

Ultracold gases and Functional renormalization

II

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Work in collaboration with

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The BCS-BEC Crossover

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- Stefan: Basics of FRG for ultracold fermions, Thermodynamics, BEC-side
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- Stefan: Basics of FRG for ultracold fermions, Thermodynamics, BEC-side
- Today: BCS-side (Particle-hole fluctuations, Rebozonization,...), Unitarity regime
Crossover can be parametrized by the dimensionless inverse s-wave scattering length $c^{-1} \rightarrow$. Experimental realization by the phenomenon of Fesbach resonances.
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**Parametrization**

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- $c^{-1} < -1$: Weakly attractive regime $\rightarrow$ Cooper pairing below $T_c \rightarrow$ BCS superfluidity
- $c^{-1} > 1$: Two-body bound state exists $\rightarrow$ Formation of molecules $\rightarrow$ below $T_c$: BEC
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- $c^{-1} < -1$: Weakly attractive regime → Cooper pairing below $T_c$ → BCS superfluidity
- $c^{-1} > 1$: Two-body bound state exists → Formation of molecules → below $T_c$: BEC
- $|c^{-1}| < 1$: Strongly correlated regime, Unitarity limit at $c^{-1} \rightarrow 0$
Universality

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- Limit of broad Feshbach resonances (experiments, e.g. with $^6\text{Li}$ and $^{40}\text{K}$)
- Universality: Thermodynamic quantities are independent of the microscopic details and can be expressed in terms of only two dimensionless parameters: The concentration $c = a k_F$ and the temperature $T/T_F$. The units are set by the density $n = k_F^3/(3\pi^2)$. 

\[ \begin{align*}
\text{h} \\
\end{align*} \]
**Microscopic action**

\[
S = \frac{1}{T} \int_0^T d\tau \int d^4x \left\{ \psi^\dagger (\partial_\tau - \Delta - \mu) \psi + \phi^*(\partial_\tau - \frac{\Delta}{2} - 2\mu + \nu) \phi \\
- h(\phi^* \psi_1 \psi_2 + h.c.) \right\}.
\]

- Grassmann field \( \psi = (\psi_1, \psi_2) \), fermions in two hyperfine states
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- detuning from the Feshbach resonance \( \nu = \mu(B - B_0) \)
Inverse Hubbard-Stratonovic Transformation I

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where \(\lambda_{\psi,\text{eff}}\) is a momentum-dependent effective four-fermion vertex.
Inverse Hubbard-Stratonovic Transformation II

In momentum space the effective four-fermion vertex reads

\[ \lambda_{\psi, \text{eff}} = -\frac{\hbar^2}{P_{\phi}(q)}, \]
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The inverse process, going from a purely fermionic theory to a theory of fermions and bosons, is called "bosonization".

![Diagram](image-url)
Momentum Dependent Four-Fermion Interaction

\[ S = \int_0^{1/T} d\tau \int_{\vec{x}} \psi^\dagger (\partial_\tau - \Delta - \mu) \psi + \frac{\lambda_{\psi,\text{eff}}}{2} (\psi^\dagger \psi)^2 , \]
Momentum Dependent Four-Fermion Interaction

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S = \frac{1}{T} \int_0^\infty d\tau \int \psi^\dagger (\partial_\tau - \Delta - \mu) \psi + \frac{\lambda_{\psi,\text{eff}}}{2} (\psi^\dagger \psi)^2 ,
\]

A generally momentum dependent four-fermion interaction is renormalized. The flow of \( \lambda_\psi \) has two contributions:

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\partial_t \lambda_\psi = \tilde{\partial}_t + \tilde{\partial}_t
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The first one is referred to as particle-particle loop, the second one as particle-hole loop.
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BCS-theory in RG language I

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BCS theory only considers the particle-particle fluctuations (first loop)  
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Valid for weak attractive interactions, finite density ($\sigma > 0$), small temperatures $T$  
Pairing of fermions is indicated by $1/\lambda_\psi \to 0$  
The pp-loop effect increases as $T$ is decreased  
The temperature at which $1/\lambda_\psi \to 0$ at the scale $k = 0$ is the BCS transition temperature $T_{c,\text{BCS}}$
Within BCS theory the outer momenta are averaged over the Fermi surface and the critical temperature is found to be

$$\frac{T_{c,\text{BCS}}}{T_F} \approx 0.61 e^{-\frac{\pi}{2k_F} |a^{-1}|}.$$ 

Here $a$ is the (vacuum) s-wave scattering length.
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Here $a$ is the (vacuum) s-wave scattering length. For $k \to 0$, $\mu \to 0$, $T \to 0$, $\nu \to 0$:

$$a = \frac{\lambda_\psi}{8\pi}$$
Gorkov’s correction to BCS-theory I

- pp-loop diverges for $T \to 0$ leading to a transition to superfluidity, ph-loop remains finite

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- Evaluate ph-loop for $T = 0$ and add it to initial value

$$\left(\lambda_{\psi,\Lambda}^{\text{eff}}\right)^{-1} = \lambda_{\psi}(k = \Lambda)^{-1} + \text{ph-loop}, \quad (2)$$
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- $T_c$ depends exponentially on the "microscopic effective coupling"
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- $T_c$ depends exponentially on the ”microscopic effective coupling”

$\Rightarrow$ Any shift in $\left(\lambda_{\psi,\Lambda}^{\text{eff}}\right)^{-1}$ results in a multiplicative factor for $T_c$.  

