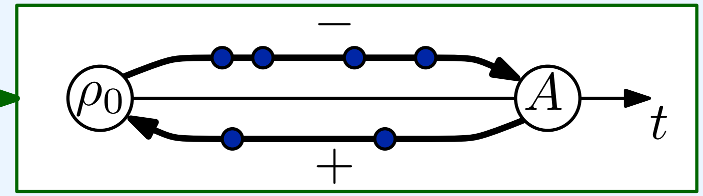
Functional RG within Keldysh formalism

Severin G. Jakobs, Mikhail Pletyukhov, Herbert Schoeller

RWTH AACHEN   JARA Jülich Aachen Research Alliance

Outline

- Introduction to Keldysh formalism →



- Properties of Green functions:
  - Causality
  - Kubo-Martin-Schwinger relations →

$$e^{\beta\Delta^{i|i'}(\omega|\omega')} G^{i|i'}_{q|q'}(\omega|\omega') \;=\; \zeta^{m^{i|i'}}\, G^{i'|i}_{\widetilde{q}'|\widetilde{q}}(\omega'|\omega)\Big|_{\widetilde{H}}$$

- The SIAM
- Γ-flow: Hybridisation as flow parameter →

$$g^{\text{Ret}}_{\Lambda}(\omega) \;=\; \frac{1}{\omega - \epsilon + \frac{i}{2}(\Gamma + \Lambda)}$$

- Results
- Conclusion

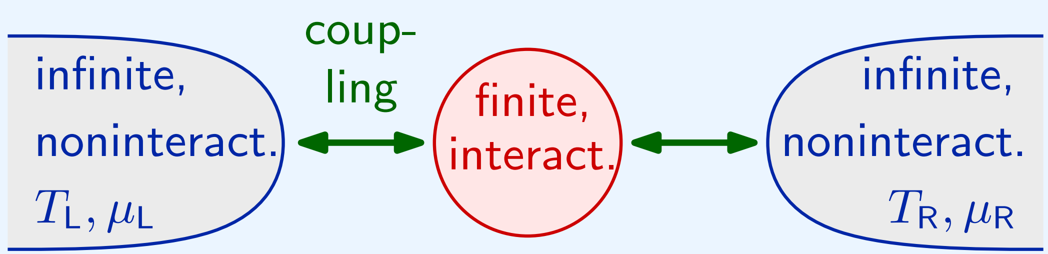
Then: C. Karrasch – fRG approach to SIAM in equilibrium

Matsubara and Keldysh formalism

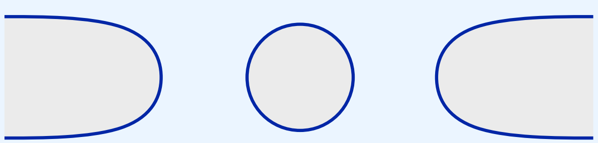
Restrictions of Matsubara formalism

- analytic continuation to real frequencies required
- does not allow for nonequilibrium

