (p_2p_4)p_{1\tau}p_3^\alpha - p_{1\tau}p_2^\alpha(p_3p_4) + p_{1\tau}p_4^\alpha(p_2p_3) + ip_{2\mu}p_{1\tau}p_{3l}p_{4m}\epsilon^{l\alpha m\mu} \\
&+ ip_2^\sigma p_1^\rho p_3^\alpha p_4^\mu\epsilon_{\sigma\tau\rho\mu} - ip_2^\sigma p_1^\rho(p_3p_4)g^{\alpha\mu}\epsilon_{\sigma\tau\rho\mu} + ip_2^\sigma p_1^\rho p_3^\mu p_4^a lpha\epsilon_{\sigma\tau\rho\mu} - p_2^\sigma p_1^\rho p_{3l}p_{4m}\epsilon_{\sigma\tau\rho\mu}\epsilon^{l\alpha m\mu}] \\
&= \text{des eine zwischen schritt noch rein? oder nur sagen was benutzt wurde..?} \\
&= 4[2(p_1p_4)(p_2p_3) - 4(p_2p_1)(p_3p_4) + 4(p_2p_1)(p_3p_4) + 2(p_2p_4)(p_1p_3) \\
&- 2((p_1p_3)(p_2p_4) - (p_1p_4)(p_2p_3))] \\
&= 4[2(p_1p_4)(p_2p_3)] = 4t_{14}t_{23}
\end{aligned}$$

where we have used, that $\epsilon_{\sigma\alpha\rho\mu}\epsilon^{l\alpha m\mu} = 2(\delta^l_\sigma\delta^m_\rho - \delta^m_\sigma\delta^l_\rho)$, $\delta^\alpha_\alpha = 4$ in 4 dimensions. We also use that any permutaion of the ϵ-tensor change the sign of it. Using the same method we get for $A^{--}B^{++} = 4_{24}t_{13}$ with $t_{ij} = (p_i - p_j)^2 \overset{m=0}{=} 2p_ip_j$.
We now immediatly jump to the 2 to 4 process calculation.

B.2. Next-to-Leading Order Process (2 to 4)

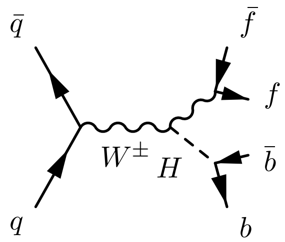


Figure B.2.: 2 to 4 process

For the 2 to 4 process the Matrix element is

$$\mathcal{M}^{\tau\tau'} = A(\overline{v}_2\gamma^\nu\omega_\tau u_1)(\overline{u}_{r1}\gamma_\nu\omega_{\tau'}v_{r2})(\overline{u}_{r3}v_{r4}) \tag{B.2}$$

with

$$A = (i)^7 e^4 g^\tau_{ff'V} g_{VH} g^{\tau'}_{ff'V} g_{Hff'} D(s_{12}, M_V) D(s_{r12}, M_V) D(s_{r34}, M_H) \tag{B.3}$$

$$\begin{aligned}
\left|\mathcal{M}^{\tau\tau' dd'}\right|^2 &= A^2(\overline{v}_2\gamma^\nu\omega_\tau u_1)(\overline{u}_{r1}\gamma_\nu\omega_{\tau'}v_{r2})(\overline{u}_{r3}v_{r4}) \\
&\times (\overline{v}_{r4}u_{r3})(\overline{v}_{r2}\omega_{-d'}\gamma^\mu u_{r1})(\overline{u}_1\omega_{-d}\gamma_\mu v_2) \\
&= A^2 tr[v_2\overline{v}_2\gamma^n u\omega_\tau u_1\overline{u}_1\omega_{-d}\gamma_\mu] \\
&\times tr[u_{r1}\overline{u}_{r1}\gamma_\nu\omega_{\tau'}v_{r2}\overline{v}_{r2}\omega_{-d'}\gamma^\mu] \\
&\times tr[v_{r4}\overline{v}_{r4}u_{r3}\overline{u}_{r3}]
\end{aligned}$$

We can see that the first two traces are exactly the same as before in the 2 to 3 process. This means that we just have to calculate the third trace.

$$\begin{aligned}
tr[v_{r4}\overline{v}_{r4}u_{r3}\overline{u}_{r3}] &= \frac{1}{4}tr[(\not{p}_{r4} - m_b)(\not{p}_{r3} + m_b)] \\
&= \frac{1}{4}tr[\not{p}_{r4}\not{p}_{r3} - m_b^2] \\
&= (p_{r4}p_{r3}) - m_b^2
\end{aligned}$$

Now we put everything together

$$\begin{aligned}|\mathcal{M}|^2 &= e^8 g_{VH}^2 g_{Hff'}^2 \left|D(s_{12}, M_V)\right|^2 \left|D(s_{r12}, M_V)\right|^2 \left|D(s_{r34}, M_H)\right|^2 \\ &\times \left[G_{--}\mathcal{M}_{--} + G_{-+}\mathcal{M}_{-+}\right] \times \left[(p_{r4}p_{r3}) - m_b^2\right]\end{aligned}$$

with

$$\begin{aligned}G_{--} &= \left|g_-^{qVq'}\right|^2 \left|g_-^{lVl'}\right|^2 + \left|g_+^{qVq'}\right|^2 \left|g_+^{lVl'}\right|^2 \\ G_{-+} &= \left|g_-^{qVq'}\right|^2 \left|g_+^{lVl'}\right|^2 + \left|g_+^{qVq'}\right|^2 \left|g_-^{lVl'}\right|^2 \\ \mathcal{M}_{--} &= 4t_{1r2}t_{2r1} \\ \mathcal{M}_{-+} &= 4t_{1r1}t_{2r2}\end{aligned}$$

B.2.1. 4 particle phase space

The phase space for 4 outgoing particles is

$$\Phi_{n'} = (2\pi)^4 \delta^4(P - p_1 - p_2 - p_3 - p_4) \prod_{i=1}^{4} \frac{\mathrm{d}^3 p_j}{(2\pi)^3 2E_j}. \tag{B.4}$$

