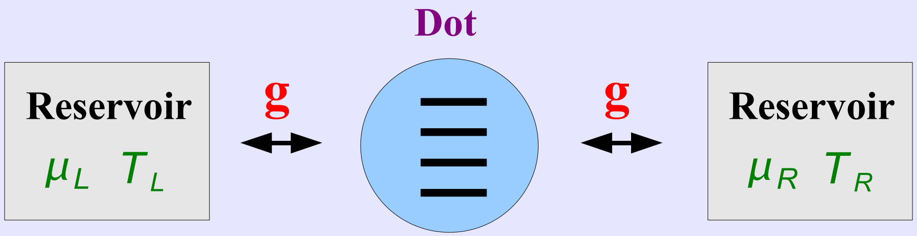
Real-time RG in frequency space: A perturbative nonequilibrium renormalization group method for dissipative quantum mechanics

H. Schoeller (RWTH Aachen)

RG in Noneq. I :



dissipative
quantum mechanics
expand in g

→ **Real-Time RG in frequency space (RTRG-FS)**

→ **Based on:**

- *H.S., König, PRL '00*
- *Korb, Reininghaus, H.S., König, PRB '07*
- *H.S., Eur. Phys. J. Special Topics 168, 179 (2009)*

technical improvements

- *H.S., Reininghaus, accep. by PRB (cond-mat/0902.1446)*
- *Schuricht, H.S., subm. to PRB (cond-mat/0905.3095)*
- *Pletyukhov, Schuricht, H.S., in preparation*

Nonequilibrium Kondo model

RG in Noneq. II :

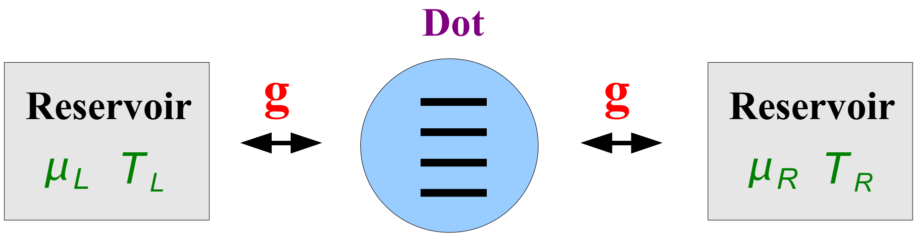
→ **Bulk systems:** $H = H_0 + V \quad , \quad H_0 \sim c^\dagger c$

→ **Functional RG (e.g. Wetterich) + Keldysh**

→ **Based on:**

- *Jakobs, diploma thesis '03*
- *Jakobs, Meden, H.S., PRL '07*
- *Gezzi, Meden, Pruschke, PRB '07*
- *Jakobs, Pletyukhov, H.S., in preparation*

Outline



1. lecture

- I. Motivation
- II. Example: Kondo model
- III. Quantum field theory in Liouville space

2. lecture

- IV. Renormalization group
- V. Analytic solution in weak coupling: 1-loop + 2-loop (generic)

3. lecture

- VI. The nonequilibrium anisotropic Kondo model at finite magnetic field
- VII. Outlook for strong coupling

Correlation functions, t-dependent evolution → talk by D. Schuricht

I. Motivation

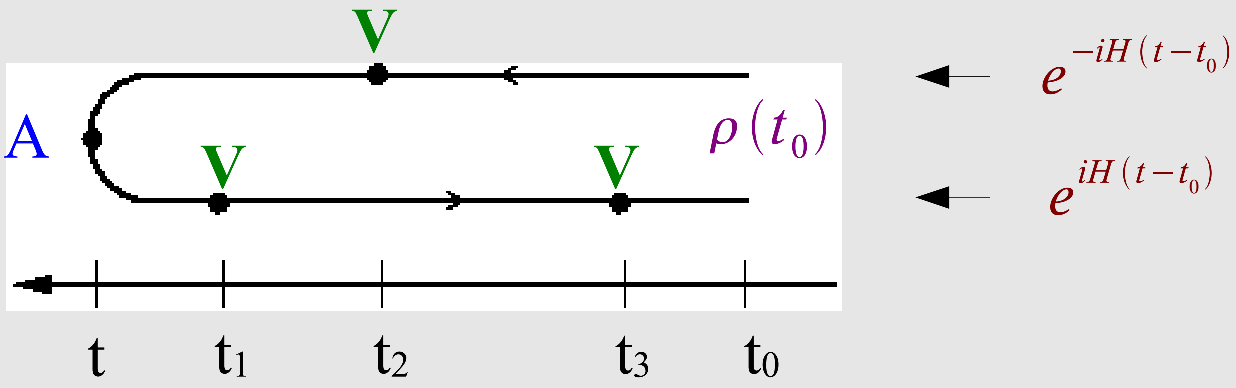
General aim :

- **System:** $H = H_0 + V$
- **Problem:** $t = t_0 : \quad \rho(t_0) = f(H_0)$ initial density matrix

  $\langle A(t) \rangle = Tr\, e^{iH(t-t_0)} A\, e^{-iH(t-t_0)} \rho(t_0) = ?$
- **Method:** Perturbative RG in V

New aspects :

Keldysh contour :



$\leftarrow e^{-iH(t-t_0)}$

$\leftarrow e^{iH(t-t_0)}$

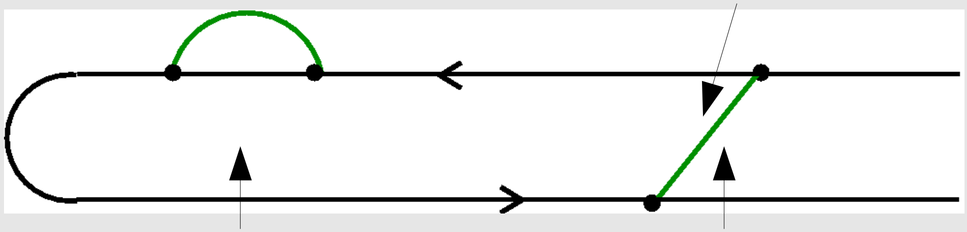
Time evolution : $\langle A(t) \rangle \sim A_{st} + A_1\, e^{iht} e^{-\Gamma t} + A_2 \left(\frac{1}{\Gamma t}\right)^{\alpha}$

nonequilibrium stationary value

exponential decay

power law decay

Keldysh structure :



reservoir contractions

dynamics in Hilbert space
quantum decay theory

dynamics in Liouville space
transport theory
relaxation and dephasing

$$\langle A(t)\rangle \sim A_{st} + A_1 e^{iht} e^{-\Gamma t} + A_2 \left(\frac{1}{\Gamma t}\right)^{\alpha}$$

→ **influenced by all diagrams!**

$$h_{\substack{decay\\theory}} \neq h_{\substack{transport\\theory}} \qquad \Gamma_{\substack{decay\\theory}} \neq \Gamma_{\substack{transport\\theory}}$$

Conventional poor man scaling approaches in nonequilibrium:

Kaminski, Nazarov, Glazman
Glazman, Pustilnik
Rosch, Paaske, Kroha, Woelfle
etc.

→ **RG only on one part of the Keldysh contour**
→ **Γ put in by hand into the RG**

RG + relaxation/dephasing:

$D = band\ width \qquad \Lambda_c = max\{V, h_0, \ldots\}$

$$J_0^k \ln^l \frac{D}{|nV - m\,h_0|} \xrightarrow[\textbf{RG}]{} J_c^k \ln^l \frac{\Lambda_c}{|nV - m\,h + i\,\Gamma|} \xrightarrow{nV = mh} J_c^k \ln^l J_c$$

bare coupling

voltage

bare magnetic field

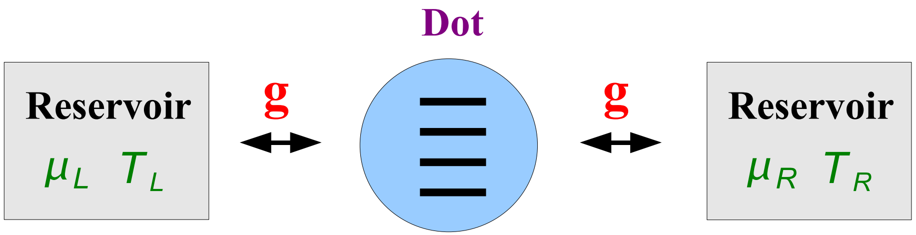
renormalized coupling
$J_c \sim \ln^{-1}(\Lambda_c / T_K)$

renormalized magnetic field

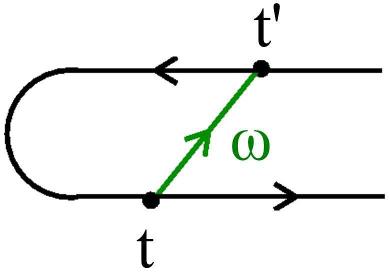
$\Gamma \sim \Lambda_c J_c^2$
renormalized relaxation/dephasing rate

Choice of cutoff function :

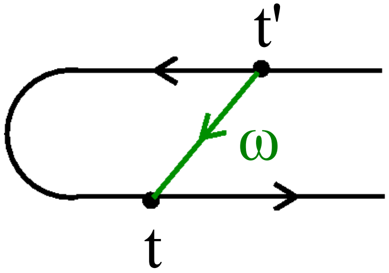
→ depends on model + diagrammatic expansion + RG formalism
→ choose such that perturbative RG is well-defined



expand in g
integrate out reservoirs



$$e^{i\omega(t-t')}f(\omega)$$



$$e^{-i\omega(t-t')}(1-f(\omega))$$

Cutoff in t-t' : $e^{i\omega(t-t')}f(\omega)\theta(|t-t'|-\frac{1}{\Lambda})$

H.S., König, '00

Cutoff in ω : $e^{i\omega(t-t')}f(\omega)\theta(\Lambda-|\omega|)$

Korb, Reininghaus,
H.S., König. '07

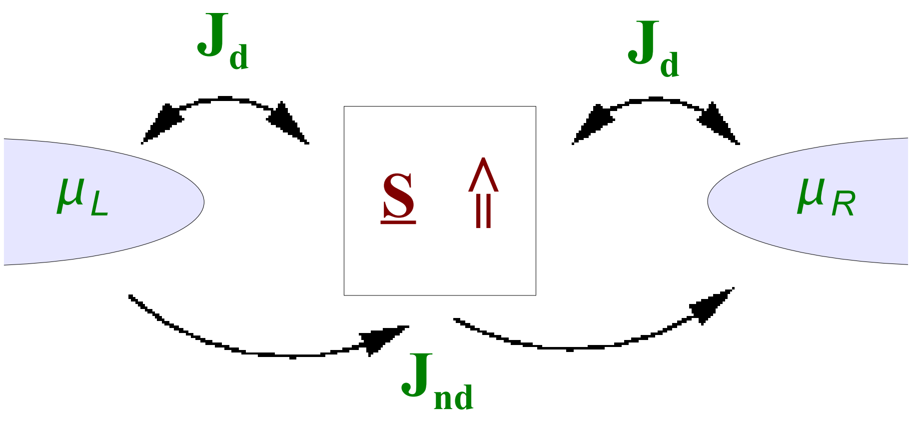
Cutoff in f(ω) : $e^{i\omega(t-t')}f_\Lambda(\omega)$

Jakobs, Meden, H.S., '07
H.S., '08
H.S., Reininghaus, '09

$$f_\Lambda(\omega) = \frac{1}{\beta}\sum_{n,\ |\omega_n|<\Lambda}\frac{e^{i\omega_n\eta}}{i\omega_n - \omega}$$

II. Example: Nonequilibrium Kondo model

isotropic
h=T=0
finite voltage V



$$H = H_{res} + V_{res \leftrightarrow dot}$$

$$H_{res} = \sum_{\alpha\sigma} \int_{-D}^{D} d\omega \ (\omega+\mu_\alpha) \ a^\dagger_{\alpha\sigma}(\omega) a_{\alpha\sigma}(\omega)$$

$$V = \sum_{\alpha\alpha'\sigma\sigma'} \int_{-D}^{D} d\omega \int_{-D}^{D} d\omega' \frac{1}{2} J_{\alpha\alpha'} \underline{S} \ \underline{\sigma}_{\sigma\sigma'} \ :a^\dagger_{\alpha\sigma}(\omega) a_{\alpha'\sigma'}(\omega'):$$

Poor man scaling :

$$J = J_d = J_{nd}$$

$$\boxed{\frac{dJ_\Lambda}{d\Lambda} = -\frac{2J_\Lambda^2}{\Lambda} \qquad J_\Lambda = \frac{1}{2 \ \ln(\Lambda/T_K)}} \rightarrow \infty$$

$$\Lambda \rightarrow T_K$$

$$T_K = D \ e^{-1/2J_0}$$
$$= \Lambda \ e^{-1/2J_\Lambda}$$

Cutoff scales :

$$\boxed{V, \ \Gamma = \pi V J^2_{\Lambda=V}}$$

$$V \gg T_K \Rightarrow \Gamma \gg T_K = V e^{-1/2J_{\Lambda=V}}$$ ***weak coupling***

Coleman et al.
Rosch et al.