typical nonequilibrium situation



$t = -\infty$

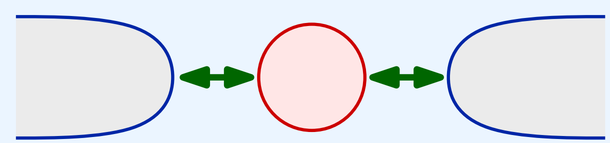


$$\rho_0 = \left(\bigotimes \rho_\alpha \right) \otimes \rho_{\text{dot}}$$

$$[H_0, \rho_0] = 0$$

$$H(t) = H_0 + V e^{-|t|0^+}$$

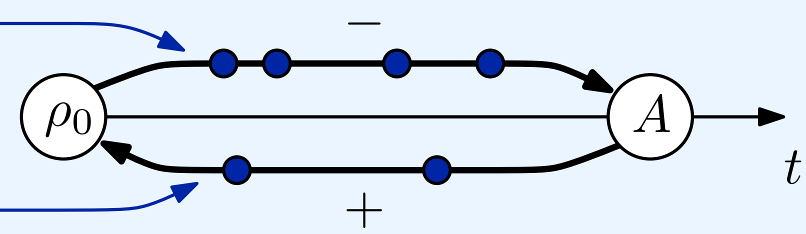
finite t

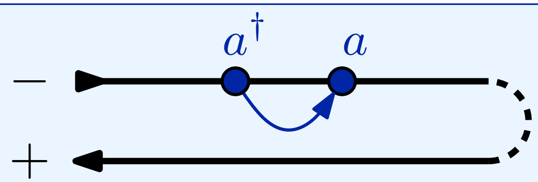


$$\rho(t) \equiv \rho \text{ (stat. state)}$$

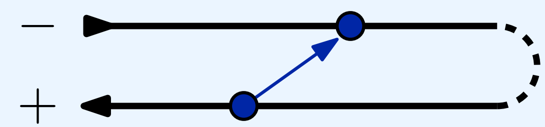
Keldysh formalism

$$\langle A(t) \rangle = \mathrm{Tr}\, \rho_0 \, U_\mathsf{I}(-\infty, t) \, A_\mathsf{I}(t) \, U_\mathsf{I}(t, -\infty)$$

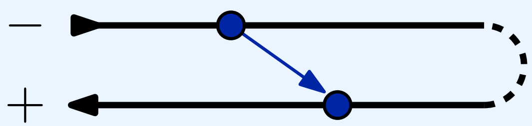




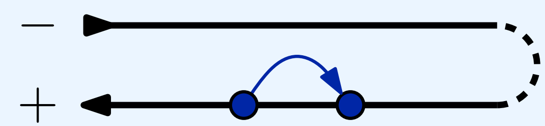
$G^{--} = G^c$



$G^{-+} = G^<$



$G^{+-} = G^>$



$G^{++} = G^{\tilde{c}}$

Keldysh rotation

$$a_{1,2} = \frac{1}{\sqrt{2}}(a_- \mp a_+)$$

$$G^{11} = 0$$

$$G^{21} = G^{\mathsf{Ret}} = \frac{1}{\omega - \epsilon - \Sigma^{\mathsf{Ret}}}$$ $\rightarrow$ quasi-part. energies, lifetimes

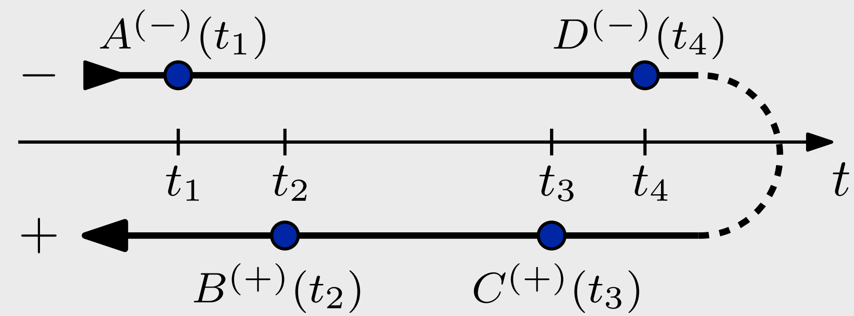
$$G^{12} = G^{\mathsf{Av}} = \left(G^{\mathsf{Ret}}\right)^\dagger$$

$$G^{22} = G^{\mathsf{K}}$$ $\rightarrow$ particle distribution:

$$g^{\mathsf{K}} = 2\pi i \, [2f(\omega) - 1] \delta(\omega - \epsilon)$$

n-particle Green Functions

Contour order:



$$T_{\mathsf{c}}\, ABCD = \zeta\, BCDA$$

$$\zeta = \begin{cases} +1, & \text{bosons} \\ -1, & \text{fermions} \end{cases}$$

$i = (i_1 \ldots i_n) =$ contour indices, $i_k = \mp$

[or: $\alpha = (\alpha_1 \ldots \alpha_n) =$ Keldysh indices, $\alpha_k = 1, 2$]