We will now insert two times „one“, with new momenta $q_{12} = p_1 + p_2$ and $q_{34} = p_3 + p_4$:

$$1 = \int \frac{\mathrm{d}^4 q_{12}}{(2\pi)^4} \frac{\mathrm{d}^4 q_{34}}{(2\pi)^4} (2\pi)^4 \delta(q_{12} - p_1 - p_2)\Theta(q_{12}^0)(2\pi)^4\delta(q_{34} - p_3 - p_4)\Theta(q_{34}^0), \tag{B.5}$$

$$1 = \int \frac{\mathrm{d}s_{12}}{2\pi} \frac{\mathrm{d}s_{34}}{2\pi} 2\pi\delta(s_{12} - q_{12}^2) 2\pi\delta(s_{34} - q_{34}^2) \tag{B.6}$$

with $\Theta(x) = 1x > 0; 0x < 0$. We can now simplify these delta functions in the following way

$$\begin{aligned}&\int \frac{\mathrm{d}^4 q_{12}}{(2\pi)^4} \frac{\mathrm{d}s_{12}}{2\pi} (2\pi)^4 \delta(q_{12} - p_1 - p_2)\Theta(q_{12}^0) 2\pi\delta(s_{12} - q_{12}^2) \\ &\quad = \int \frac{\mathrm{d}^3 q_{12}}{(2\pi)^3} \frac{\mathrm{d}s_{12}}{2\pi} \frac{1}{2E_{12}} (2\pi)^4 \delta(q_{12} - p_1 - p_2),\end{aligned} \tag{B.7}$$

by using

$$q_{12}^2 = E_{12} - \vec{q}_{12}^{\,2} \tag{B.8}$$

$$\Rightarrow s_{12} - q_{12}^2 = (s_{12} + \vec{q}_{12}^{\,2}) - E_{12}^2 = 0$$

$$\Rightarrow E_{12} = q_{12}^0 \tag{B.9}$$

with $q_{12}^0 > 0$ (because of $\Theta(q_{12}^0)$.
The phase space now is of the following form:

$$\int \mathrm{d}\Phi_4 = \int \prod_{i=1}^{4} \frac{\mathrm{d}^3 p_i}{(2\pi)^3 2E_i} \frac{\mathrm{d}s_{12}}{2\pi} \frac{\mathrm{d}^3 q_{12}}{(2\pi)^3 2E_{12}} \frac{\mathrm{d}s_34}{2\pi} \frac{\mathrm{d}^3 q_{34}}{(2\pi)^3 2E_{34}} \\ \times (2\pi)^4 \delta(P - q_{12} - q_{34})(2\pi)^4 \delta(q_{12} - p_1 - p_2)(2\pi)^4 \delta(q_{34} - p_3 - p_4) \tag{B.10}$$

We know that the term $\frac{\mathrm{d}^3 p_i}{(2\pi)^3 2E_i}$ is Lorenz invariant, which means that we can carry out these integrals in any frame. So we can take the „rest frame“ of q_{12}. And in this frame we know that the 2-particle phase space (2.19) holds,

$$\int \mathrm{d}\Phi_2(p_1, p_2) = \int \frac{\mathrm{d}^3 p_1}{(2\pi)^3 2E_1} \frac{\mathrm{d}^3 p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta(E_{12} - E_1 - E_2) \delta(\vec{p}_1 + \vec{p}_2) \\ = \frac{\beta_{12}}{8\pi} \int \frac{\mathrm{d}\cos\theta_{12}}{2} \frac{\mathrm{d}\phi_{12}}{2\pi} \tag{B.11}$$

with

$$\beta_{12} = \frac{\lambda^{\frac{1}{2}}(s_{12}, m_1^2, m_2^2)}{s_{12}}.$$

The same applies for q_{34}. So the only integrals left are the integrals over s_{ij} and q_{ij},

$$\int \Phi_4 = \int \frac{\mathrm{d}s_{12}}{2\pi} \frac{\mathrm{d}^3 q_{12}}{(2\pi)^3 2E_{12}} \frac{\mathrm{d}s_34}{2\pi} \frac{\mathrm{d}^3 q_{34}}{(2\pi)^3 2E_{34}} \\ \times \frac{\beta_{12}}{8\pi} \int \frac{\mathrm{d}\cos\theta_{12}}{2} \frac{\mathrm{d}\phi_{12}}{2\pi} \frac{\beta_{34}}{8\pi} \int \frac{\mathrm{d}\cos\theta_{34}}{2} \frac{\mathrm{d}\phi_{34}}{2\pi}. \tag{B.12}$$

The integrals over q_{12} and q_{34} can be solved by assuming each of them is a „particle“ of mass $\sqrt{s_{12}}$ and $\sqrt{s_{34}}$. So we can choose the „rest frame“ of P,

$$\int \mathrm{d}\Phi_2(q_{12}, q_{34}) = \int \frac{\mathrm{d}^3 q_{12}}{(2\pi)^3 2E_{12}} \frac{\mathrm{d}^3 q_{34}}{(2\pi)^3 2E_{34}} (2\pi)^4 \delta(P - q_{12} - q_{34}) \\ = \frac{\beta}{8\pi} \int \frac{\mathrm{d}\cos\theta}{2} \frac{\mathrm{d}\phi}{2\pi} \tag{B.13}$$

with

$$\beta = \frac{\lambda^{\frac{1}{2}}(s, s_{12}, s_{34})}{s}.$$

Finally we get the following equation for the 4 particle phase space:

$$
\begin{aligned}
\int \mathrm{d}\Phi_4 &= \int \frac{s_{12}}{2\pi}\frac{s_{34}}{2\pi}\frac{\beta}{8\pi}\int \frac{\mathrm{d}\cos\theta}{2}\frac{\mathrm{d}\phi}{2\pi} \\
&\quad\times \frac{\beta}{8\pi}\int \frac{\mathrm{d}\cos\hat{\theta}_{12}}{2}\frac{\mathrm{d}\hat{\phi}_{12}}{2\pi}\frac{\beta}{8\pi}\int \frac{\mathrm{d}\cos\hat{\theta}_{34}}{2}\frac{\mathrm{d}\hat{\phi}_{34}}{2\pi} \\
&= \int \frac{s_{12}}{2\pi}\frac{s_{34}}{2\pi}\mathrm{d}\Phi_2(q_{12},q_{34})\mathrm{d}\Phi_2(\hat{p}_1,\hat{p}_2)\mathrm{d}\Phi_2(\hat{p}_3,\hat{p}_4),
\end{aligned} \tag{B.14}
$$

where the momenta labelled with $\hat{}$ are covered in the rest frame of s_{12} or s_{34}. What we obmitted up until now are the borders of the s_{12} and s_{34} integrals or in other words the physical region of the energies. In order to solve these integrals, consider the following:

$$
\begin{aligned}
s_{12} &= (p_1+p_2)^2 = M_{12}^2 \\
s_{34} &= (p_3+p_4)^2 = M_{34}^2
\end{aligned}
$$

and

$$
\begin{aligned}
M_{12} &\geq m_1 + m_2 \\
M_{34} &\geq m_3 + m_4.
\end{aligned}
$$

Hence we conclude, that $m_1 + m_2$ respectively $m_3 + m_4$ have to be the lower borders for the integrals. Additionally we know

$$
\begin{aligned}
&\sqrt{s} = M_{12} + m_{34} \\
\Rightarrow\ &M_{12} \leq \sqrt{s} - m_3 - m_4 \\
&M_{34} \leq \sqrt{s} - m_1 - m_2.
\end{aligned}
$$

This (physical) region is called Goldhaber plot or Goldhaber triangle.