Kaminski, Nazarov, Glazman
Rosch et al., Kehrein

Current :

$$\boxed{I = \frac{e^2}{h}\frac{3\pi^2}{2} V J^2_{\Lambda=V} = \frac{e^2}{h}\frac{3\pi^2}{8}\frac{V}{\ln^2(V/T_K)}}$$

cut off at V !

Finite magnetic field h :

+ other terms ~ $\boxed{(V-h) J^3_{\Lambda=V} \ln \frac{V}{|V-h+i\Gamma|}}$

Γ visible !

Result of RTRG:

E → Laplace variable
frequency-dependence not indicated

$$\frac{dJ}{d\Lambda}=-\frac{2J^2}{\Lambda}$$

Γ needed, otherwise J_d , $J_{nd} \rightarrow \infty$ for $\Lambda \rightarrow T_K$

$$\frac{d}{d\Lambda}J_d(E) = -\frac{1}{\Lambda+\Gamma(E)+ih(E)-iE}\,J_d(E)^2 - \frac{1}{2}\sum_{\pm}\frac{1}{\Lambda+\Gamma(E\pm V)+ih(E\pm V)-i(E\pm V)}\,J_{nd}(\pm E)^2$$

$$\frac{d}{d\Lambda}J_{nd}(E) = -\frac{1}{\Lambda+\Gamma(E)+ih(E)-iE}\,J_d(E)J_{nd}(E) - \frac{1}{\Lambda+\Gamma(E+V)+ih(E+V)-i(E+V)}\,J_d(E+V)J_{nd}(E)$$

RG for current rates are cut off at $\Lambda = V >> \Gamma$

$$\frac{d}{d\Lambda}I = -12\pi^2\Im\left\{\ln\left(\frac{2\Lambda+\Gamma(V)+ih(V)-iV}{\Lambda+\Gamma(V)+ih(V)-iV}\right)\right\}\,J_l\;K_{LR}$$

$$I = \frac{e^2}{h}\frac{3\pi^2}{2}V J^2_{\Lambda=V}$$

(Γ not visible)

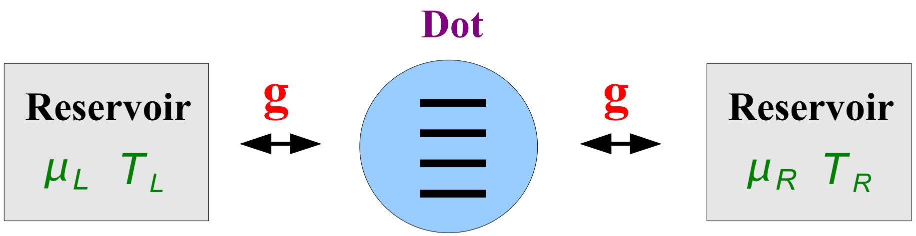
Laplace variable E included => time-dependence can be calculated

$$L_D^{eff}(E) = (h(E)-i\Gamma(E))\,\frac{1}{2}\left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & \uparrow\uparrow \\ -1 & 1 & 0 & 0 & \downarrow\downarrow \\ 0 & 0 & 2 & 0 & \uparrow\downarrow \\ 0 & 0 & 0 & 2 & \downarrow\uparrow \end{array}\right)$$

$$\tilde{\rho}_D(z) = \frac{i}{z-L_D^{eff}(z)}\,\rho_D(t_0)$$

III. Quantum field theory in Liouville space

Generic model :



Dot

Reservoir $\mu_L \; T_L$

g

g

Reservoir $\mu_R \; T_R$

$$H = H_{res} + H_{dot} + V$$

reservoirs *dot* *interaction dot ↔ reservoirs*

$$H_{res} = \sum_\mu \int d\omega \ (\omega+\mu_\alpha) \ a^\dagger_\mu(\omega) a_\mu(\omega)$$

$$H_{dot} = \sum_s E_s |s\rangle\langle s|$$

$$V = \frac{1}{n!} \, g_{12\ldots n} \, :a_1 a_2 \ldots a_n:$$

dot operator
→ interaction vertex

field operators
of the reservoirs

$\mu = \alpha \, \sigma \, n \ldots.$

α → reservoir index
σ → spin index
n → channel index

E_s → can contain strong interaction!

$1 \equiv \eta \, \mu \, \omega$

η = ± → creation/annihilation operator
μ → α σ n ....
ω → frequency

Note: **generic form necessary since RG generates this form!**

Basic idea:

$$\rho_D(t) = Tr_{res}\,\rho(t)$$

reduced dot density matrix

Isolated dot:

$$\dot{\rho}_D(t) \;=\; -i[H_D, \rho_D(t)] \;=\; -i\,L_D\,\rho_D(t)$$

$$\rho_D(t) \;=\; e^{-i L_D(t-t_0)}\rho_D(t_0)$$

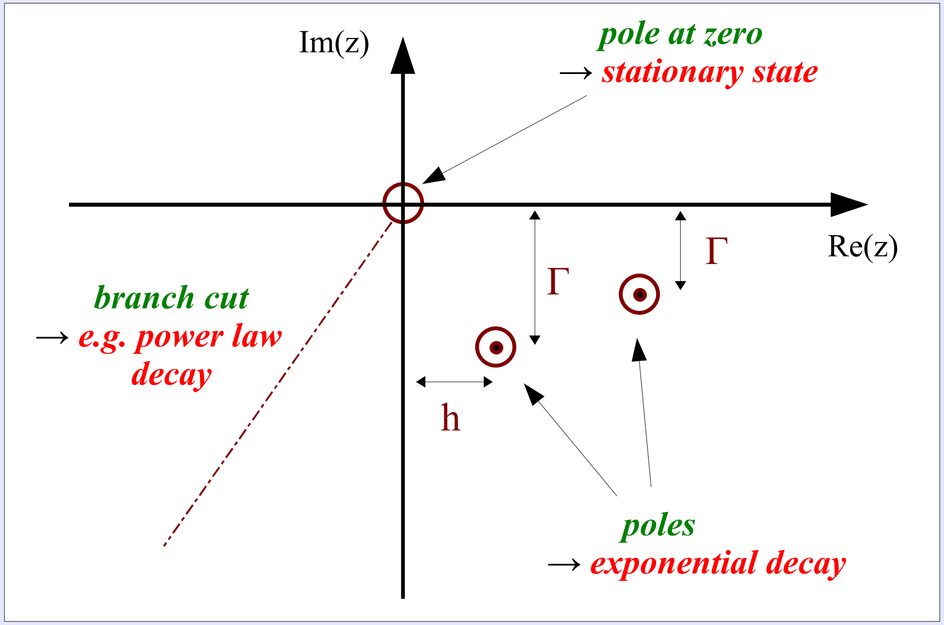
$$\tilde{\rho}_D(z) \;=\; \int_{t_0}^{\infty} dt\, e^{iz(t-t_0)}\rho_D(t) \;=\; \frac{i}{z-L_D}\rho_D(t_0)$$

Dot + reservoirs:

$$\tilde{\rho}_D(z) \;=\; \frac{i}{z-L_D^{eff}(z)}\rho_D(t_0)$$

$$L_D^{eff}(i\eta)\,\rho_D^{st} \;=\; 0$$

stationary solution



- **formulate RG for** $L_D^{eff}(z)$
- **result:**

  h, Γ → cutoff scales for vertices

- **problem: zero eigenvalue**

  → does not occur in RG for suitable cutoff function

Dynamics of the dot density matrix :

$$\tilde{\rho}_D(z) = \int_{t_0}^{\infty} dt\, e^{iz(t-t_0)}\; Tr_{res}\; e^{-iL(t-t_0)}\; \rho(t_0) = Tr_{res}\, \frac{i}{z-L_{res}-L_D-L_V}\; \rho_D(t_0) \prod_{\alpha} \rho_{res}^{eq}(\mu_\alpha, T_\alpha)$$

grandcanonical distribution

$$L_{res} = [H_{res}, \bullet] \qquad L_D = [H_D, \bullet]$$

$$L_V = [V, \bullet] = \frac{1}{n!}\, \sigma^{p_1..p_n}\; G_{1...n}^{p_1..p_n}\; :A_1^{p_1} \ldots A_n^{p_n}:$$

p_i → Keldysh indices

$$V = \frac{1}{n!}\, g_{1...n}\; :a_{1.}..a_n:$$

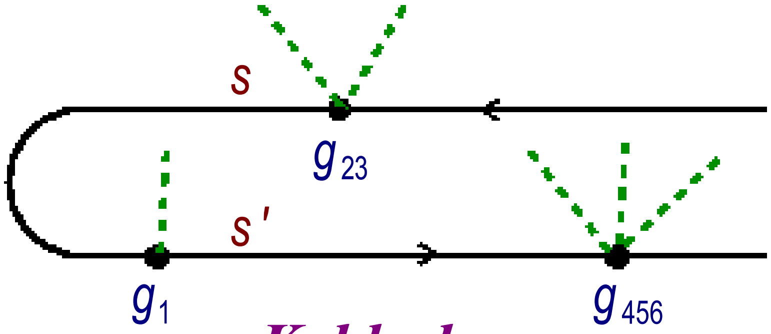
sign operator

dot Liouville operator

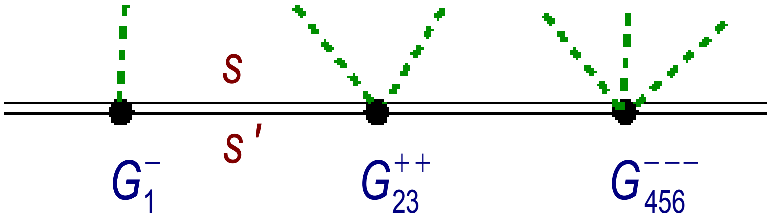
reservoir Liouville field operators

$$A_1^p = \begin{Bmatrix} a_1 \;\bullet & for\ p=+ \\ \bullet\; a_1 & for\ p=- \end{Bmatrix}$$

$$G_{1...n}^{pp...p} = \begin{Bmatrix} 1 & for\ \ n\ \ even \\ \sigma^p & for\ \ n\ \ odd \end{Bmatrix} \begin{Bmatrix} g_{1...n}\;\bullet & for\ p=+ \\ -\;\bullet\; g_{1...n} & for\ p=- \end{Bmatrix}$$



Keldysh



Liouville space

Expand in L_V and integrate out reservoirs :

$$\tilde{\rho}_D(z) = i \; Tr_{res} \; \frac{1}{z-L_{res}-L_D-L_V} \; \rho_D(t_0) \; \prod_\alpha \rho_{res}^{eq}(\mu_\alpha, T_\alpha)$$

$$\rightarrow i \; Tr_{res} \; \frac{1}{z-L_{res}-L_D} \; L_V \; \frac{1}{z-L_{res}-L_D} \; L_V \dots . \; L_V \; \frac{1}{z-L_{res}-L_D} \rho_D(t_0) \; \prod_\alpha \rho_{res}^{eq}(\mu_\alpha, T_\alpha)$$

$$\frac{1}{n!} \; \sigma^{p_1 .. p_n} \quad G_{1..n}^{p_1..p_n} \quad :A_1^{p_1} \dots A_n^{p_n}:$$

Shift all reservoir field operators to the right by using:

$$A_1^p \;\; L_{res} \; = \; (L_{res} - \bar{\mu}_1 - \bar{\omega}_1) \;\; A_1^p \qquad \bar{\mu}_1 \; = \; \eta_1 \mu_1 \qquad \bar{\omega}_1 \; = \; \eta_1 \omega_1$$

Integrate out reservoirs by using Wick's theorem and

$$Tr_{res} L_{res} \; = \; 0$$

$$L_{res} \rho_{res}^{eq} \; = \; 0$$

$$\boxed{\tilde{\rho}_D(z) \; \rightarrow \; \frac{i}{S} \; (\pm)^{N_p} \; \left(\prod \gamma\right) \; \frac{1}{z-L_D} \; G \; \frac{1}{z+X_1-L_D} \; G \dots . \; \frac{1}{z+X_r-L_D} \; G \; \frac{1}{z-L_D} \rho_D(t_0)}$$

symmetry factor

fermionic sign factor

reservoir contractions

chemical potentials + frequencies

Diagrammatic rules :