$t = (t_1 \ldots t_n) =$ times

$$G^{i|i'}_{q|q'}(t|t') \;=\; (-i)^n \left\langle\, T_{\mathsf{c}}\; a^{(i_1)}_{q_1}(t_1) \ldots a^{(i_n)}_{q_n}(t_n)\; a^{(i'_n)\,\dagger}_{q'_n}(t'_n) \ldots a^{(i'_1)\,\dagger}_{q'_1}(t'_1) \,\right\rangle$$

$$G^{i|i'}_{q|q'}(\omega|\omega') \;=\; \int dt_1 \ldots dt'_n\; e^{i(\omega\cdot t - \omega'\cdot t')}\; G^{i|i'}_{q|q'}(t|t')$$

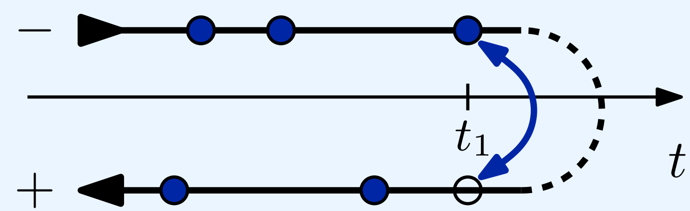
$q = (q_1 \ldots q_n) =$ states

$\omega \cdot t = \omega_1 t_1 + \cdots + \omega_n t_n$

Causality

$$G^{-,i_2\dots i_n|i'}(t|t') \;=\; G^{+,i_2\dots i_n|i'}(t|t'),$$

if $t_1 > t_2 \dots t'_n$



Keldysh rotation $a_{1,2} = \frac{1}{\sqrt{2}}(a_- \mp a_+)$

$$G^{1,\alpha_2\dots\alpha_n|\alpha'}(t|t') \;=\; 0 \quad \text{if } t_1 > t_2 \dots t'_n$$

More general:

If Keldysh index associated with largest time equals 1, **then** $G^{\alpha|\alpha'}(t|t') = 0$.

Fourier transform

Special case: $G^{1\dots1|1\dots1} \equiv 0$

Time translational invariance:

$$G(\omega|\omega') = 2\pi\delta(\omega_1 + \dots + \omega_n - \omega'_1 - \dots - \omega'_n)\; G(t_1\!=\!0, \omega_2 \dots \omega_n|\omega')$$

no contribution from decay poles for $t_2 \dots t'_n \to \infty$

$$G^{21\dots1|1\dots1}(t_1\!=\!0, \omega_2 \dots \omega_n|\omega') = \int_{-\infty}^{t_1=0} dt_2 \dots dt'_n \; e^{i(+\omega\cdot t - \omega'\cdot t')}\; G^{21\dots1|1\dots1}(t|t')$$

is analytic in the **lhp of $\omega_2 \dots \omega_n$** and the **uhp of $\omega'_1 \dots \omega'_n$**

Causality

- $G^{1\dots1|1\dots1} \equiv 0$
- $G^{21\dots1|1\dots1}(\omega_2 \dots \omega_n|\omega')$ analytic in

$$\begin{cases} \text{lhp of } \omega_2 \dots \omega_n \\ \text{uhp of } \omega'_1 \dots \omega'_n \end{cases}$$

- $2n$ **fully retarded GFs:**

$$G^{21\dots1|1\dots1}, \ \dots, \ G^{1\dots1|1\dots12}$$

For vertex functions exchange $1 \leftrightarrow 2$.