We can generalize this procedure for a n particle final state. For that we first assume two groups of invariant masses squared $M_l^2 = (p_1 + ... + p_l)^2$ and $M_{n-l}^2 = (p_{l+1} + ... + p_n)^2$. For illustration, the Feynman diagram of the process is shown in figure B.3.

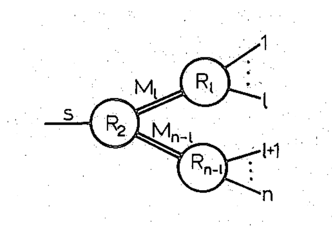


Figure B.3.: Tree diagram [2]

The corresponding cross section is

$$\Phi_n(s) = \int \mathrm{d}M_l^2 \mathrm{d}M_{n-l}^2 \Phi_2(s; M_l^2, M_{n-l}^2)\Phi_l(M_l^2; m_1^2, ..., m_l^2) \\ \times \Phi_{n-l}(M_{n-l}^2; m_{l+1}^2, ..., m_n^2) \quad [2, \text{p.182 eq.4.7}] \tag{B.15}$$

with the physical region

$$\begin{aligned} M_l &\geq m_1 + ... + m_l \\ M_{n-l} &\geq m_{l+1} + ... + m_n \\ M_l + M_{n-l} &\geq \sqrt{s}. \end{aligned} \tag{B.16}$$

In order to get a better understanding for the conditions we look at the physical region of the phase space which is shown in figure B.4.

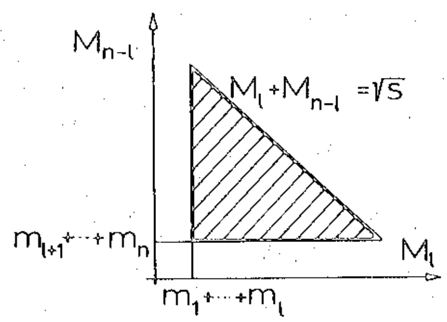


Figure B.4.: Physical region in the invariants $M_l = [(p_1 + ... + p_l)^2]^{1/2}, M_{n-l} = [(p_{l+1} + ...p_n)^2]^{1/2}$[2]

C. Passarino-Veltman Reduction of Tensor Integrals

When we want to calculate the NL order of process we get to oneloop tensor integrals with power of the loop momentum in the nominator. With the technique introduced by Passarino and Veltman [14] we reduce these tensor integral to scalar n-point functions. Ine this work we used FeynCalc [11] to simplify the NL order process to scalar integrals.

In this section we give the general definition of the integrals and momentas in the Denner notation [1]. We use the Denner notation because it is principally the same as used in FeynCalc [11].

$$T^{N}_{\mu_1..\mu_P}(p_1,..,p_{N-1},m_0,..,m_{N-1}) = \frac{(4\pi)^{4-D}}{i\pi^2}\int \mathrm{d}^D q \frac{q_{\mu_1}\cdots q_{\mu_P}}{D_0 D_1 \cdots D_{N-1}} \tag{C.1}$$

With the denominator parts coming from the propagators in the Feynman diagrams

$$D_0 = q^2 - m_0^2 + i\epsilon, \quad D_i = (q+p_i)^2 - m_i^2, \quad i = 1,...N-1. \tag{C.2}$$

The momentas are

$$p_{i0} = p_i \text{ and } p_{ij} = p_i - p_j.$$

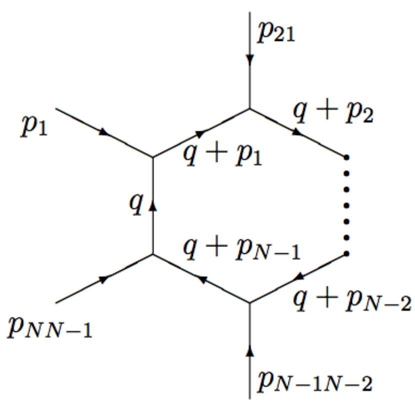


Figure C.1.: Conventions for the N-point-integral

The tensor integrals are invariant under arbitrary permutations of the propagators D_i and totally symmetric in the Lorentz indices μ_k. The infinitesimal imaginary part, $i\epsilon$, is needed to regulate the singularities of the integrand. The parameter μ has the dimension of mass and serves to keep the dimension of the integrals fixed for varying D.

The standard way of denoting the tensor integrals is the with the N-th charachter of the alphabet, i.e. $T^1 \equiv A$, $T^2 \equiv B$. The scalar integral carries the index zero.

Knowing the Lorentz covarince of the integrals, the decomposition of the lowest order integrals reads

$$
\begin{aligned}
B_\mu &= p_{1\mu} B_1, && \text{(C.3)}\\
B_{\mu\nu} &= g_{\mu\nu} B_{00} + p_{1\mu} p_{1\nu} B_{11}, \\
C_\mu &= \sum_{i=1}^{2} p_{i\mu} C_i, && \text{(C.4)}\\
C_{\mu\nu} &= g_{00} C_{00} + \sum_{i,j=1}^{2} p_{i\mu} p_{j\nu} C_{ij}, \\
C_{\mu\nu\rho} &= \sum_{i=1}^{2} (g_{\mu\nu} p_{i\rho} + g_{\nu\rho} p_{i\mu} + g_{\mu\rho} p_{i\nu}) C_{00i} + \sum_{i,j,k=1}^{2} p_{i\mu} p_{j\nu} p_{k\rho} C_{ijk} \\
D_\mu &= \sum_{i=1}^{3} p_{i\mu} D_i, && \text{(C.5)}\\
D_{\mu\nu} &= g_{00} D_{00} + \sum_{i,j=1}^{3} p_{i\mu} p_{j\nu} D_{ij}, \\
D_{\mu\nu\rho} &= \sum_{i=1}^{2} (g_{\mu\nu} p_{i\rho} + g_{\nu\rho} p_{i\mu} + g_{\mu\rho} p_{i\nu}) D_{00i} + \sum_{i,j,k=1}^{3} p_{i\mu} p_{j\nu} p_{k\rho} D_{ijk} \\
D_{\mu\nu\rho\sigma} &= (g_{\mu\nu} g_{\rho\sigma} + g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho}) D_{0000} \\
&\quad + \sum_{i,j=1}^{3} \left(g_{\mu\nu} p_{i\rho} p_{j\sigma} + g_{\nu\rho} p_{i\mu} p_{j\sigma} + g_{\mu\rho} p_{i\nu} p_{j\sigma} \right. \\
&\qquad \left. + g_{\mu\sigma} p_{i\nu} p_{j\rho} + g_{\nu\sigma} p_{i\mu} p_{j\rho} + g_{\rho\sigma} p_{i\mu} p_{j\nu} \right) E_{00ij} \\
&\quad + \sum_{i,j,k,l=1}^{3} p_{i\mu} p_{j\nu} p_{k\rho} p_{k\sigma} && \text{(C.6)}
\end{aligned}
$$

C.1. Scalar Integrals

In this section are presented the results for the scalar integrals in D-dimensions.