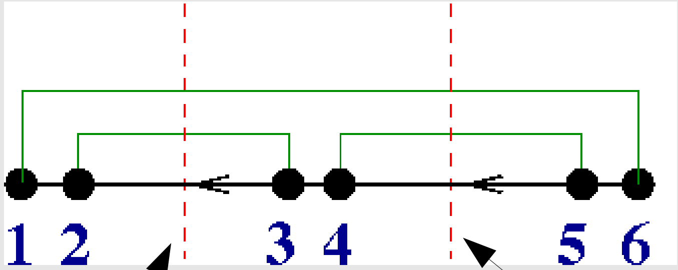
$$\tilde{\rho}_D(z) \;\rightarrow\; \frac{i}{S}\;(\pm)^{N_p}\;\left(\prod\gamma\right)\;\frac{1}{z-L_D}\;G\;\frac{1}{z+X_1-L_D}\;G\,\ldots\;\frac{1}{z+X_r-L_D}\;G\;\frac{1}{z-L_D}\rho_D(t_0)$$

symmetry factor

fermionic sign factor

reservoir contractions

chemical potentials + frequencies



$X = \bar{\mu}_1+\bar{\mu}_2+\bar{\omega}_1+\bar{\omega}_2$

$X = \bar{\mu}_1+\bar{\mu}_4+\bar{\omega}_1+\bar{\omega}_4$

$$\longrightarrow\; i\,\gamma_{16}^{p_1 p_6}\gamma_{23}^{p_2 p_3}\gamma_{45}^{p_4 p_5}\frac{1}{z-L_D}G_{12}^{p_1 p_2}\Pi_{12}G_{34}^{p_3 p_4}\Pi_{14}G_{56}^{p_5 p_6}\frac{1}{z-L_D}\rho_D(t_0)$$

$$\Pi_{ij} \;=\; \frac{1}{z+\bar{\mu}_i+\bar{\mu}_j+\bar{\omega}_i+\bar{\omega}_j-L_D}$$

$$\bar{\mu}_i \;=\; \eta_i\mu_i$$

$$\bar{\omega}_i \;=\; \eta_i\omega_i$$

Reservoir contraction :

$$\gamma_{11'}^{pp'} \;=\; \delta_{\eta,-\eta'}\delta_{\mu\mu'}\delta(\omega-\omega')\;\;p'\left\{\begin{matrix}\eta\\1\end{matrix}\right\}\frac{D^2}{D^2+\omega^2}\;\;f(\eta p'\omega)$$

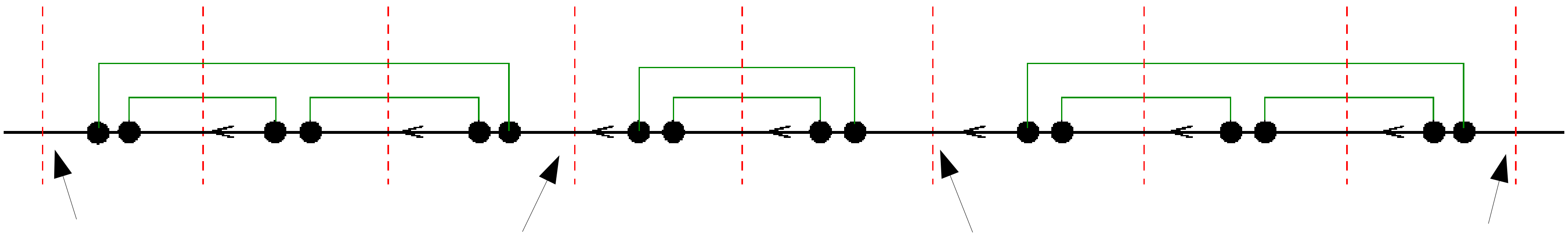
contains the explicit dependence on the Keldyh indices !!!

band width cutoff

Fermi/Bose - function

$$f(-\omega)=\mp(1\pm f(\omega))$$

Effective dot Liouville operator :



$$\frac{1}{z-L_D} \qquad \Sigma(z) \qquad \frac{1}{z-L_D} \qquad \Sigma(z) \qquad \frac{1}{z-L_D} \qquad \Sigma(z) \qquad \frac{1}{z-L_D}$$

$$\tilde{\rho}_D(z) = \frac{i}{z-L_D-\Sigma(z)}\rho_D(t_0) = \frac{i}{z-L_D^{eff}(z)}\rho_D(t_0)$$

$$L_D^{eff}(z) = L_D + \Sigma(z)$$

$\Sigma(z) \rightarrow$ **sum over all irreducible diagrams contains relaxation/dephasing**

irreducible

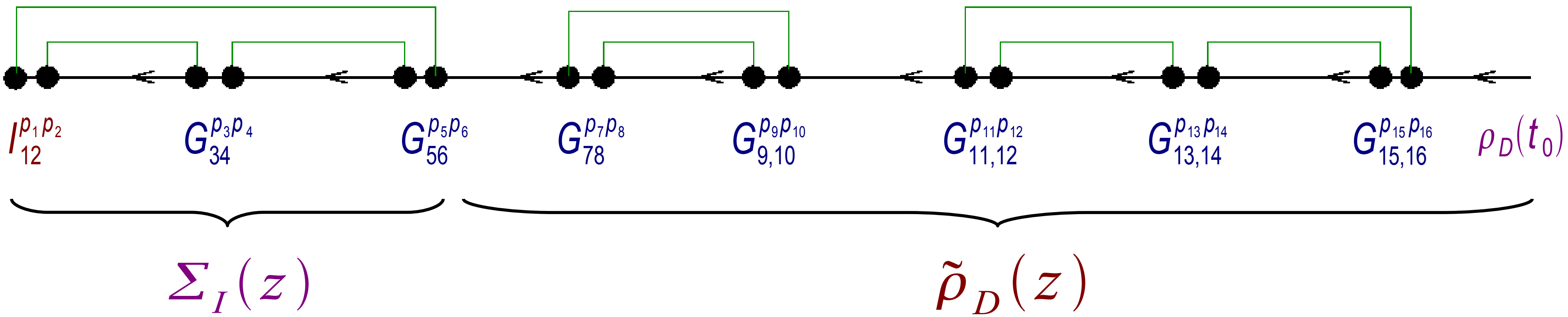
$$\Sigma(z) \rightarrow \frac{1}{S}(\pm)^{N_p}\left(\prod \gamma\right)_{irr} G \, \frac{1}{z+X_1-L_D} \, G \dots \frac{1}{z+X_r-L_D} \, G$$

Time space :

$$\frac{d}{dt}\rho_D(t) + i[L_D, \rho_D(t)] = -i\int_{t_0}^{t} dt'\, \Sigma(t-t')\rho_D(t')$$

$$\tilde{\rho}_D(z) = \int_{t_0}^{\infty} dt\, e^{iz(t-t_0)}\rho_D(t) \qquad \Sigma(z) = \int_0^{\infty} dt\, e^{izt}\,\Sigma(t)$$

Observables (e.g. current):



$$\langle I\rangle(z) \quad = \quad -i \;\; Tr_D \;\; \Sigma_I(z)\tilde{\rho}_D(z) \quad = \quad Tr_D \;\; \Sigma_I(z)\frac{1}{z-L_D(z)-\Sigma(z)}\rho_D(t_0)$$

$$\tilde{\rho}_D(z) \quad = \quad \frac{i}{z-L_D-\Sigma(z)}\rho_D(t_0) \quad = \quad \frac{i}{z-L_D^{eff}(z)}\rho_D(t_0)$$

$$\begin{Bmatrix}\Sigma_I(z)\\ \Sigma(z)\end{Bmatrix} \;\to\; \frac{1}{S}\;(\pm)^{N_p}\;\left(\prod\gamma\right)_{irr}\begin{Bmatrix}I\\ G\end{Bmatrix}\;\frac{1}{z+X_1-L_D}\;G\ldots.\;\frac{1}{z+X_r-L_D}\;G$$

irreducible

IV. Renormalization group in Liouville space

Fermions

RG step one (discrete):

- Keldysh indices no longer appear
- zero eigenvalue no longer appears
- perturbative treatment

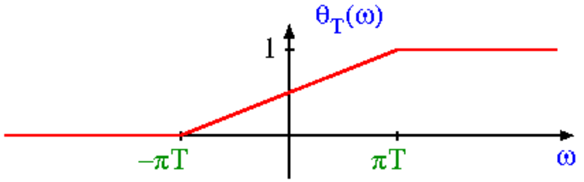
$$f(\omega) = \underset{\text{integrate out}}{\underset{\uparrow}{\frac{1}{2}}} + \underset{\text{antisymmetric}}{\underset{\uparrow}{f(\omega) - \frac{1}{2}}}$$

RG step two (continuous):

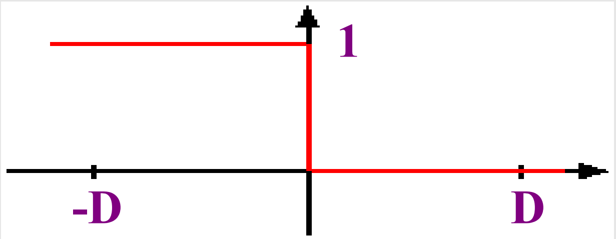
integrate out logarithmic divergencies

cutoff on imaginary frequency axis ***(Jakobs, Meden, H.S., PRL '07)***

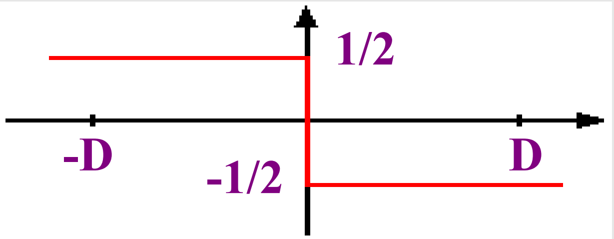
$$f(\omega) - \frac{1}{2} \quad \rightarrow \quad \boxed{f_\Lambda(\omega) = \frac{1}{\beta} \sum_n \frac{\theta_T(\Lambda - |\omega_n|)}{i\omega_n - \omega}}$$



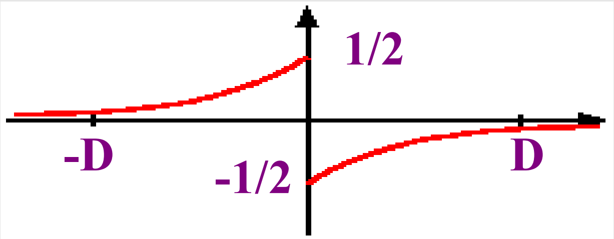
T=0 :



$$f(\omega) = \theta(-\omega)$$



$$f(\omega) - \frac{1}{2}$$



$$f_\Lambda(\omega) \sim -\frac{1}{\pi} \arctan(\Lambda / \omega)$$

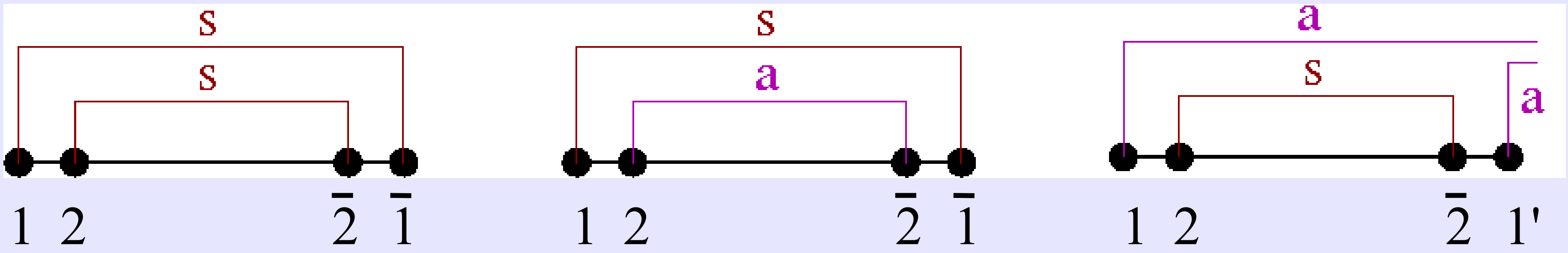
RG step one (discrete):

integrate out

$$\gamma^{pp'}_{11'} = \delta_{\eta,-\eta'}\delta_{\mu\mu'}\delta(\omega-\omega')\; p'\frac{D^2}{D^2+\omega^2}\; f(\eta p'\omega) \;=\; \delta_{1\bar{1}'}p'\gamma^s_1 + \delta_{1\bar{1}'}\gamma^a_1$$

$$\gamma^s_1 = \frac{1}{2}\,\frac{D^2}{D^2+\omega^2} \qquad \gamma^a_1 = \frac{D^2}{D^2+\omega^2}\left(f(\bar{\omega})-\frac{1}{2}\right) \qquad \bar{\omega} = \eta\omega$$