- $\gamma^{2\dots2|2\dots2} \equiv 0$
- $\gamma^{12\dots2|2\dots2}(\omega'_2 \dots \omega'_n|\omega)$ analytic in

$$\begin{cases} \text{lhp of } \omega'_2 \dots \omega'_n \\ \text{uhp of } \omega_1 \dots \omega_n \end{cases}$$

- $2n$ **fully retarded VFs:**

$$\gamma^{12\dots2|2\dots2}, \ \dots, \ \gamma^{2\dots2|2\dots21}$$

Example: $n = 1$

$$G^{1|1} \equiv 0$$

$$G^{\text{Ret}}(\omega') = G^{2|1}(t=0|\omega') \text{ analytic in uhp of } \omega'$$

$$G^{\text{Av}}(\omega) = G^{1|2}(\omega|t'=0) \text{ analytic in lhp of } \omega$$

$$\Sigma^{2|2} \equiv 0$$

$$\Sigma^{\text{Ret}}(\omega) = \Sigma^{1|2}(t'=0|\omega) \text{ analytic in uhp of } \omega$$

$$\Sigma^{\text{Av}}(\omega') = \Sigma^{2|1}(\omega'|t=0) \text{ analytic in lhp of } \omega'$$

Kubo Martin Schwinger conditions

[For real bosons: Chou et. al., Phys. Rep. **118**, 1 (1985)]

Equilibrium: $\rho = \dfrac{e^{-\beta H}}{\mathrm{Tr}\, e^{-\beta H}} \qquad (\mu = 0)$

$$\Rightarrow \quad A(t - i\beta) = e^{\beta H} A(t) e^{-\beta H} = \rho^{-1} A(t) \rho$$

$$G^{i|i'}(t - i\beta_+ | t' - i\beta_+) = (-i)^n \zeta^P \,\mathrm{Tr}\, \rho\rho^{-1} \left(\begin{array}{c} +\text{-operators, anti} \\ \text{time ordered} \end{array} \right) \rho \left(\begin{array}{c} -\text{-operators,} \\ \text{time ordered} \end{array} \right)$$

$$= (-i)^n \zeta^P \,\mathrm{Tr}\, \rho \left(\begin{array}{c} -\text{-operators,} \\ \text{time ordered} \end{array} \right) \left(\begin{array}{c} +\text{-operators, anti} \\ \text{time ordered} \end{array} \right)$$

$$= \zeta^{m^{i|i'}} \widetilde{G}^{\bar{i}|\bar{i}'}(t|t')$$

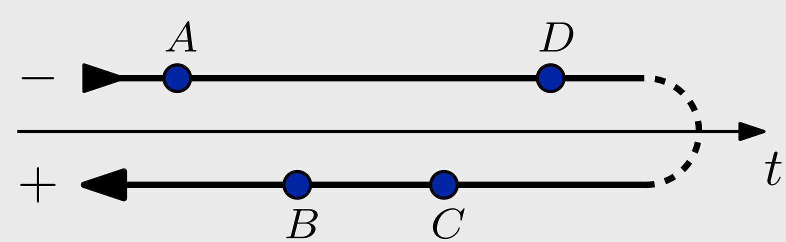
add $-i\beta$ to all times on the $+$-branch

$$m^{i|i'} = \sum_{i'_k = +} 1 - \sum_{i_k = +} 1$$

$$\widetilde{G}^{i|i'}_{q|q'}(t|t') = (-i)^n \left\langle \widetilde{T}_{\mathsf{c}}\, a_1 \ldots a_n\, a_n^\dagger \ldots a_1^\dagger \right\rangle$$

Tilde order:

[Wang, Heinz, Phys. Rev. D **66**, 025008 (2002)]



$$T_{\mathsf{c}}\, ABCD = \zeta\, BCDA$$

$$\widetilde{T}_{\mathsf{c}}\, ABCD = \zeta\, CBAD$$

Kubo Martin Schwinger conditions

[For real bosons: Chou et. al., Phys. Rep. **118**, 1 (1985)]

$$e^{\beta\Delta^{i|i'}(\omega|\omega')} G^{i|i'}(\omega|\omega') = \zeta^{m^{i|i'}} \widetilde{G}^{\bar{i}|\bar{i}'}(\omega|\omega') \quad \text{with} \quad \Delta^{i|i'}(\omega|\omega') = \sum_{i'_k=+} \omega'_k - \sum_{i_k=+} \omega_k$$