C.1.1. $A_0(M)$

$$\begin{aligned}
A_0(M) &= \mu^{4-D}\int \frac{\mathrm{d}^D q}{(2\pi)^D}\frac{(4\pi)^{\frac{d}{2}}}{i}\frac{1}{q^2-M^2} \\
&= \mu^{4-D}\int \frac{\mathrm{d}^D q}{i\pi^{\frac{D}{2}}}\frac{1}{q^2-M^2} \\
&= -\mu^{4-D}M^{-1+\frac{D}{2}}\frac{\Gamma(1-\frac{d}{2})}{\Gamma(1)} \\
&\overset{D=4-2\epsilon}{=} -\mu^{2\epsilon}M^2(M^2)^{-\epsilon}\Gamma(-1+\epsilon) \\
&= -(1+2\epsilon\ln(\mu))M^2(1-\epsilon\ln(M^2))\frac{-1}{1!}\left[\frac{1}{\epsilon}-\gamma_E+1+O(\epsilon)\right] \\
&\overset{\epsilon\to 0}{=} M^2\left[\frac{1}{\epsilon}-\ln(M^2)+2\ln(\mu)+1\right]
\end{aligned} \tag{C.7}$$

In the last step we only get rid of the terms proportional to ϵ. We can clearly see that the A_0 function contains no Sudakov logarithms.

C.1.2. $B_0(p^2, M_0, M_1)$

We turn next to the scalar one-point function, B_0.

$$B_0(p^2, M_0, M_1) = \int \frac{\mathrm{d}^D q}{i\pi^{\frac{D}{2}}}\frac{1}{(q^2-M_0^2)((q+p)^2-M_1^2)} \tag{C.8}$$

The solution after Feynman parameterization and Wick rotation is (from [16]):

$$\begin{aligned}
B_0(p^2, M_0, M_1) &\overset{D=4-2\epsilon}{=} \mu^{2\epsilon}\left[\frac{1}{\epsilon}+2-\ln(p^2)+\sum_{i=1}^{2}\left[\gamma_i\ln(\frac{\gamma_i-1}{\gamma_i})-ln(\gamma_i-1)\right]\right] \\
&\overset{\epsilon\to 0}{=} \left[\frac{1}{\epsilon}+2-\ln(p^2)+\sum_{i=1}^{2}\left[\gamma_i\ln(\frac{\gamma_i-1}{\gamma_i})-ln(\gamma_i-1)\right]+2\ln\mu\right]
\end{aligned} \tag{C.9}$$

with

$$\gamma_{1,2} = \frac{p^2-M_1^2+M_0^2\pm\sqrt{(p^2-M_1^2+M_0^2)^2-4p^2(M_0^2)}}{2p^2}$$

In Section 3 we take a closer look at this result in the high-energy limit to check if there are Sudakov logarithms appearing.

C.1.3. $C_0(p_1, p_2, M_0, M_1, M_2)$

We move now to the general solution of the C_0 integral where we give the result of $C_0(p_1, p_2, M_0, M_2, M_3)$. A detailed calculation is in Appendix D. The general definition of the C_0 integral is

$$\begin{aligned}
C_0(p_1, p_2, M_0, M_1, M_2) &= \mu^{4-D} \int \frac{\mathrm{d}^D q}{i\pi^{\frac{D}{2}}} \frac{1}{(q^2 - M_0^2)((q+p_1)^2 - M_1^2)((q+p_2)^2 - M_2^2)} \\
&= -\mu^{4-D} \int \frac{\mathrm{d}^D q}{i\pi^{\frac{D}{2}}} \int_0^1 \mathrm{d}x' \int_0^{x'} \mathrm{d}y \frac{1}{(\overline{q}^2 + D^*)^3} \\
&= -\mu^{4-D} \int_0^1 \int_0^{x'} (-1)^{-3} (D^*)^{-3+\frac{D}{2}} \frac{\Gamma(3 - \frac{D}{2})}{2} \\
&\overset{D=4-2\epsilon}{=} \mu^{2\epsilon} \int_0^1 \int_0^{x'} (D^*)^{-(1+\epsilon)} \frac{\Gamma(1+\epsilon)}{2} \\
&= \int_0^1 \int_0^{x'} \frac{1}{2D^*} (1 + 2\epsilon \ln(\mu))(1 - \epsilon \ln(D^*))(1 + \epsilon\gamma_E) \\
&\overset{\epsilon \to 0}{=} \int_0^1 \int_0^{x'} \frac{1}{2D^*}
\end{aligned} \tag{C.10}$$

with

$$D^* = \left[-(-x'p_2 + yp_1 + p_2)^2 + x'(-p_2^2 - M_0^2 + M_2^2) + y(p_1^2 + M_0^2 - M_1^2) + p_2^2 - M_2^2\right]$$

For the end result we need several variables listed below:

$$\begin{aligned}
a &= -p_2^2 \\
b &= -p_1^2 \\
c &= 2p_2p_1 \\
d &= p_2^2 - m_0^2 + m_2^2 \\
e &= p_1^2 + m_0^2 = m_1^2 - 2p_2p_1 \\
f &= -m_2^2 - i\epsilon
\end{aligned}$$

$$\begin{aligned}
y_{01} &= \frac{\alpha c + 2a + d + \alpha e}{c + 2\alpha b} \\
y_{02} &= \frac{d + \alpha e}{c + 2\alpha b + \alpha c + 2a} \\
y_{03} &= \frac{d + \alpha e}{\alpha c + 2a} \\
a_1 &= \frac{1}{c + 2\alpha b} \\
a_2 &= \frac{1 - \alpha}{c + 2\alpha b + \alpha c + 2a} \\
a_3 &= \frac{\alpha}{\alpha c + 2a}
\end{aligned}$$

$$
\begin{aligned}
y_{11} \text{ and } y_{21} \text{ are the roots of } 0 &= by^2 + y(e+c) + a + d + f \\
y_{12} \text{ and } y_{22} \text{ are the roots of } 0 &= y^2(a+b+c) + y(e+c) + f \\
y_{13} \text{ and } y_{23} \text{ are the roots of } 0 &= ay^2 + yd + f \\
\text{and } \alpha \text{ which is the real solution of } 0 &= \alpha^2 b + \alpha c + a
\end{aligned}
$$