Gorkov’s correction to BCS-theory II

Screening of the interaction between two fermions by the particle-hole fluctuations is a quantitative effect and lowers the critical temperature as compared to BCS theory by a multiplicative factor.
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$$T_c = \frac{1}{(4e)^{1/3}} T_{c,\text{BCS}} \approx \frac{1}{2.2} T_{c,\text{BCS}}.$$ 

This is the Gorkov effect (1963).
Bosonization

In a bosonized language, the fermionic interaction is described by boson exchange

\[ \partial_t \overset{\sim}{=} \tilde{\partial}_t + \ldots \]
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The phase transition to the superfluid phase is indicated by the vanishing of the bosonic mass term \( m^2 = 0 \) (SSB)
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In this setting, where the bosonization took place only on the microscopic scale, we do not account for particle-hole fluctuations.
Bosonization is destroyed by the RG flow

We neglected so far, that the term

$$\int_{\tau, \vec{x}} \lambda \psi \psi_1 \psi_1 \psi_2 \psi_2,$$

is re-generated by the flow.
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$\lambda_\psi$ contributes to the effective interaction between fermions

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→ Connection between divergence of \( \lambda_{\psi, \text{eff}} \) and SSB?
Rebosonization I

Idea:

- Bosonize at microscopic scale with a field $\phi_\Lambda$, $\Rightarrow \lambda_\psi,\Lambda = 0$
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(Re-)appearance of a $\lambda_{\psi}$ by the flow of the box diagrams can be absorbed by the introduction of scale dependent fields $\phi_k$
Rebosonization II

For scale dependent fields we obtain a modified flow equation (Gies & Wetterich, 2001)

\[ \partial_k \Gamma_k [\chi_k] = \frac{1}{2} \text{STr} \left[ \left( \Gamma_k^{(2)} + R_k \right)^{-1} \partial_k R_k \right] + \int \frac{\delta \Gamma_k}{\delta \chi_k} \partial_k \chi_k. \]
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The $k$-dependence can be chosen arbitrarily.

We choose the following scale dependence for the bosonic fields.

\[ \partial_k \bar{\phi}_k(q) = (\psi_1 \psi_2)(q) \partial_k \nu. \]

With $\partial_k \nu$ to be determined.
Rebosonization III

In consequence the flow equations in SYM get modified

\[ \partial_k \bar{h} = \partial_k \bar{h} \bigg|_{\bar{\phi}_k} - \bar{m}^2 \partial_k \nu, \]
\[ \partial_k \lambda_\psi = \partial_k \lambda_\psi \bigg|_{\bar{\phi}_k} - 2 \bar{h} \partial_k \nu. \]
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We can choose \( \partial_k \nu \) such that the flow of \( \lambda_\psi \) vanishes. Then we have \( \lambda_\psi = 0 \) on all scales.
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\[ \Rightarrow \]

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\[
\Rightarrow
\]

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\]

Now, the four-fermion interaction is purely given by the boson exchange and ph-fluctuations are incorporated via the second term in the latter equation.
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We can determine the transition temperature (critical temperature $T_c$) from normal fluidity to superfluidity in this system:
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- Microphysics does not depend on temperature.
- Start the flow in the UV at defined $T$ and look in the IR if it ends up in the symmetric phase or the spontaneously broken phase.

- The temperature for which $m_0^2 \to 0$ as $k \to 0$ is $T_c$. This is directly related to the divergence of the effective four-fermion coupling

$$\lambda_{\psi,\text{eff}} \propto \frac{-\hbar^2}{m^2} \to \infty.$$
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We cannot investigate $\Gamma_k$ in its full generality but we have to truncate it, i.e. choose a suitable approximation, where we allow for the RG running of all parameters.
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$$\Gamma_k[\chi] = \int_{\tau,\vec{x}} \left\{ \psi^\dagger (\partial_\tau - \Delta - \mu) \psi + \phi^* (Z_{\phi,k} \partial_\tau - \frac{\Delta}{2}) \phi \right\} + U_k(\rho, \mu) - h_k (\phi^* \psi_1 \psi_2 + h.c.) \right\}$$
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\]

For the effective potential, we use an expansion around the $k$-dependent location of the minimum $\rho_0(k)$.

\[
U_k(\rho, \mu) = m_k^2 (\rho - \rho_0) + \frac{1}{2} \lambda_k (\rho - \rho_0)^2
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We cannot investigate $\Gamma_k$ in its full generality but we have to truncate it, i.e. choose a suitable approximation, where we allow for the RG running of all parameters.

\[
\Gamma_k[\chi] = \int \left\{ \psi^\dagger (\partial_\tau - \Delta - \mu) \psi + \phi^*(Z_{\phi,k} \partial_\tau - \frac{\Delta}{2}) \phi 
+ U_k(\rho, \mu) - h_k (\phi^* \psi_1 \psi_2 + h.c.) \right\}
\]

For the effective potential, we use an expansion around the $k$-dependent location of the minimum $\rho_0(k)$.

\[
U_k(\rho, \mu) = m_k^2 (\rho - \rho_0) + \frac{1}{2} \lambda_k (\rho - \rho_0)^2 
+ U(\rho_0, \mu_0) - n_k (\mu - \mu_0) + \alpha_k (\mu - \mu_0)(\rho - \rho_0)
\]
Critical Temperature
Thanks for your attention
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Questions?