Fourier transform

$$G^{i|i'}(t - i\beta_+|t' - i\beta_+) = (-i)^n \zeta^P \,\mathrm{Tr}\, \rho\rho^{-1} \begin{pmatrix} +\text{-operators, anti} \\ \text{time ordered} \end{pmatrix} \rho \begin{pmatrix} -\text{-operators,} \\ \text{time ordered} \end{pmatrix}$$

$$= (-i)^n \zeta^P \,\mathrm{Tr}\, \rho \begin{pmatrix} -\text{-operators,} \\ \text{time ordered} \end{pmatrix} \begin{pmatrix} +\text{-operators, anti} \\ \text{time ordered} \end{pmatrix}$$

$$= \zeta^{m^{i|i'}} \widetilde{G}^{\bar{i}|\bar{i}'}(t|t')$$

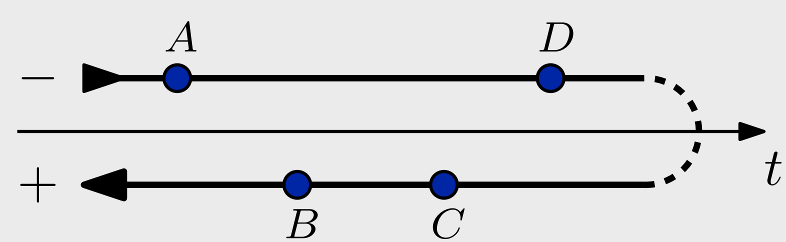
add $-i\beta$ to all times on the $+$-branch

$$m^{i|i'} = \sum_{i'_k=+} 1 - \sum_{i_k=+} 1$$

$$\widetilde{G}^{i|i'}_{q|q'}(t|t') = (-i)^n \left\langle \widetilde{T}_\mathsf{c}\, a_1 \ldots a_n\, a_n^\dagger \ldots a_1^\dagger \right\rangle$$

Tilde order:

[Wang, Heinz, Phys. Rev. D **66**, 025008 (2002)]



$$T_\mathsf{c}\, ABCD = \zeta\, BCDA$$

$$\widetilde{T}_\mathsf{c}\, ABCD = \zeta\, CBAD$$

Fluctuation dissipation theorem

$$e^{\beta\Delta^{i|i'}(\omega|\omega')}\, G^{i|i'}(\omega|\omega') = \zeta^{m^{i|i'}}\, \widetilde{G}^{\bar{i}|\bar{i}'}(\omega|\omega') \quad \text{with} \quad \Delta^{i|i'}(\omega|\omega') = \sum_{i'_k=+} \omega'_k - \sum_{i_k=+} \omega_k$$