Resum diagrams which are irreducible with respect to γ^s_1
→ well-defined perturbation theory !



$$\frac{1}{2}\gamma^s_1\gamma^s_2 G^{pp}_{12}\Pi_{12}G^{p'p'}_{\bar{2}\bar{1}} \qquad p'\gamma^s_1\gamma^a_2 G^{pp}_{12}\Pi_{12}G^{p'p'}_{\bar{2}\bar{1}} \qquad p'\gamma^s_2 G^{pp}_{12}\Pi_{12}G^{p'p'}_{\bar{2}1'} - (1\leftrightarrow 1')$$

$$L^a_D(z) \qquad (G^a)^{pp'}_{11'}(z)$$

New diagrammatic expansion with effective quantities:

$$\begin{Bmatrix} \Sigma_I(z) \\ \Sigma(z) \end{Bmatrix} \rightarrow \frac{1}{S} \; (\pm)^{N_p} \; \left(\prod \gamma^a\right)_{irr} \begin{Bmatrix} \bar{I}^a(z) \\ \bar{G}^a(z) \end{Bmatrix} \times$$

$$\times \; \frac{1}{z+X_1-L_D^a(z+X_1)} \; \bar{G}^a(z+X_1) \dots . \; \frac{1}{z+X_r-L_D^a(z+X_r)} \; \bar{G}^a(z+X_r)$$

$\gamma_1^a = \frac{D^2}{D^2+\omega^2} \left(f(\bar{\omega})-\frac{1}{2}\right)$ independent of Keldysh indices

=> only $\bar{G}^a_{1\dots n} = \sum_{p_1..p_n} (G^a)^{p_1..p_n}_{1\dots n}$ occurs => no Keldysh indices anymore!!

Conservation of probability: $Tr_D L_D^a(z) = Tr_D L_D^{eff}(z) = Tr_D \bar{G}^a_{1\dots n} = 0$

$\langle 0|ss'\rangle = \delta_{ss'}$ => $\langle 0|L_D^a(z) = \langle 0|L_D^{eff}(z) = \langle 0|\bar{G}^a_{1\dots n} = 0$

=> **the eigenvalue zero can never occur in the resolvents !!**

RG step two (continuous):

$$\gamma_1^a = \frac{D^2}{D^2+\omega^2}\left(f(\bar{\omega})-\frac{1}{2}\right) \rightarrow \gamma_1^\Lambda = \frac{D^2}{D^2+\omega^2} f_\Lambda(\bar{\omega})$$

$$f_\Lambda(\omega) = \frac{1}{\beta}\sum_n \frac{\theta_T(\Lambda-|\omega_n|)}{i\omega_n - \omega} \qquad \bar{\omega} = \eta\,\omega$$

$$\gamma_1^\Lambda = d\Lambda \frac{d\gamma_1^\Lambda}{d\Lambda} + \gamma_1^{\Lambda-d\Lambda}$$

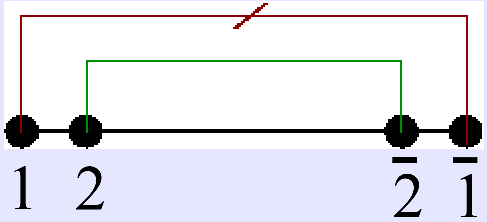
integrate out

Resum diagrams which are irreducible with respect to $d\Lambda \dfrac{d\gamma_1^\Lambda}{d\Lambda}$

$$L_D^{\Lambda-d\Lambda}(z) = L_D^\Lambda(z) - dL_D^\Lambda(z)$$

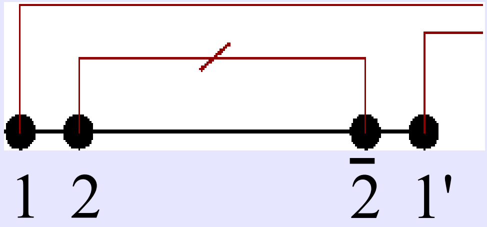
Lowest order (1-loop) for the Kondo problem:

$$-dL_D^\Lambda(z)$$



$$d\Lambda \frac{d\gamma_1^\Lambda}{d\Lambda}\gamma_2^\Lambda \bar{G}_{12}^\Lambda(z)\,\Pi_{12}^\Lambda\,\bar{G}_{\bar{2}\bar{1}}^\Lambda(z_{12}+\bar{\omega}_{12})$$

$$-d\,\bar{G}_{11'}^\Lambda(z)$$



$$d\Lambda \frac{d\gamma_2^\Lambda}{d\Lambda}\bar{G}_{12}^\Lambda(z)\,\Pi_{12}^\Lambda\,\bar{G}_{\bar{2}1'}^\Lambda(z_{12}+\bar{\omega}_{12})-(1\leftrightarrow 1')$$

$$\Pi_{12\ldots n}^\Lambda = \frac{1}{z_{12\ldots n}+\bar{\omega}_{12\ldots n}-L_D^\Lambda(z_{12\ldots n}+\bar{\omega}_{12\ldots n})}$$

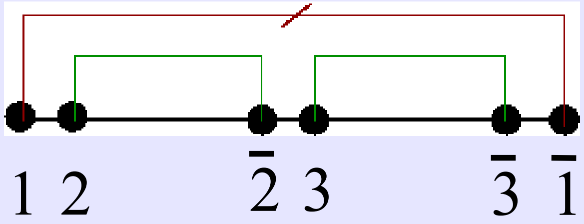
$$z_{12\ldots n} = z+\sum_{i=1}^n \eta_i\,\mu_i$$

$$\bar{\omega}_{12\ldots n} = \sum_{i=1}^n \eta_i\,\omega_i$$

Third order (2-loop) for the Kondo problem:

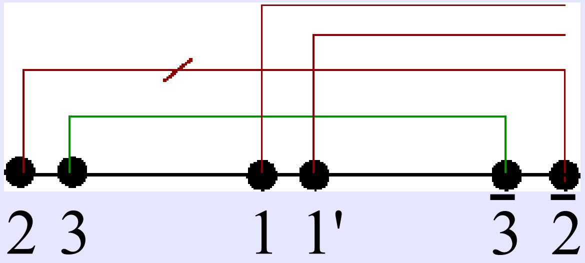
$-dL_D^\Lambda(z) \quad \rightarrow$

$$d\Lambda \frac{d\gamma_1^\Lambda}{d\Lambda} \gamma_2^\Lambda \gamma_3^\Lambda \bar{G}_{12}^\Lambda(z)\, \Pi_{12}^\Lambda\, \bar{G}_{\bar{2}3}^\Lambda(z_{12}+\bar{\omega}_{12})\, \Pi_{13}^\Lambda\, \bar{G}_{\bar{3}\bar{1}}^\Lambda(z_{13}+\bar{\omega}_{13})$$

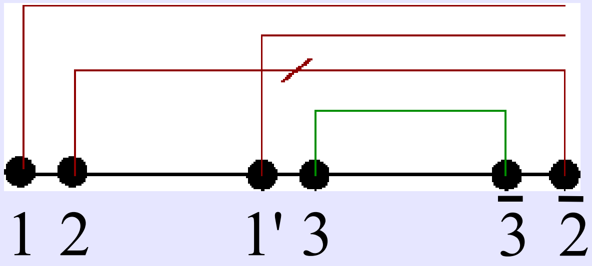


$-d\,\bar{G}_{11'}^\Lambda(z) \quad \rightarrow$

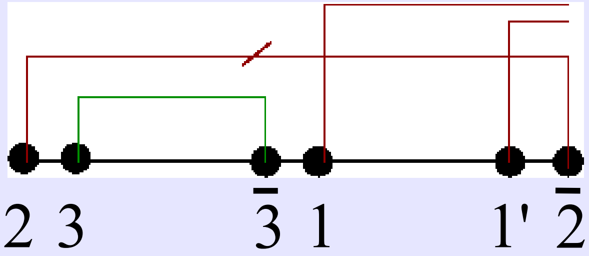
$$d\Lambda \frac{d\gamma_2^\Lambda}{d\Lambda} \gamma_3^\Lambda \bar{G}_{23}^\Lambda(z)\, \Pi_{23}^\Lambda\, \bar{G}_{11'}^\Lambda(z_{23}+\bar{\omega}_{23})\, \Pi_{11'23}^\Lambda\, \bar{G}_{\bar{3}\bar{2}}^\Lambda(z_{11'23}+\bar{\omega}_{11'23})$$



$$-d\Lambda \frac{d\gamma_2^\Lambda}{d\Lambda} \gamma_3^\Lambda \bar{G}_{12}^\Lambda(z)\, \Pi_{12}^\Lambda\, \bar{G}_{1'3}^\Lambda(z_{12}+\bar{\omega}_{12})\, \Pi_{11'23}^\Lambda\, \bar{G}_{\bar{3}\bar{2}}^\Lambda(z_{11'23}+\bar{\omega}_{11'23}) - (1 \leftrightarrow 1')$$



$$d\Lambda \frac{d\gamma_2^\Lambda}{d\Lambda} \gamma_3^\Lambda \bar{G}_{23}^\Lambda(z)\, \Pi_{23}^\Lambda\, \bar{G}_{\bar{3}1}^\Lambda(z_{23}+\bar{\omega}_{23})\, \Pi_{12}^\Lambda\, \bar{G}_{1'\bar{2}}^\Lambda(z_{12}+\bar{\omega}_{12}) - (1 \leftrightarrow 1')$$



Frequency integrations:

$\int_{-\infty}^{\infty} d\bar{\omega}_i = ?$ $\bar{\omega}_i = \eta_i \omega_i$

η = + → creation op.
η = - → annihilation op.

$$\frac{d L_D^\Lambda(z)}{d\Lambda} = -\frac{d\gamma_1^\Lambda}{d\Lambda}\gamma_2^\Lambda \bar{G}_{12}^\Lambda(z)\, \Pi_{12}^\Lambda\, \bar{G}_{\bar{2}\bar{1}}^\Lambda(z_{12}+\bar{\omega}_{12}) + \dots$$

$$\frac{d \bar{G}_{11'}^\Lambda(z)}{d\Lambda} = -\frac{d\gamma_2^\Lambda}{d\Lambda} \bar{G}_{12}^\Lambda(z)\, \Pi_{12}^\Lambda\, \bar{G}_{\bar{2}1'}^\Lambda(z_{12}+\bar{\omega}_{12}) - (1 \leftrightarrow 1') + \dots$$

$$\Pi_{12}^\Lambda = \frac{1}{z_{12}+\bar{\omega}_{12}-L_D^\Lambda(z_{12}+\bar{\omega}_{12})}$$

$$z_{12} = z+\eta_1\mu_1+\eta_2\mu_2$$

$$\bar{\omega}_{12} = \bar{\omega}_1+\bar{\omega}_2$$

$\frac{1}{z-L_D^\Lambda(z)}$ → analytic in upper half plane

=> $\bar{G}_{12}^\Lambda(z)$, Π_{12}^Λ , $\bar{G}_{\bar{2}\bar{1}}^\Lambda(z_{12}+\bar{\omega}_{12})$ → analytic in $\bar{\omega}_1, \bar{\omega}_2$ in upper half plane

Close integration in upper half of the complex plane :

$$\gamma_i^\Lambda = -\frac{D^2}{D^2+\bar{\omega}_i^2}\, T \sum_n \frac{\theta_T(\Lambda-|\omega_n|)}{\bar{\omega}_i - i\omega_n}$$

$$\frac{d\gamma_i^\Lambda}{d\Lambda} = -\frac{D^2}{D^2+\bar{\omega}_i^2}\, \frac{1}{2\pi}\left(\frac{1}{\bar{\omega}_i - i\Lambda_T} + \frac{1}{\bar{\omega}_i + i\Lambda_T}\right)$$

$\bar{\omega}_i = iD$
start RG at
$\Lambda_0 \sim D$

$\bar{\omega}_i = i\omega_n$
$0 < \omega_n < \Lambda$

$\bar{\omega}_i = iD$
start RG at
$\Lambda_0 \sim D$

$\bar{\omega}_i = i\Lambda_T$
Matsubara frequency
closest to Λ

Λ_T occurs in all resolvents !