The final result is written as the following equation.

$$
\begin{aligned}
C_0(p_1, p_2, M_0, M_1, M_2) = \frac{1}{2}\, [a_1 S_3(y_{01}, y_{11}, y_{21}) - a_2 S_3(y_{02}, y_{12}, y_{22}) \\
- a_3 S_3(y_{03}, y_{13}, y_{23})]
\end{aligned} \tag{C.11}
$$

$$
\begin{aligned}
S_3(y_0, y_1, y_2) = {} & R(y_0, y_1) + R(y_0, y_2) + \\
& \left[\eta(-y_1, -y2) - \eta(y_0 - y_1, y_0 - y_2) - \eta(a - i\epsilon, \frac{1}{a - i\delta})\right] \ln\left(\frac{1 - y_0}{y_0}\right) \\
R(y_0, y_1) = {} & Li_2\left(\frac{y_1 - 1}{y_1 - y_0}\right) - Li_2\left(\frac{y_1}{y_1 - y_0}\right) + \ln\left(\frac{1 - y_0}{y_1 - y_0}\right)(\ln(1 - y_1) - \ln(y_0 - y_1)) \\
& - \ln\left(\frac{-y_0}{y_1 - y_0}\right)(\ln(-y_1) - \ln(y_0 - y_1))
\end{aligned}
$$

This result is not very handy so for further calculations we use the symmetric form from the denner paper [1]. First we give the result in the Denner notation and give the translation the our FeynCalc [11] notation later.

$$
\begin{aligned}
C_0(\hat{p}_{10}, \hat{p}_{20}, m_0, m_1 m_2) = \frac{1}{\hat{\alpha}} \sum_{i=0}^{2} [\sum_{\sigma=\pm} & [Li_2(\frac{\hat{y}_{0i} - 1}{\hat{y}_{i\sigma}}) - Li_2(\frac{\hat{y}_{0i}}{\hat{y}_{i\sigma}}) \\
& + \eta(1 - x_{i\sigma}, \frac{1}{\hat{y}_{i\sigma}}) \ln(\frac{\hat{y}_{0i} - 1}{\hat{y}_{i\sigma}}) - \eta(-x_{i\sigma}, \frac{1}{\hat{y}_{i\sigma}}) \ln(\frac{y_{0i}}{y_{i\sigma}})] \\
& - [\eta(-x_{i+}, -x_{i-}) - \eta(\hat{y}_{i+}, \hat{y}_{i-}) - 2\pi i \Theta(-\hat{p}_{jk}^2)\Theta(-Im(\hat{y}_{i+}\hat{y}_{i-}))] \ln(\frac{1 - \hat{y}_{i0}}{-\hat{y}_{i0}})]
\end{aligned}
$$

with $(i, j, k = 0, 1, 2$ and cyclic)

$$
\begin{aligned}
\hat{y}_{0i} &= \frac{1}{2\hat{\alpha}\hat{p}_{jk}^2} \left[\hat{p}_{jk}^2(\hat{p}_{jk}^2 - \hat{p}_{ki}^2 - \hat{p}_{ij}^2 + 2m_i^2 - m_j^2 - m_k^2)\right. \\
&\quad \left. - (\hat{p}_{ki}^2 - \hat{p}_{ij}^2)(m_j^2 - m_k^2) + \hat{\alpha}(\hat{p}_{jk}^2 - m_j^2 + m_k^2)\right] \\
x_{i\pm} &= [\hat{p}_{jk}^2 - m_j^2 + m_k^2 \pm \hat{\alpha}_i] \\
\hat{y}_{i\pm} &= \hat{y}_{0i} - x_{i\pm} \\
\hat{\alpha} &= \kappa(\hat{p}_{10}^2, \hat{p}_{21}^2, \hat{p}_{20}^2) \\
\hat{\alpha}_i &= \kappa(\hat{p}_{jk}^2, m_j^2, m_k^2)(1 + i\epsilon\hat{p}_{jk}^2)
\end{aligned}
$$

and with Källén-Lehmann function

$$\kappa(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 2(xy + yz = zx}.$$

The dilogarithm or Spence function is defined as the integral

$$Li_2(x) = \int_0^1 \frac{\mathrm{d}t}{t} \ln(1 - xt) \qquad |\arg(1 - xt)| < \pi.$$

D. Detailed calculation of the C_0 integral

In the following section we introduce a detailed calculation of the C_0 integral which appears mostly in the vertex corrections. We include all masses and momenta to get the most general result.

$$C\left(p\right)=\mu^{4-D}\int\frac{\mathrm{d}^{D}q}{i\pi^{\frac{D}{2}}}\frac{1}{\left(q^{2}-m_{0}^{2}\right)\left(\left(q+p_{1}\right)^{2}-m_{1}^{2}\right)\left(\left(q+p_{2}\right)^{2}-m_{2}^{2}\right)}\tag{D.1}$$

The first step is again introducing Feynman parameters and performing a Wick rotation. After this we get:

$$\begin{aligned}
C_0(p_1,p_2,M_0,M_1,M_2)&=\mu^{4-D}\int\frac{\mathrm{d}^{D}q}{i\pi^{\frac{D}{2}}}\frac{1}{(q^2-M_0^2)((q+p_1)^2-M_1^2)((q+p_2)^2-M_2^2)}\\
&=-\mu^{4-D}\int\frac{\mathrm{d}^{D}q}{i\pi^{\frac{D}{2}}}\int_0^1\mathrm{d}x'\int_0^{x'}\mathrm{d}y\frac{1}{(\overline{q}^2+D^*)^3}\\
&=-\mu^{4-D}\int_0^1\mathrm{d}x'\int_0^{x'}\mathrm{d}y(-1)^{-3}(D^*)^{-3+\frac{D}{2}}\frac{\Gamma(3-\frac{D}{2})}{2}\\
&\overset{D=4-2\epsilon}{=}\mu^{2\epsilon}\int_0^1\mathrm{d}x'\int_0^{x'}\mathrm{d}y(D^*)^{-(1+\epsilon)}\frac{\Gamma(1+\epsilon)}{2}\\
&\overset{\mu\to 1}{=}\int_0^1\mathrm{d}x'\int_0^{x'}\mathrm{d}y\frac{1}{2D^*}(1+2\epsilon\ln(\mu))(1-\epsilon\ln(D^*))(1+\epsilon\gamma_E)\\
&\overset{\epsilon\to 0}{=}\int_0^1\mathrm{d}x'\int_0^{x'}\mathrm{d}y\frac{1}{2D^*}
\end{aligned}$$

with

$$D^*=\left[-(-x'p_2+yp_1+p_2)^2+x'(-p_2^2-M_0^2+M_2^2)+y(p_1^2+M_0^2-M_1^2)+p_2^2-M_2^2\right]$$