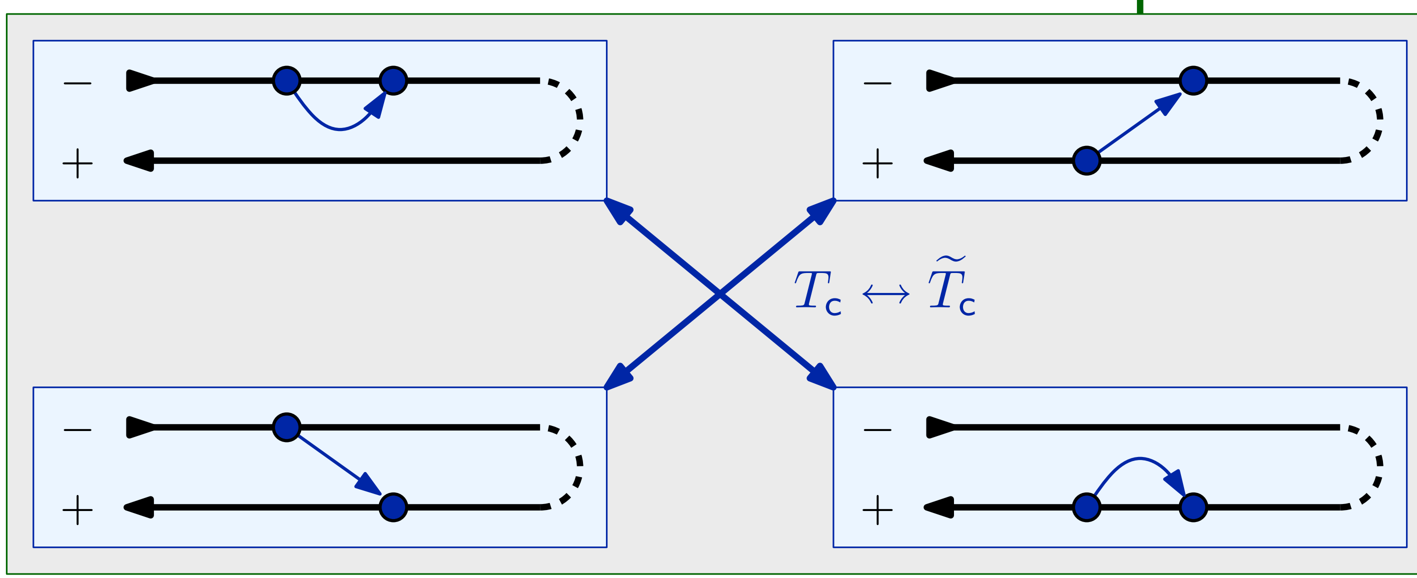
Special case $n = 1$: $\widetilde{G}^{\bar{i}|\bar{i}'} = G^{i'|i}$

$$\Rightarrow \quad e^{\beta\Delta^{i|i'}(\omega|\omega')}\, G^{i|i'}(\omega|\omega') = \zeta^{m^{i|i'}} G^{i'|i}(\omega|\omega'), \qquad n = 1$$

$$\Rightarrow \quad G^{<}(\omega) = \zeta e^{-\beta\omega} G^{>}(\omega)$$

$$\Rightarrow \quad G^{\mathsf{K}}(\omega) = \big[1 + 2\,\zeta\, n_\zeta(\omega)\big]\big[G^{\mathsf{Ret}}(\omega) - G^{\mathsf{Av}}(\omega)\big]$$

$$n_\zeta(\omega) = \frac{1}{e^{\beta\omega} - \zeta}$$



Time reversal

$$e^{\beta\Delta^{i|i'}(\omega|\omega')}\, G^{i|i'}(\omega|\omega') = \zeta^{m^{i|i'}}\, \widetilde{G}^{\bar{i}|\bar{i}'}(\omega|\omega') \quad \text{(KMS)}$$

R time reversal operator, antiunitary

$$\widetilde{q} := Rq$$
$$\widetilde{A} := RAR^\dagger$$

Transformation of creators and annihilators:

$$\widetilde{a_q^\dagger} := a_{\widetilde{q}}^\dagger$$
$$\widetilde{a}_q := a_{\widetilde{q}}$$

Transformation of density matrix $\rho_H = e^{-\beta H} / \mathrm{Tr}\, e^{-\beta H}$:

$$\widetilde{\rho}_H = \rho_{\widetilde{H}}$$

$$\mathrm{Tr}\, \rho_H A(t) = \Big[\mathrm{Tr}\, R\, \rho_H A(t)\, R^\dagger\Big]^* = \Big[\mathrm{Tr}\, \rho_{\widetilde{H}} \widetilde{A}(-t)\Big|_{\widetilde{H}}\Big]^*$$

$$\Rightarrow \quad G^{i|i'}_{q|q'}(t|t') = (-i)^n\, \mathrm{Tr}\, \rho_H T_{\mathsf{c}}\, a_1 \ldots a_n a_n^\dagger \ldots a_1^\dagger = \ldots = \widetilde{G}^{\bar{i}'|\bar{i}}_{\widetilde{q}'|\widetilde{q}}(-t'|-t)\Big|_{\widetilde{H}}$$

$$\Rightarrow \quad e^{\beta\Delta^{i|i'}(\omega|\omega')} G^{i|i'}_{q|q'}(\omega|\omega') = \zeta^{m^{i|i'}}\, G^{i'|i}_{\widetilde{q}'|\widetilde{q}}(\omega'|\omega)\Big|_{\widetilde{H}}$$