RG equations in Matsubara space :

Analytic continuation in upper half plane : $\omega_n, \omega_{n_1}, \omega_{n_2} \; > \; 0$

$$L_D^\Lambda(E, \omega_n) \;\equiv\; L_D^\Lambda(E + i\,\omega_n)$$

$$\bar{G}^\Lambda_{\eta_1\nu_{1,}\eta_2\nu_2}(E, \omega_n, \omega_{n_1}, \omega_{n_2}) \;\equiv\; \bar{G}^\Lambda_{\eta_1\nu_1\omega_{1,}\,\eta_2\nu_2\omega_2}(E + i\,\omega_n)\big|_{\eta_i\omega_i \to i\,\omega_{n_i}}$$

$$\frac{d\,L_D^\Lambda(E, \omega_n)}{d\,\Lambda} \;=\; \bar{G}^\Lambda_{12}(E, \omega_n, \Lambda_T, \omega_{n_2})\,\Pi^\Lambda(E_{12}, \Lambda_T + \omega_n + \omega_{n_2})$$
$$\bar{G}^\Lambda_{\bar{2}\bar{1}}(E_{12}, \Lambda_T + \omega_n + \omega_{n_2}, -\,\omega_{n_2}, -\,\Lambda_T) \;+\; \ldots$$

$$\frac{d\,\bar{G}^\Lambda_{12}(E, \omega_n, \omega_{n_1}, \omega_{n_2})}{d\,\Lambda} \;=\; i\,\bar{G}^\Lambda_{13}(E, \omega_n, \omega_{n_1}, \Lambda_T)\,\Pi^\Lambda(E_{13}, \Lambda_T + \omega_n + \omega_{n_1})$$
$$\bar{G}^\Lambda_{\bar{3}2}(E_{13}, \Lambda_T + \omega_n + \omega_{n_1}, -\,\Lambda_T, \omega_{n_2}) \;-\; (1 \leftrightarrow 2) \;+\; \ldots$$

$$\Pi^\Lambda(E, \omega_n) \;=\; \frac{1}{E + i\,\omega_n - L_D^\Lambda(E + i\,\omega_n)}$$

$$E_{12} \;=\; E + \eta_1\mu_1 + \eta_2\mu_2$$

Sum over Matsubara frequencies :

$$2\pi T \sum\nolimits_n \theta_T(\Lambda - \omega_n)\,\theta(\omega_n) \;\underset{T=0}{\to}\; \int_0^\Lambda d\,\omega$$

Final result :

$$L_D^{eff}(E) \;=\; L_D^{\Lambda=0}(E, \omega_n = 0)$$

Cutoff scales and logarithmic enhancements :

Resolvents: $\frac{1}{z-L_D^{\Lambda}(z)}$ $\qquad z = E+\sum_k \eta_k \mu_k + i\Lambda_T + i(\omega_n+\omega_{n_2}+\ldots+\omega_{n_l})$

$$\frac{1}{z-L_D^{\Lambda}(z)} = \frac{1}{z-\lambda_i(z)}|x_i(z)\rangle\langle\bar{x}_i(z)|$$

$$L_D^{\Lambda}(z)|x_i(z)\rangle=\lambda_i(z)|x_i(z)\rangle$$

$$\langle\bar{x}_i(z)|L_D^{\Lambda}(z)=\lambda_i(z)\langle\bar{x}_i(z)|$$

$= \frac{a_i}{z-z_i}|x_i(z_i)\rangle\langle\bar{x}_i(z_i)| +$ analytic terms in z $\qquad z_i=\lambda_i(z_i) \qquad a_i=\left(1-\frac{d\lambda_i}{dz}(z_i)\right)^{-1}$

$z_i=h_i-i\Gamma_i\ ,\ \Gamma_i>0$ → poles of $\frac{1}{z-L_D^{\Lambda}(z)}$

$$\frac{1}{z-z_i} = \frac{1}{i\Lambda_T+i(\omega_n+\omega_{n_2}+\ldots+\omega_{n_l})+i\Gamma_i+E+\sum_k \eta_k\mu_k-h_i} \qquad 0<\omega_{n_k}<\Lambda$$

all positive!!

Cutoff scale : $\Lambda \sim max\left\{T, \Gamma_i, \left|E+\sum_k \eta_k\mu_k-h_i\right|\right\} > \Gamma_i$

Logarithmic enhancements : $E+\sum_k \eta_k\mu_k-h_i = 0$

V. Analytic solution in weak coupling

→ for T=0 and for spin/orbital fluctuations:

$$V = \frac{1}{2} g_{12} :a_1 a_2:$$

Cutoff scale : $max\left\{\Gamma_i , \left|E + \sum_k \eta_k \mu_k - h_i\right|\right\}$

Define : $\Lambda_c = max\left\{|E|, |\mu_\alpha|, |h_i|\right\}$ ~ maximal value for $\Delta = E + \sum_k \eta_k \mu_k - h_i$

Weak coupling : $\Lambda_c \gg T_K \quad <=> \quad J_\Lambda \sim G_{12} \ll 1$

T_K → scale of strong coupling (Kondo temperature)

Λ > Λc : $J_\Lambda \ll 1 \qquad \Gamma_i^\Lambda \sim \Lambda J_\Lambda^2 \ll \Lambda$

=> cutoff scales are not important and can be treated perturbatively

Expand around leading order solution without cutoff scales :

$$\frac{d\bar{G}_{11'}^{(1)}}{d\Lambda} = \frac{1}{\Lambda}\left\{\bar{G}_{12}^{(1)} \bar{G}_{21'}^{(1)} - (1 \leftrightarrow 1')\right\}$$

poor man scaling equation

$$L_D^\Lambda(E, \omega_n) = L_D^{(0)} + L_D^{(1)}(E, \omega_n) + L_D^{(2)}(E, \omega_n) + \ldots$$

$$\bar{G}_{12}^\Lambda(E, \omega_n, \omega_{n_1}, \omega_{n_2}) = \bar{G}_{12}^{(1)} + \bar{G}_{12}^{(2)}(E, \omega_n, \omega_{n_1}, \omega_{n_2}) + \ldots$$

$L_D^{(n)} \sim J_\Lambda^n$

$\bar{G}_{12}^{(n)} \sim J_\Lambda^n$

$0 < \Lambda < \Lambda_c$:

RG of Γ_i^Λ is cut off by Λ_c : $\boxed{\Gamma_i^\Lambda \sim \Lambda_c J_c^2}$ $J_c = J_{\Lambda=\Lambda_c}$

- $\Gamma_i^\Lambda \sim \Lambda J_\Lambda^2$ is flowing to smaller values
- some terms on the r.h.s of the RG equation for Γ_i^Λ will contain $|\Delta| = \left|E + \sum_k \eta_k \mu_k - h_i\right| \sim \Lambda_c$
- otherwise go to higher order $\Gamma_i^\Lambda \sim \Lambda_c J_c^k$

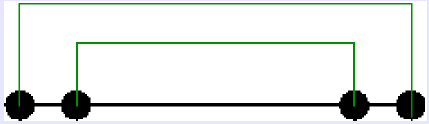
=> the minimal cutoff scale is $\Gamma \sim \Lambda_c J_c^2$

$$J_\Lambda \sim J_c\,\left(1 + J_c \ln\frac{\Lambda_c}{|\Lambda + i\Gamma|} + \dots\right) \xrightarrow[\Lambda\to 0]{} J_c\,\left(1 + \underset{\uparrow}{J_c \ln J_c} + \dots\right) \ll 1$$

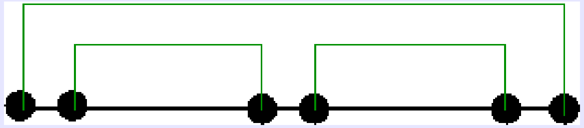
perturbative correction !

=> perturbation theory in $J_c \ll 1$ is justified !!

$$L_D^{eff}(z) = L_D(z)\big|_{\Lambda=\Lambda_c} +$$



$+$



$+ \dots$

Contraction : evaluate with $\gamma_1^{\Lambda_c} = f_{\Lambda_c}(\omega) = \frac{1}{2\pi}\int_{-\Lambda_c}^{\Lambda_c} d\omega' \, \frac{1}{i\omega' - \omega} = -\frac{1}{\pi}\arctan(\Lambda_c/\omega)$

Vertex : use vertex at Λ_c $\bar{G}_{12}^{\Lambda_c}(E, \omega_n, \omega_{n_1}, \omega_{n_2})$

Resolvents: take full Liouvillian in denominator $\frac{1}{z - L_D^{eff}(z)}$

=> we obtain a self-consistent equation for $L_D^{eff}(z)$

only the physical values $h_i^{\Lambda=0}, \Gamma_i^{\Lambda=0}$ enter the final solution!

Typical form of the results :

$$\Delta = E + \sum_k \eta_k \mu_k - h_i$$

$\Lambda > \Lambda_c$:

$$\bar{G}^{\Lambda}_{12}(E,\omega_n,\omega_{n_1},\omega_{n_2}) = \bar{G}^{(1)}_{12} + \bar{G}^{(2a_2)}_{12} + i\,\bar{G}^{(2a_1)}_{12} + \bar{G}^{(2b)}_{12}(E,\omega_n,\omega_{n_1},\omega_{n_2}) + \dots$$

poor man scaling

2-loop correction

important for rates

frequency dependent part in 2nd order
→ contains cutoff scales
→ important for final logarithms

$$\bar{G}^{(2a_1)}_{12} \, , \, \bar{G}^{(2b)}_{12} \sim J^2_{\Lambda}$$

$$\bar{G}^{(1)}_{12} + \bar{G}^{(2a_2)}_{12} \sim J_{\Lambda} + J^2_{\Lambda} - J^2_0 + J^2_{\Lambda} \ln J_{\Lambda} - J^2_{\Lambda} \ln J_0 \rightarrow \tilde{J}_{\Lambda}$$

change of
$T_K \rightarrow \tilde{T}_K$

unimportant corrections

important correction!

initial coupling

$$L^{\Lambda}_{D}(E,\omega) \sim L^{(0)}_{D} + (\Delta + i\,\omega) J_{\Lambda} + i(\Delta + i\,\omega) J^2_{\Lambda} + i\,\Lambda F\left(\frac{\Delta + i\,\omega}{\Lambda}\right) J^2_{\Lambda} + \dots$$

important for the renormalized energies

important for the rates

$0 < \Lambda < \Lambda_c$:

$J_c = J_{\Lambda=\Lambda_c}$ $\quad \Delta = E + \sum_k \eta_k \mu_k - h_i$ $\quad \Gamma_i \sim \Lambda_c J_c^2$

$$L_D^{eff}(z) \sim (\Delta J_c + i \Delta J_c^2)\ (1 + J_c \ln \frac{\Lambda_c}{|\Delta + i \Gamma_i|} + \ldots)$$

real part

imaginary part
starts one order higher!

logarithmic enhancement
near resonance

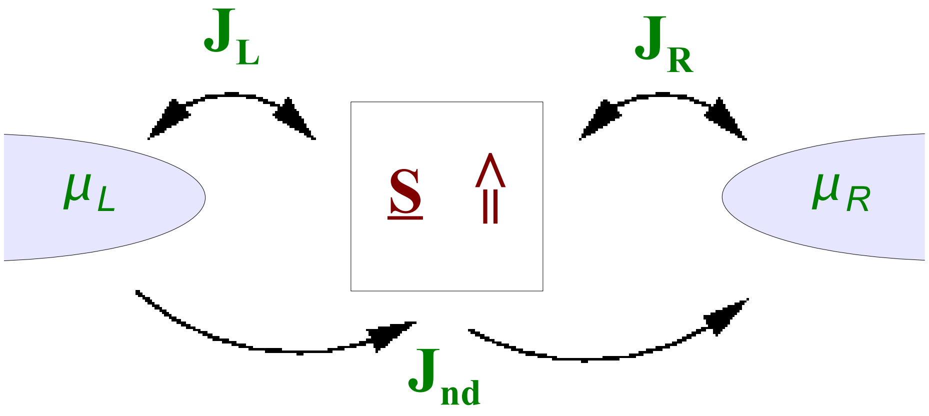
At resonance :

$$J_c \ln \frac{\Lambda_c}{|\Delta + i \Gamma_i|} \xrightarrow{\Delta=0} J_c \ln \frac{\Lambda_c}{\Gamma_i} \sim J_c \ln J_c$$

perturbative correction !