We now have to integrate the expression twice whereas the integration variable y depends on the other variable x'.

$$
\begin{aligned}
C_0 &= \frac{1}{2}\int_0^1 \mathrm{d}x' \int_0^{x'} \mathrm{d}y \left[-(-x'p_2 + yp_1 + p_2)^2 + x'(-p_2^2 - M_0^2 + M_2^2) + y(p_1^2 + M_0^2 - M_1^2) + p_2^2 - M_2^2\right]^{-1} \\
&= \frac{1}{2}\int_0^1 \mathrm{d}x' \int_0^{x'} \mathrm{d}y \Big[-x'^2 p_2^2 - y^2 p_1^2 + 2x'yp_2p_1 + x'\left(p_2^2 - m_0^2 + m_2^2\right) \\
&\qquad\qquad + y\left(p_1^2 + m_0^2 - m_1^2 - 2p_2p_1\right) - m_2^2 - i\epsilon\Big]^{-1} \\
&= \frac{1}{2}\int_0^1 \mathrm{d}x' \int_0^{x'} \mathrm{d}y \left[ax'^2 + by^2 + cx'y + dx' + ey + f\right]^{-1} \\
&= \frac{1}{2}\int_0^1 \mathrm{d}x' \int_{-\alpha x'}^{(1-\alpha)x'} \mathrm{d}y' \left[ax'^2 + (y' + \alpha x')^2 b + x'(y' + \alpha x')c + dx' + (y' + \alpha x')e + f\right]^{-1} \\
&= \frac{1}{2}\int_0^1 \mathrm{d}x' \int_{-\alpha x'}^{(1-\alpha)x'} \mathrm{d}y' \left[x'^2 \underbrace{(a + \alpha c + b\alpha^2)}_{=0} + by'^2 + x'y'(2\alpha b + c) + x'(d + \alpha e) + y'e + f\right]^{-1}
\end{aligned}
$$

In the last two steps we performed a shift with $y' = y + \alpha x'$ where we defined α in a way that it solves the equation $b\alpha^2 + c\alpha + a = 0$. With that definition of α the integration of the x' variable becomes a easy to solve linear integral. For now we assume that a, b and c such that α is real. This means f ist the only variable with an negative imaginary part.

To perform the x' integration we have to manipulate the integration borders and introduce a second shift with $y' = (1 - \alpha)x'$.

$$
\begin{aligned}
C_0 &= \frac{1}{2}\left(\int_0^1 \mathrm{d}x' \int_0^{(1-\alpha)x'} \mathrm{d}y' - \int_0^1 \mathrm{d}x' \int_0^{-\alpha)x'} \mathrm{d}y'\right) \\
&\qquad \left[by'^2 + x'y'(2\alpha b + c) + x'(d + \alpha e) + y'e + f\right]^{-1} \\
&\overset{y=(1-\alpha)x'}{=} \frac{1}{2}\left(\int_0^{1-\alpha} \mathrm{d}y \int_{\frac{y}{1-\alpha}}^1 \mathrm{d}x' - \int_0^{-\alpha} \mathrm{d}y \int_{-\frac{y}{\alpha}}^1 \mathrm{d}x'\right) \\
&\qquad \left[x' \underbrace{(d + \alpha e + y(2\alpha b + c))}_{A} + \underbrace{(by^2 + ye + f)}_{B}\right]^{-1} \\
&= \frac{1}{2}\int_0^{1-\alpha} \mathrm{d}y \int_{\frac{y}{1-\alpha}}^1 \mathrm{d}x' \frac{1}{Ax + B} - \frac{1}{2}\int_0^{-\alpha} \mathrm{d}y \int_{-\frac{y}{\alpha}}^1 \mathrm{d}x' \frac{1}{Ax + B} \\
&= \frac{1}{2A}\left[\int_0^{1-\alpha} \left[\ln\left(1 + \frac{B}{A}\right) - \ln\left(\frac{y}{1-\alpha} + \frac{B}{A}\right)\right]\right. \\
&\qquad \left. - \left[\ln\left(1 + \frac{B}{A}\right) - \ln\left(-\frac{y}{\alpha} + \frac{B}{A}\right)\right]\right]
\end{aligned}
$$

$$
\begin{aligned}
C_0 &= \frac{1}{2A}\left[\int_0^{1-\alpha} \mathrm{d}y \ln\left(\frac{(A+B)(1-\alpha)}{yA+B(1-\alpha)}\right) - \int_0^{-\alpha} \mathrm{d}y \ln\left(\frac{(A+B)\alpha}{-yA+B\alpha}\right)\right] \\
&= \frac{1}{2}\int_0^{1-\alpha} \mathrm{d}y \frac{1}{d+\alpha e+y(2\alpha b+c)} \ln\left(\frac{(d+\alpha e+y(2\alpha b+c)+by^2+ye+f)(1-\alpha)}{y(d+\alpha e+y(2\alpha b+c))+(by^2+ye+f)(1-\alpha)}\right) \\
&\quad -\frac{1}{2}\int_0^{-\alpha} \mathrm{d}y \frac{1}{d+\alpha e+y(2\alpha b+c)} \ln\left(\frac{(d+\alpha e+y(2\alpha b+c)+by^2+ye+f)\alpha}{-y(d+\alpha e+y(2\alpha b+c))+(by^2+ye+f)\alpha}\right)
\end{aligned}
$$

Now we shift the integration limits for the two integrals. The first one gets the shift $y = (1-\alpha)y'$. The second one performs the shift $y = -\alpha y'$.

$$
\begin{aligned}
C_0 = \frac{1}{2}\int_0^1 \mathrm{d}y' &\frac{1-\alpha}{d+\alpha e+y'(1-\alpha)(2\alpha b+c)} \\
&\cdot \ln\left[\frac{b(1-\alpha)^2 y'^2 + y'(1-\alpha)(c+2\alpha b+e)+d+e\alpha+f}{b(1-\alpha)^2+(2\alpha b+c)(1-\alpha))y'^2+(d+\alpha e+(1-\alpha)e)y'+f}\right] \\
+\frac{1}{2}\int_0^1 \mathrm{d}y' &\frac{\alpha}{d+\alpha e-y'\alpha(2\alpha b+c)} \ln\left[\frac{b\alpha^2 y'^2+y'(-\alpha)(c+2\alpha b+e)+d+e\alpha+f}{b(\alpha^2+(2\alpha b+c)(-\alpha)y'^2+(d+\alpha e-\alpha e)y'+f}\right]
\end{aligned} \tag{D.2}
$$