VI. The nonequilibrium anisotropic Kondo model at finite magnetic field

H. Schoeller and F. Reininghaus (RWTH Aachen) → arXiv:0902.1446



- Coulomb blockade regime
- finite magnetic field **h**
- anisotropic case: J^z_L, J^z_R, J^z_{nd}
  $J^\perp_L, J^\perp_R, J^\perp_{nd}$

→ molecular magnets !

Romeike, Wegewijs, Hofstetter & Schoeller, *Phys. Rev. Lett. 96, 196601 (2006)*

Specific for our approach:

- combination of nonequilibrium RG with relaxation/dephasing rates: **generic approach**
- 2-loop calculation: analysis of ***all*** subleading terms

  Conductance + magnetic susceptibility
  → similar to Rosch, Paaske, Kroha, Wölfle PRL '03

  + redefinition of the Kondo temperature

$$T_K \rightarrow \sqrt{J_0^\perp}\; T_K$$

  + derivation of the line shape

- calculation of parameters characterizing time evolution up to first logarithmic correction

  Spin relaxation/dephasing rates: $\Gamma_1 \;\; \Gamma_2$
  Renormalized g-factor: g

- analysis of the anisotropic case

Analytic solution in weak coupling

Weak coupling :

$$\Lambda_c \;=\; max\{V\,,h\} \;\gg\; T_K$$

$\Lambda > \Lambda_c$: expand exact RG equations systematically around leading order solution $J^z_{\alpha\alpha'}$ $J^\perp_{\alpha\alpha'}$ from poor man scaling

$$\frac{d\,\bar{G}^{(1)}_{11'}}{d\,\Lambda} \;=\; \frac{1}{\Lambda}\left\{\bar{G}^{(1)}_{12}\,\bar{G}^{(1)}_{21'} - (1 \leftrightarrow 1')\right\}$$

poor man scaling equation

$$L^\Lambda_D(E\,,\omega_n) \;=\; L^{(0)}_D \;+\; L^{(1)}_D(E\,,\omega_n) \;+\; L^{(2)}_D(E\,,\omega_n) \;+\ldots$$

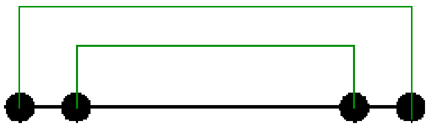
$$\bar{G}^\Lambda_{12}(E\,,\omega_n\,,\omega_{n_1}\,,\omega_{n_2}) \;=\; \bar{G}^{(1)}_{12} \;+\; \bar{G}^{(2)}_{12}(E\,,\omega_n\,,\omega_{n_1}\,,\omega_{n_2}) \;+\; \ldots$$

$$L^{(n)}_D \sim J^n_\Lambda$$

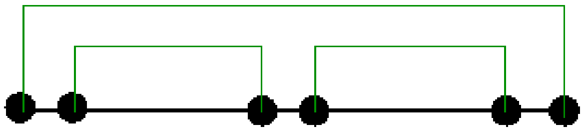
$$\bar{G}^{(n)}_{12} \sim J^n_\Lambda$$

$0 < \Lambda < \Lambda_c$: perturbation theory in renormalized vertices $J^z_{\alpha\alpha'}|_{\Lambda=\Lambda_c}$ $J^\perp_{\alpha\alpha'}|_{\Lambda=\Lambda_c}$

$$L^{eff}_D(z) \;=\; L_D(z)|_{\Lambda=\Lambda_c} +$$



$$+$$



$$+\;\ldots$$

Poor man scaling :

$$\Lambda \gg \Lambda_c = max\{V, h\} \gg T_K$$

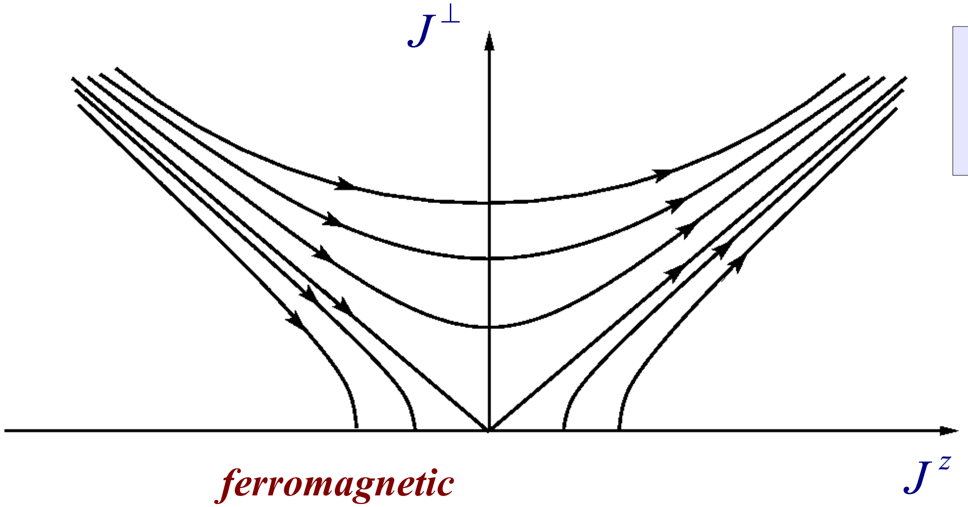
$J^z, J^\perp$ → matrices in reservoir space

$J^z_{\alpha\alpha'}, J^\perp_{\alpha\alpha'}$

$$\frac{d}{d\Lambda} J^z = -\frac{1}{\Lambda} J^\perp J^\perp$$

$$\frac{d}{d\Lambda} J^\perp = -\frac{1}{2\Lambda} (J^z J^\perp + J^\perp J^z)$$

$$\frac{d}{d\Lambda} h = \frac{h}{2\Lambda} Tr\, J^\perp J^\perp$$



strong coupling fixed point

$J^z = J^\perp = \infty$

spin singlett

ferromagnetic fixed point

$J^z < 0 \quad J^\perp = 0$

$J^z_{\alpha\alpha'} = J^\perp_{\alpha\alpha'} = J$

$$\frac{dJ}{d\Lambda} = -\frac{2J^2}{\Lambda}$$

$$J = \frac{1}{2\ln(\Lambda/T_K)}$$

$h = (1-J)h_0$

$T_K = D\ e^{-1/2J_0}$

Isotropic case:

$$J^z_{\alpha\alpha'} = J^\perp_{\alpha\alpha'} = J \;=\; \frac{1}{2\ln(max(V,h)/T_K)} \qquad T=0 \qquad h=(1-J)h_0$$

$V<\tilde{h}$:

$$I[e/h] \;=\; (\pi^2/2)J^2 V \;+\; \pi^2 J^3 (V-\tilde{h})\ln\frac{\tilde{h}}{|V-\tilde{h}+i\Gamma_2|}$$

$$M \;=\; -1/2$$

$$\Gamma_1 \;=\; 2\pi J^2\tilde{h} \;-\; 2\pi J^3 (V-\tilde{h})\ln\frac{\tilde{h}}{|V-\tilde{h}+i\Gamma_2|}$$

$$\Gamma_2 \;=\; (\pi/2)J^2(V+2\tilde{h})$$

$$\tilde{h} \;=\; h \;+\; (1/2)J^2(V-\tilde{h})\ln\frac{\tilde{h}}{|V-\tilde{h}+i\Gamma_1|}$$

I , Γ_1 , $\tilde{h}$ →
logarithmic enhancement at
$V \;=\; \tilde{h}$

V=0 : no logarithmic enhancement
since spin-flip needed for renormalization
=> cutoff given by h

$V>\tilde{h}$:

$$I[e/h] \;=\; (3\pi^2/2)J^2 V \;-\; 2\pi^2 J^2\tilde{h}^2/(V+\tilde{h}) \;+\; 4\pi^2 J^3 (V-\tilde{h})\ln\frac{V}{|V-\tilde{h}+i\Gamma_2|}$$

$$M \;=\; -\frac{\tilde{h}}{V+\tilde{h}}\;\left(1 \;+\; 2J\frac{V}{V+\tilde{h}}\ln\frac{V}{|\tilde{h}+i\Gamma_2|} \;-\; J\frac{V-\tilde{h}}{\tilde{h}}\ln\frac{V}{|V-\tilde{h}+i\Gamma_2|}\right)$$

$$\Gamma_1 \;=\; \pi J^2(V+\tilde{h}) \;+\; 2\pi J^3\tilde{h}\ln\frac{V}{|\tilde{h}+i\Gamma_2|}$$

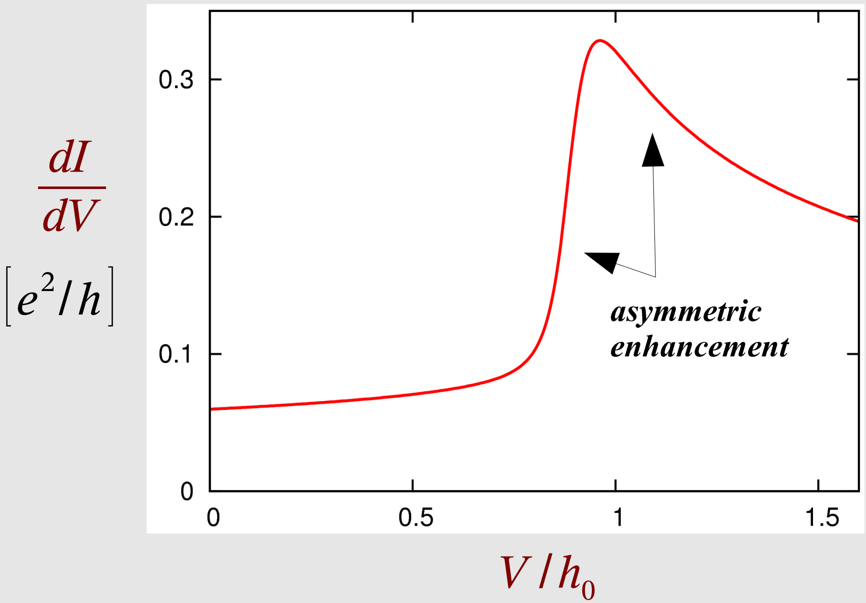
$$\Gamma_2 \;=\; (\pi/2)J^2(2V+\tilde{h}) \;+\; \pi J^3\tilde{h}\ln\frac{V}{|\tilde{h}+i\Gamma_1|} \;+\; \pi J^3(V-\tilde{h})\ln\frac{V}{|V-\tilde{h}+i\Gamma_1|}$$

$$\tilde{h} \;=\; h \;-\; J^2\tilde{h}\ln\frac{V}{|\tilde{h}+i\Gamma_1|} \;+\; (1/2)J^2(V-\tilde{h})\ln\frac{V}{|V-\tilde{h}+i\Gamma_1|}$$

I , M , Γ_2 , $\tilde{h}$ →
logarithmic enhancement at
$V \;=\; \tilde{h}$

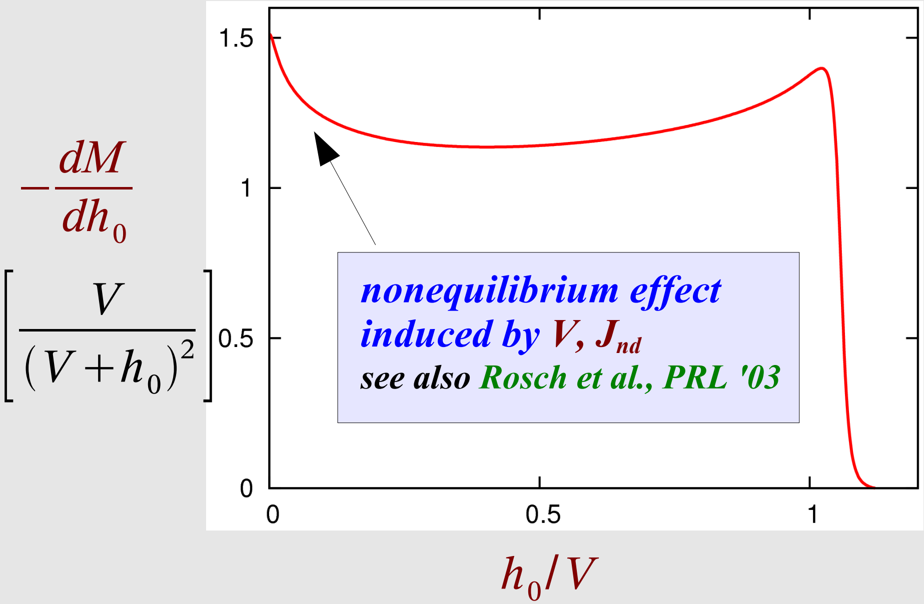
M , Γ_1 , Γ_2 , $\tilde{h}$ →
logarithmic enhancement at
$\tilde{h}=0$