Again we shift the limits with $y' = \frac{y}{1-\alpha}$ (first integral) and $y' = -\frac{y}{\alpha}$. Additionally we define the variable $N = d + \alpha e + y(2\alpha b + c)$.

$$
\begin{aligned}
C_0 &= \frac{1}{2}\int_0^{1-\alpha} \mathrm{d}y \frac{1}{N} \ln\left[\frac{by^2+ye+f+N}{by^2+ey+f+\frac{y}{1-\alpha}N}\right] \\
&\quad -\frac{1}{2}\int_0^{-\alpha} \mathrm{d}y \frac{1}{N} \ln\left[\frac{by^2+ye+f+N}{by^2+ey+f-\frac{y}{\alpha}N}\right]
\end{aligned} \tag{D.3}
$$

$$
\begin{aligned}
&= \frac{1}{2}\int_{-\alpha}^{1-\alpha} \mathrm{d}y \frac{1}{N} \ln\left[by^2+ye+f+N\right] \\
&\quad -\frac{1}{2}\int_0^{1-\alpha} \frac{1}{N} \ln\left[by^2+ey+f+\frac{y}{1-\alpha}N\right] \\
&\quad +\frac{1}{2}\int_0^{-\alpha} \frac{1}{N} \ln\left[by^2+ey+f-\frac{y}{\alpha}N\right]
\end{aligned} \tag{D.4}
$$

Notice that $1/N$ has a singularity at $y_0 = -\frac{d+\alpha e}{c+2\alpha b}$. To make sure that the residual at y_0 is zero we add to every integral the term

$$
-\ln\left[b{y_0}^2 + ey_0 + f\right].
$$

It can be done in the way that totally we add zero. C_0 looks then like

$$
\begin{aligned}
C_0 = \frac{1}{2}\Bigg[\int_{-\alpha}^{1-\alpha} \mathrm{d}y \frac{1}{N}\left[\ln\left[by^2 + ye + f + N\right] - \ln\left[b{y_0}^2 + ey_0 + f\right]\right] \\
- \int_0^{1-\alpha} \frac{1}{N}\left[\ln\left[by^2 + ey + f + \frac{y}{1-\alpha}N\right] - \ln\left[b{y_0}^2 + ey_0 + f\right]\right] \\
+ \int_0^{-\alpha} \frac{1}{N}\left[\ln\left[by^2 + ey + f - \frac{y}{\alpha}N\right] - \ln\left[b{y_0}^2 + ey_0 + f\right]\right]\Bigg]
\end{aligned} \tag{D.5}
$$

This additional term allows us to study the integral with complex α. Now we substitute $y' = y - \alpha$, $y = \frac{y'}{(1-\alpha)}$ and $y = -\frac{y'}{\alpha}$.

$$
\begin{aligned}
C_0 = \frac{1}{2}\Bigg[\int_0^1 \mathrm{d}y \frac{1}{y(c+2\alpha b) + \alpha c + 2a + d + \alpha e}\left[\ln\left[by^2 + y(e+c) + a + d + f\right]\right. \\
\left. - \ln\left[b{y_0}^2 + ey_0 + f\right]\right] \\
- \int_0^1 \frac{1-\alpha}{y(c + 2\alpha b + \alpha c + 2a) + d + \alpha e}\left[\ln\left[y^2(a+b+c) + y(e+d) + f\right] - \ln\left[b{y_0}^2 + ey_0 + f\right]\right] \\
- \int_0^1 \frac{\alpha}{y(\alpha c + 2a) + d + e\alpha}\left[\ln\left[ay^2 + dy + f\right] - \ln\left[b{y_0}^2 + ey_0 + f\right]\right]\Bigg]
\end{aligned} \tag{D.6}
$$

To get the denominators in front back to zero we define $y_1 = y_0 + \alpha$, $y_2 = \frac{y_0}{1-\alpha}$ and $y_3 = -\frac{y_0}{\alpha}$.

$$
\begin{aligned}
C_0 &= \frac{1}{2}\Bigg[\int_0^1 \mathrm{d}y \frac{1}{y(c+2\alpha b) + \alpha c + 2a + d + \alpha e}\left[\ln\left[by^2 + y(e+c) + a + d + f\right]\right. \\
&\qquad \left. - \ln\left[b{y_1}^2 + y_1(e+c) + a + d + f\right]\right] \\
&\quad - \int_0^1 \frac{1-\alpha}{y(c + 2\alpha b + \alpha c + 2a) + d + \alpha e}\left[\ln\left[y^2(a+b+c) + y(e+d) + f\right]\right. \\
&\qquad \left. - \ln\left[{y_2}^2(a+b+c) + y_2(e+d) + f\right]\right] \\
&\quad - \int_0^1 \frac{\alpha}{y(\alpha c + 2a) + d + e\alpha}\left[\ln\left[ay^2 + dy + f\right] - \ln\left[a{y_3}^2 + dy_3 + f\right]\right]\Bigg] \qquad \text{(D.7)} \\
&= \frac{1}{2}\Bigg[\frac{1}{c + 2\alpha b}\int_0^1 \mathrm{d}y \frac{1}{y - y - y_{01}}\left[\ln\left[by^2 + y(e+c) + a + d + f\right]\right. \\
&\qquad \left. - \ln\left[b{y_1}^2 + y_1(e+c) + a + d + f\right]\right] \\
&\quad - \frac{1-\alpha}{c + 2\alpha b + \alpha c + 2a}\int_0^1 \frac{1-\alpha}{y - y_{02}}\left[\ln\left[y^2(a+b+c) + y(e+d) + f\right]\right. \\
&\qquad \left. - \ln\left[{y_2}^2(a+b+c) + y_2(e+d) + f\right]\right] \\
&\quad - \frac{\alpha}{\alpha c + 2a}\int_0^1 \frac{\alpha}{y - y_{03}}\left[\ln\left[ay^2 + dy + f\right] - \ln\left[a{y_3}^2 + dy_3 + f\right]\right]\Bigg] \qquad \text{(D.8)}
\end{aligned}
$$