Conductance:



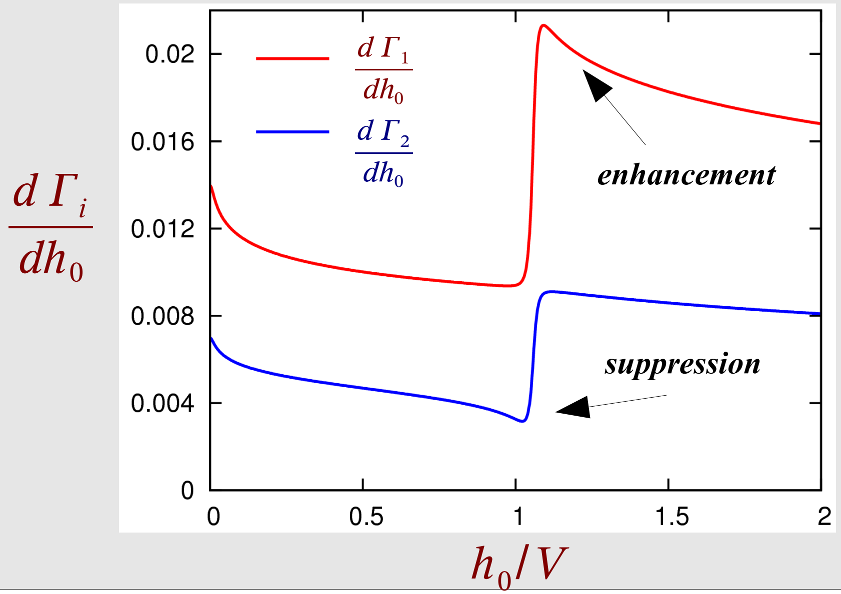
asymmetric enhancement

Magnetic susceptibility



nonequilibrium effect
induced by V, J_{nd}
see also Rosch et al., PRL '03

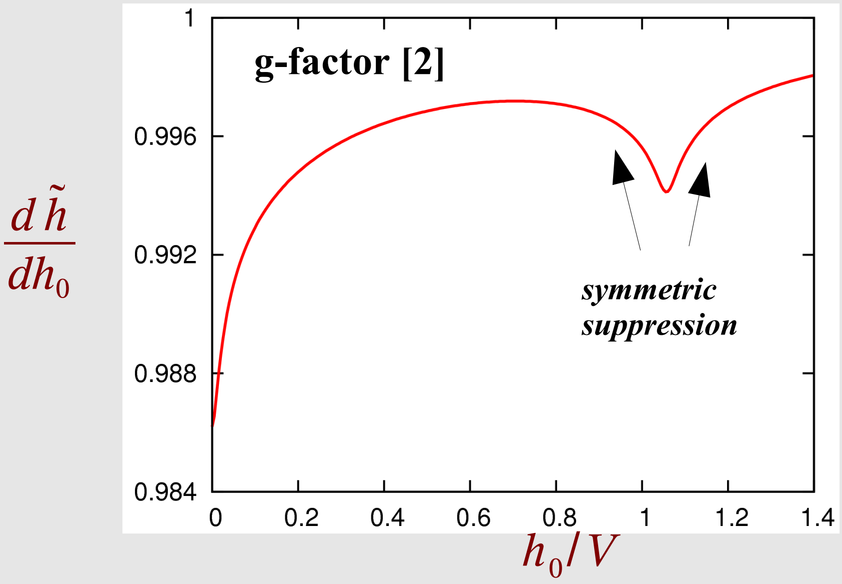
Spin relaxation/dephasing rates:



enhancement

suppression

g-factor:



symmetric suppression

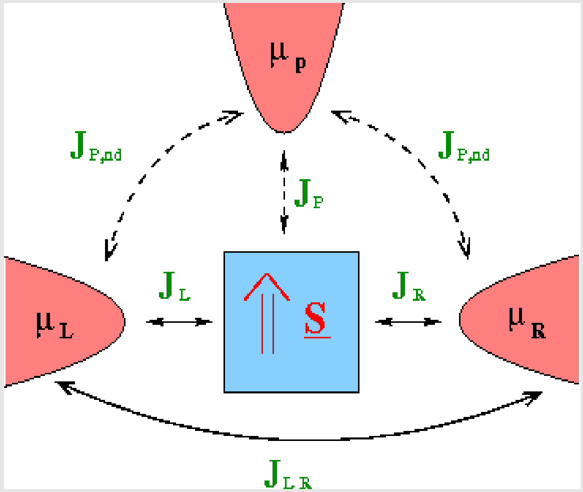
Experimental measurement of renormalized g-factor

Weakly coupled probe lead:

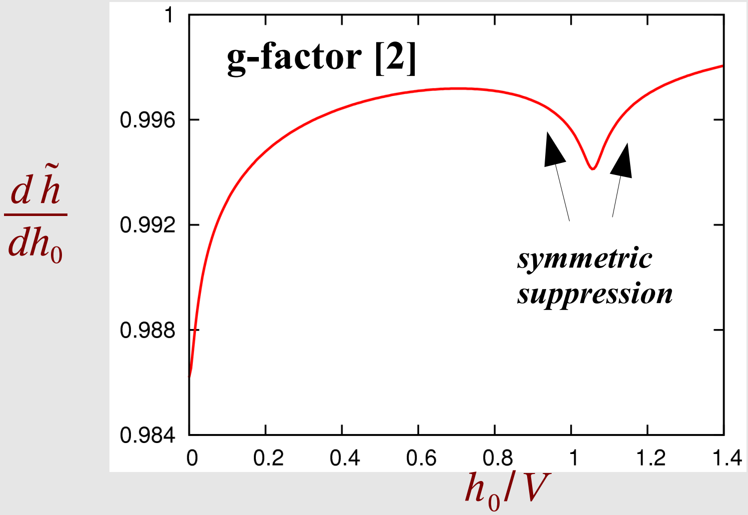
$$J_{P,nd} = \sqrt{2x_P}\ \ J$$

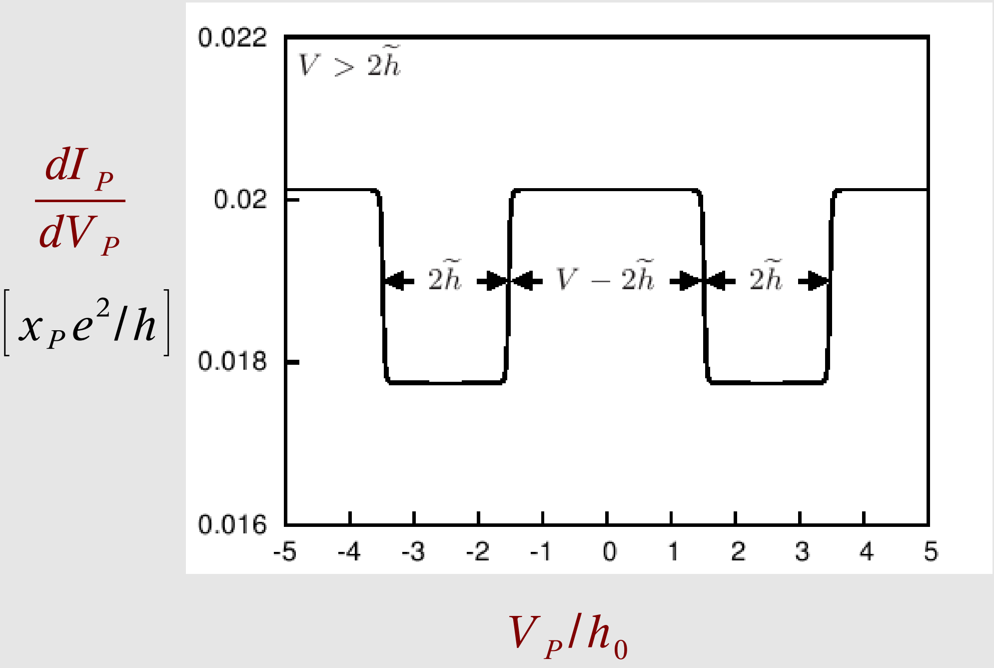
$$J_{P,nd} = \sqrt{2x_P}\ \ J$$

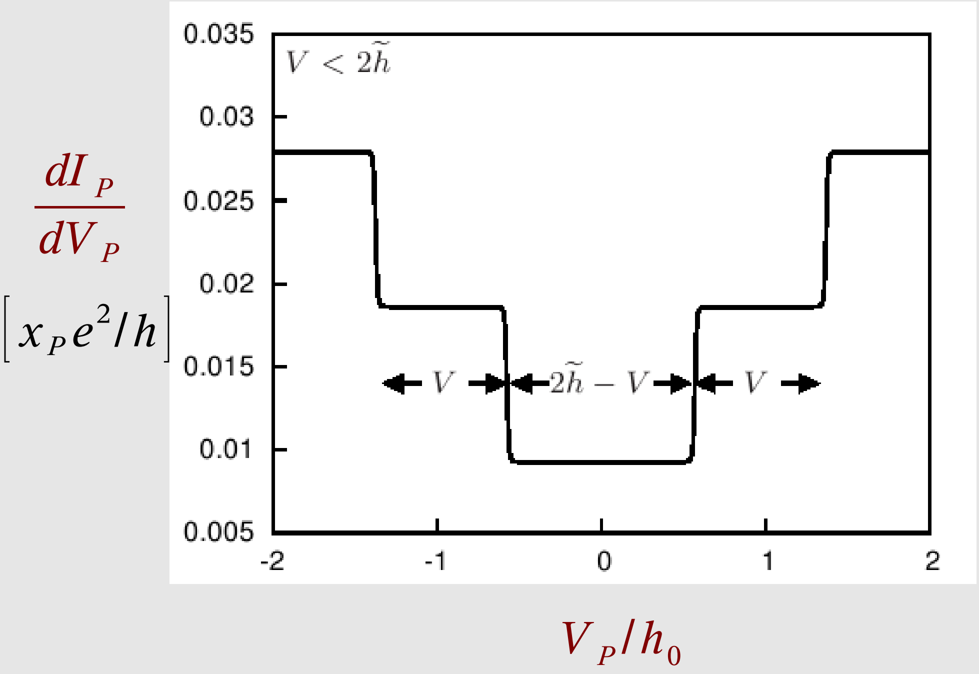
$$x_P \ \ll\ 1$$



g-factor:







Nonequilibrium effects ~ ln(V/h) for V >> h

$$J^{z/\perp}_{\alpha\alpha'} \equiv J^{z/\perp}_{\alpha\alpha'}\big|_{\Lambda=V}$$

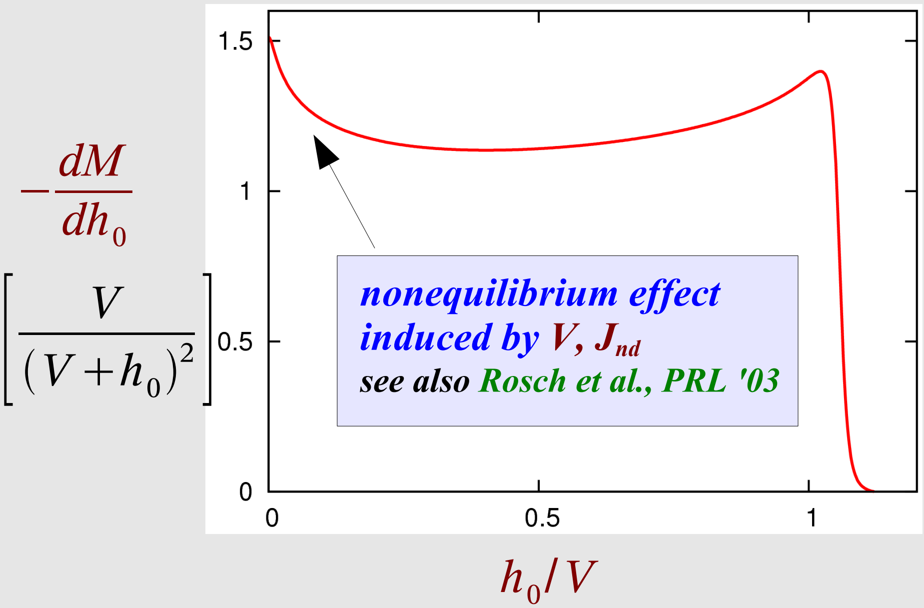
$$M = -\frac{1}{2}\,\frac{(J^{\perp}_{\alpha})^2\tilde{h} + 2J^{z}_{\alpha}(J^{\perp}_{\alpha})^2\tilde{h}\ln\frac{V}{|\tilde{h}+i\Gamma_2|} + 2(J^{\perp}_{nd})^2\tilde{h} + \boxed{2J^{\perp}_{\alpha}J^{z}_{nd}J^{\perp}_{nd}\tilde{h}\ln\frac{V}{|\tilde{h}+i\Gamma_2|}}}{(J^{\perp}_{\alpha})^2\tilde{h} + 2J^{z}_{\alpha}(J^{\perp}_{\alpha})^2\tilde{h}\ln\frac{V}{|\tilde{h}+i\Gamma_2|} + 2(J^{\perp}_{nd})^2\tilde{h} + 2(J^{\perp}_{nd})^2 V}$$

noneq. effect
→ induced by J_{nd}

(see also: *Rosch, Paaske, Kroha, Wölfle, PRL '03)*

logarithmic terms increase with J^z

Magnetic susceptibility



nonequilibrium effect
induced by V, J_{nd}
see also Rosch et al., PRL '03

Anisotropic case:

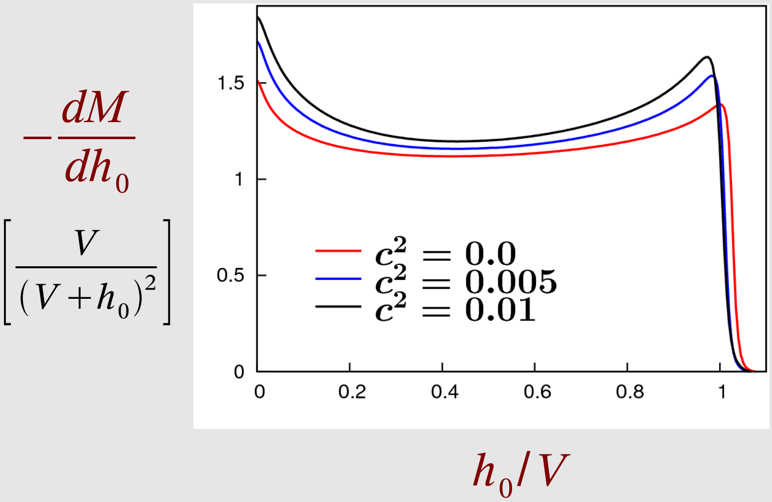
$J^z_{\alpha\alpha'} = J^z \quad , \quad J^\perp_{\alpha\alpha'} = J^\perp$ $T = 0$ $h = (1 - J) h_0$

$c^2 = (J^z)^2 - (J^\perp)^2$ keep $\mathrm{T_K}$ = const and **vary c^2**

$V > \tilde{h}$:

$$M = -\frac{\tilde{h}}{V+\tilde{h}} \left(1 + 2J^z \frac{V}{V+\tilde{h}} \ln \frac{V}{|\tilde{h} + i\Gamma_2|} - J^z \frac{V-\tilde{h}}{\tilde{h}} \ln \frac{V}{|V - \tilde{h} + i\Gamma_2|}\right)$$

Magnetic susceptibility



logarithmic terms increase with J^z

→ important for molecular magnets

Anisotropic case:

$J^z_{\alpha\alpha'}=J^z$, $J^\perp_{\alpha\alpha'}=J^\perp$ $T=0$ $h=(1-J)h_0$

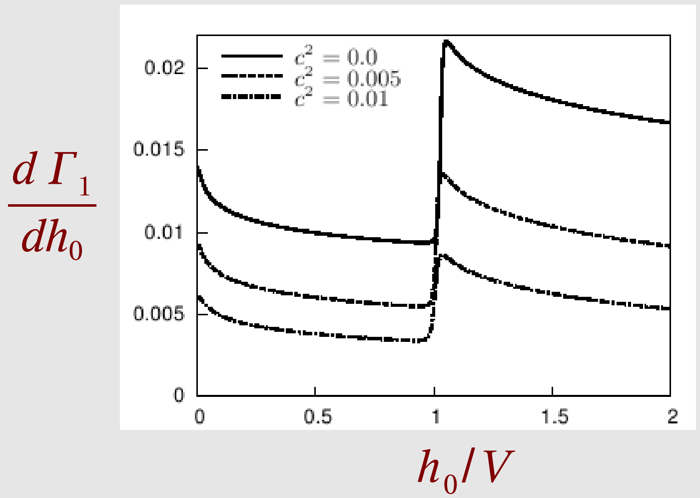
$c^2=(J^z)^2-(J^\perp)^2$ keep T_K = const and **vary c^2**

$V>\tilde{h}$:

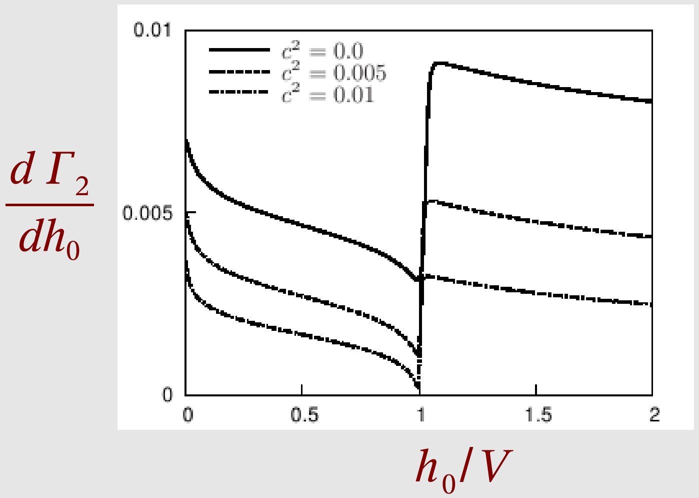
$$\Gamma_1 = \pi(J^\perp)^2(V+\tilde{h}) + 2\pi J^z(J^\perp)^2\tilde{h}\ln\frac{V}{|\tilde{h}+i\Gamma_2|}$$

$$\Gamma_2 = (\pi/2)(J^z)^2V + (\pi/2)(J^\perp)^2(V+\tilde{h}) + \pi J^z(J^\perp)^2\tilde{h}\ln\frac{V}{|\tilde{h}+i\Gamma_1|} + \pi J^z(J^\perp)^2(V-\tilde{h})\ln\frac{V}{|V-\tilde{h}+i\Gamma_1|}$$

Spin relaxation rate:



Spin dephasing rate:



less pronounced features

sharper features

VII. Outlook for strong coupling

$O(J^2)$, h=0, isotropic case
frequency dependence neglected

$$\frac{d}{d\Lambda}J_d(E) = -\frac{1}{\Lambda+\Gamma(E)+ih(E)-iE}\,J_d(E)^2 - \frac{1}{2}\sum_{\pm}\frac{1}{\Lambda+\Gamma(E\pm V)+ih(E\pm V)-i(E\pm V)}\,J_{nd}(\pm E)^2$$

$$\frac{d}{d\Lambda}J_{nd}(E) = -\frac{1}{\Lambda+\Gamma(E)+ih(E)-iE}\,J_d(E)J_{nd}(E) - \frac{1}{\Lambda+\Gamma(E+V)+ih(E+V)-i(E+V)}\,J_d(E+V)J_{nd}(E)$$

$$\frac{d}{d\Lambda}\Gamma(E) = \sum_{\alpha\alpha'} 2\ \ln\left(\frac{2\Lambda-\Gamma(E+\mu_\alpha-\mu_{\alpha'})-ih(E+\mu_\alpha-\mu_{\alpha'})-i(E+\mu_\alpha-\mu_{\alpha'})}{\Lambda-\Gamma(E+\mu_\alpha-\mu_{\alpha'})-ih(E+\mu_\alpha-\mu_{\alpha'})-i(E+\mu_\alpha-\mu_{\alpha'})}\right)\ J_{\alpha\alpha'}(E)\ \ J_{\alpha'\alpha}(E+\mu_\alpha-\mu_{\alpha'})$$

Current rate :

$$\frac{d}{d\Lambda}I = -12\pi^2\Im\left\{\ln\left(\frac{2\Lambda+\Gamma(V)+ih(V)-iV}{\Lambda+\Gamma(V)+ih(V)-iV}\right)\right\}\ J_I\ \ K_{LR}$$

Weak coupling regime :

$$V \gg T_K = \Lambda e^{-1/2J}$$

=> $\Gamma \gg T_K$ => $J \ll 1$

$$\Gamma = \pi(J_{nd}^2)_{\Lambda=V}V$$

$$I = \frac{e^2}{h}\frac{3\pi^2}{2}(J_{nd}^2)_{\Lambda=V}V = \frac{e^2}{h}\,\frac{3\pi^2}{8}\,\frac{V}{\ln^2(V/T_K)}$$

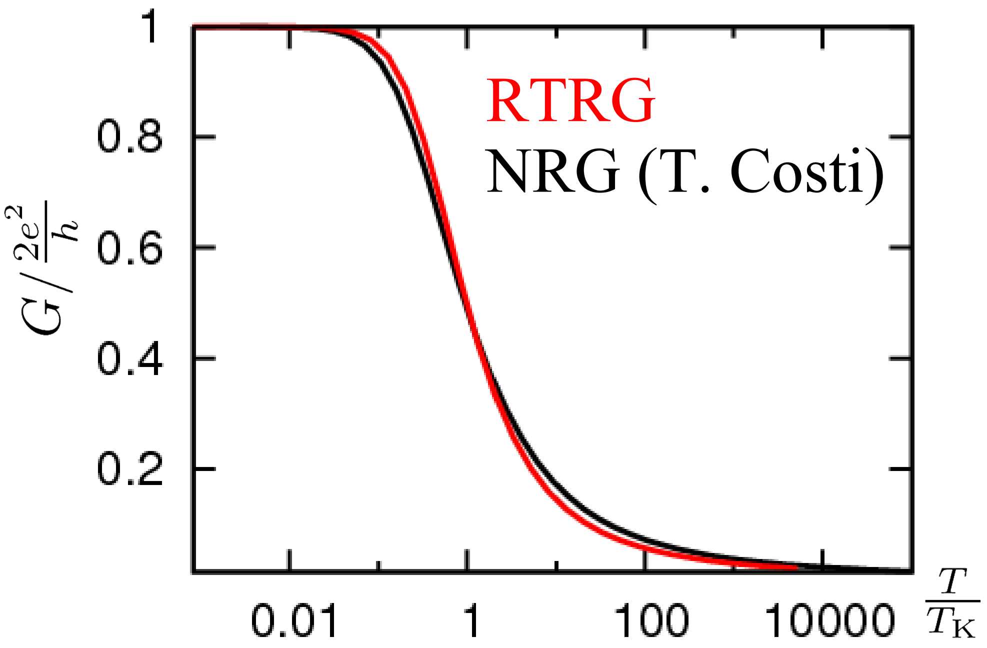
Strong coupling regime :

$T, V < T_K \qquad J \sim 1$

Idea : T,V < T_K => $\boxed{\Gamma \sim T_K J^2 \sim T_K}$ => cutoff provided by T_K

Problem : prefactor of $\Gamma = c\, T_K$ can not be determined by perturbative RG

Trick :
- adjust initial value of $\Gamma_{\Lambda=D}$ such that $G_{T=V=0} = 2e^2/h$
- use this initial value for arbitrary T, V



good agreement with
numerical renormalization group

Summary

- RG-method in Liouville space
  - → nonequilibrium RG on the Keldysh contour
  - → full time evolution + stationary state+ correlation functions (D. Schuricht)
  - → theory renormalized field + relaxation/dephasing rates
  - → analytic solution in weak coupling
  - → theory for line shape at resonance
  - → technical advantages: → Keldysh indices can be avoided
    - → formulation on Matsubara axis
    - → generic cutoff by rates

- Application to the nonequilibrium Kondo model
  - → anisotropic case + finite magnetic field
  - → full 2-loop calculation
  - → current, magnetization: theory for Γ_1 , Γ_2 , g + line shape
  - → Γ_1 , Γ_2 up to $O(J^3 \ln)$, g up to $O(J^2 \ln)$
  - → new proposal to measure g(h/V) in 3-terminal setup
  - → several nonequilibrium induced effects
  - → correlation functions + time evolution (D. Schuricht)
  - → strong coupling?