To write it in a nicer way we define some more variables and introduce the function

$$
S_3(y_0, y_1, y_2,) = \int_0^1 \mathrm{d}y \frac{1}{y - y_0}\left[\ln(ay^2 + by + c) - \ln(a{y_0}^2 + by_0 + c)\right]
$$

$$
\begin{aligned}
y_{01} &= -\frac{\alpha c + 2a + d + \alpha e}{c + 2\alpha b} \\
y_{02} &= -\frac{d + \alpha c}{c + 2\alpha b + \alpha c + 2a} \\
y_{03} &= -\frac{d + \alpha e}{\alpha c + 2a} \\
a_1 &= \frac{1}{c + 2\alpha b} \\
a_2 &= \frac{1 - \alpha}{c + 2\alpha b + \alpha c + 2a} \\
a_3 &= \frac{\alpha}{\alpha c + 2a}
\end{aligned}
$$

$$
\begin{aligned}
&y_{11} \text{ and } y_{21} \text{ are the roots of } 0 = by^2 + y(e + c) + a + d + f \\
&y_{12} \text{ and } y_{22} \text{ are the roots of } 0 = y^2(a + b + c) + y(e + c) + f \\
&y_{13} \text{ and } y_{23} \text{ are the roots of } 0 = ay^2 + yd + f
\end{aligned}
$$

The solution reads now

$$
C_0 = \frac{1}{2}\left[a_1 S_3(y_{01}, y_{11}, y_{21}) - a_2 S_3(y_{02}, y_{12}, y_{22}) - a_3 S_3(y_{03}, y_{13}, y_{23})\right]
$$

The next step is to solve the S_3 integral. For that we first look at the following function:

$$
R(y_0, y_1) = \int_0^1 \mathrm{d}y \frac{1}{y - y_0}\left[\ln(y - y1) - \ln(y - y_0)\right]
$$

If we use that

$$
\begin{aligned}
Li_2(x) &= -\int_0^1 \mathrm{d}t \frac{\ln(1 - xt)}{t} \\
Li_2(x) &= -Li_2(1 - x) + \frac{\pi^2}{6} - \ln(x)\ln(1 - x) \\
\text{and } \ln(ab) &= \ln(a) + \ln(b) + \eta(a, b) \\
\text{with } \eta(a, b) &= 2\pi i\left[\theta(-\Im(a))\theta(-\Im(b))\theta(\Im(ab)) - \theta(\Im(a))\theta(\Im(b))\theta(-\Im(ab))\right]
\end{aligned}
$$

then we get

$$
R(y_0, y_1) = Li_2\left(\frac{y_1 - 1}{y_1 - y_0}\right) - Li_2\left(\frac{y_1}{y_1 - y_0}\right) + \ln\left(\frac{1 - y_0}{y_1 - y_0}\right)\left(\ln(1 - y_1) - \ln(y_0 - y_1)\right) \tag{D.9}
$$

$$
- \ln\left(\frac{-y_0}{y_1 - y_0}\right)\left(\ln(-y_1) - \ln(y_0 - y_1)\right). \tag{D.10}
$$

For the general calculation of the

$$S_3(y_0, y_1, y_2,) = \int_0^1 \mathrm{d}y \frac{1}{y - y_0} \left[\ln(ay^2 + by + c) - \ln(a{y_0}^2 + by_0 + c)\right]$$

function we assume that
1) a is real
2) b, c, y_0 may be complex
3) $\Im(ay^2 + by + c)$ has the same sign in the range $[0, 1]$
We know that

$$ay^2 + by + c = a(y - y_1)(y - y_2) \text{ with } y_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2ac}$$

Notice that

$$-(y_1 + y_2) = \frac{b}{a} \text{ and } y_1 y_2 = \frac{c}{a}$$

So the imaginary part for $y = 0$ reads

$$a\Im(y_1 y_2)$$

for $y = 1$ it is

$$-a\Im(y_1 + y_2) + a\Im(y_1 y_2).$$

The sign of $-a\Im(y_1 + y_2) + a\Im(y_1 y_2)$ must be the same as in $a\Im(y_1 y_2)$ as claimed in our third assumption.The third condition also implies either

$$y\Im(b) + \Im(b) > 0 \Rightarrow -\Im(b) > 0, \Im(c) > 0$$

or

$$y\Im(b) + \Im(b) < 0 \Rightarrow -\Im(b) < 0, \Im(c) < 0$$

We now introduce the infinitesimal parameters ϵ and δ with opposite signs then $\Im(a(y - y_1)(y - y_2))$ and $\Im(a(y_0 - y_1)(y_0 - y_2))$ have also opposite signs.

Now,

$$\ln(a(y - y_1)(y - y_2)) \to \ln((a - i\epsilon)(y - y_1)(y - y_2)) = \ln((a - i\epsilon) \ln((y - y_1)(y - y_2)).$$

If we use $\ln(ab) = \ln(a) + \ln(b) + \eta(a, b)$ again for the case that $a(y - y_1)(y - y_2)$ and $(a - i\epsilon)$ have the same signs of the imaginary parts. Therefore

$$\ln(a(y_0 - y_1)(y_0 - y_2)) \to \ln((a - i\delta)(y_0 - y_1)(y_0 - y_2)) = \ln((a - i\delta) \ln((y_0 - y_1)(y_0 - y_2))$$

With these properties we can write

$$
\begin{aligned}
S_3(y_0, y_1, y_2) &= \int_0^1 \mathrm{d}y \frac{1}{y-y_0} \left[\ln((y-y_1)(y-y_2)) \right. \\
&\qquad \left. - \ln((y_0-y_1)(y_0-y_2)) + \ln(a-i\epsilon) - \ln(a-i\delta)\right] \\
&= \int_0^1 \mathrm{d}y \frac{1}{y-y_0} \left[\ln((y-y_1)(y-y_2)) \right. \\
&\qquad \left. - \ln((y_0-y_1)(y_0-y_2)) - \eta\left(a-i\epsilon, \frac{1}{a-i\delta}\right)\right] \\
&= \int_0^1 \mathrm{d}y \frac{1}{y-y_0} \left[\ln((y-y_1)(y-y_2)) - \ln((y_0-y_1)(y_0-y_2))\right] \\
&\qquad - \int_0^1 \mathrm{d}y \frac{1}{y-y_0} \eta\left(a-i\epsilon, \frac{1}{a-i\delta}\right) \\
&= \int_0^1 \mathrm{d}y \frac{1}{y-y_0} \left[\ln((y-y_1)(y-y_2)) - \ln((y_0-y_1)(y_0-y_2))\right] \\
&\qquad - \eta\left(a-i\epsilon, \frac{1}{a-i\delta}\right) \ln\left(\frac{1-y_0}{y_0}\right) \\
&= R(y_0, y_1) + R(y_0, y_2) \\
&\qquad + \left[\eta(y_1, y_2)\eta(y_0 y_1, y_0 y_2) - \eta\left(a-i\epsilon, \frac{1}{a-i\delta}\right)\right] \ln\left(\frac{1-y_0}{y_0}\right)
\end{aligned}
$$

This finishes the caculation of the C_0 integral. In our case we assume that all masses and momenta squared are real so all η functions disappear and just dilogaritmic functions are left.

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Declaration

Ich versichere, dass ich diese Arbeit selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

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Karin Firnkes