Papers claiming to do *without time reversal* (real boson fields):

- Carrington, Hou, Sowiak – Phys. Rev. D **62**, 065003 (2000)
- Wang, Heinz – Phys. Rev. D **66**, 025008 (2002)

$\rightarrow$ not correct

KMS for time reversal invariant GFs

$$e^{\beta\Delta^{i|i'}(\omega|\omega')} G^{i|i'}_{q|q'}(\omega|\omega') = \zeta^{m^{i|i'}} G^{i'|i}_{\widetilde{q}'|\widetilde{q}}(\omega'|\omega)\Big|_{\widetilde{H}}$$

Define *time reversal invariant GF*: $G_{q|q'} = G_{\widetilde{q}|\widetilde{q}'}\Big|_{\widetilde{H}}$

$$\Rightarrow \quad e^{\beta\Delta^{i|i'}(\omega|\omega')} G^{i|i'}(\omega|\omega') = (-1)^n \zeta^{m^{i|i'}} G^{\bar{i}|\bar{i}'}(\omega|\omega')^*$$

not ok:

$$H = \int d^3x\, \psi^\dagger_{\boldsymbol{x}} \frac{\left[-i\nabla - e\boldsymbol{A}(\boldsymbol{x})\right]^2}{2m} \psi_{\boldsymbol{x}}$$

$q = \boldsymbol{x}$

ok:

$$H = \sum_\sigma (\epsilon_0 + \sigma B) a^\dagger_\sigma a_\sigma + \sum_\sigma \int d^3p\, \epsilon_p \psi^\dagger_{\sigma\boldsymbol{p}} \psi_{\sigma\boldsymbol{p}} + \sum_\sigma \int d^3p \left[V_p a^\dagger_\sigma \psi_{\sigma\boldsymbol{p}} + \text{h.c.}\right] + V a^\dagger_\uparrow a_\uparrow a^\dagger_\downarrow a_\downarrow$$

$q = \boldsymbol{x}, \sigma$ or $\boldsymbol{p}, \sigma$

Example: Fermions, $n = 1$

$$\Delta^{-|-}(\omega|\omega') = 0, \quad m^{-|-} = 0$$

$$\Rightarrow \; G^{-|-}(\omega|\omega') = -G^{+|+}(\omega|\omega')^*$$

$$\Delta^{+|-}(\omega|\omega') = -\omega, \quad m^{+|-} = -1$$

$$\Rightarrow \; e^{-\beta\omega} G^{+|-}(\omega|\omega') = -\zeta G^{-|+}(\omega|\omega')^*$$

$G_{q|q'} = G_{q'|q}$

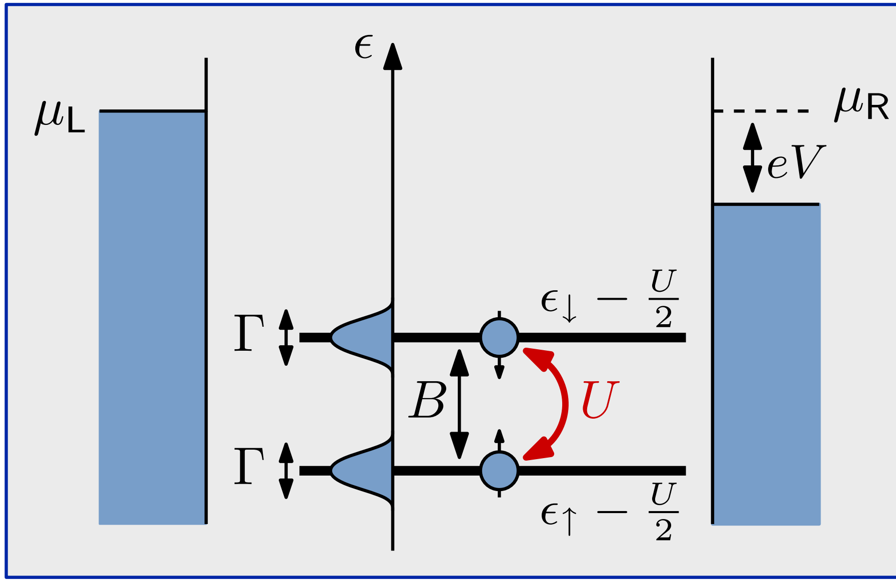
Compare:

$$G^c(\omega) = -G^{\widetilde{c}}(\omega)^\dagger$$

$$e^{-\beta\omega} G^>(\omega) = \zeta G^<(\omega) = -\zeta G^<(\omega)^\dagger$$

The Single Impurity Anderson Model

$$H_{\text{imp}} = \sum_\sigma \epsilon_\sigma n_\sigma + U(n_\uparrow - \tfrac{1}{2})(n_\downarrow - \tfrac{1}{2})$$
$$= \sum_\sigma (\epsilon_\sigma - \tfrac{U}{2}) n_\sigma + U n_\uparrow n_\downarrow + c$$



$$H_{\text{leads}} = \sum_{\alpha=\text{R,L}} \sum_\sigma \int dk\, \epsilon_k n_{\alpha k \sigma}$$

$$H_{\text{coup}} = \sum_{\alpha,\sigma} \int dk\, V_{\alpha k} d_\sigma^\dagger c_{\alpha k \sigma} + \text{h.c.}$$

GF *are* time reversal invariant: $G_\sigma = G_{\widetilde{\sigma}}\big|_{\widetilde{H}}$

Hybridisation function

$$\Gamma_\alpha(\omega) \overset{!}{=} 2\pi \sum_\sigma \int dk\, |V_{\alpha k}|^2\, \delta(\omega - \epsilon_k) \equiv \Gamma_\alpha$$

$$\Gamma = \Gamma_{\text{L}} + \Gamma_{\text{R}}$$

$$g_\sigma^{\text{Ret, Av}}(\omega) = \frac{1}{\omega - (\epsilon_\sigma - \frac{U}{2}) \pm i\Gamma/2}$$

$$g_\sigma^{\text{K}}(\omega) = F(\omega)\left[g^{\text{Av}}(\omega) - g^{\text{Ret}}(\omega)\right]$$

$$F(\omega) = \sum_\alpha \frac{\Gamma_\alpha}{\Gamma}\left[2 f_\alpha(\omega) - 1\right]$$

Hybridisation as flow parameter

Flow parameter:

$$\Gamma \longrightarrow \Gamma_\Lambda = \Gamma + \Lambda$$

$$\Gamma_\alpha \longrightarrow \Gamma_\alpha^{(\Lambda)} = \Gamma_\alpha + \frac{\Gamma_\alpha}{\Gamma}\Lambda$$

with Λ flowing $\infty \longrightarrow 0$

$$g_\Lambda^{\text{Ret, Av}}(\omega) = \frac{1}{\omega - (\epsilon - \frac{U}{2}) \pm \frac{i}{2}\Gamma_\Lambda}$$

$$g_\Lambda^{\text{K}}(\omega) = F(\omega)\left[g_\Lambda^{\text{Av}}(\omega) - g_\Lambda^{\text{Ret}}(\omega)\right]$$

$$F_\Lambda(\omega) = \sum_\alpha \frac{\Gamma_\alpha^{(\Lambda)}}{\Gamma_\Lambda}\left[2f_\alpha(\omega) - 1\right] = F(\omega)$$

Single scale propagator:

$$s_\Lambda^{\text{Ret}} = -\frac{i}{2}\left(g_\Lambda^{\text{Ret}}\right)^2 = \left(s_\Lambda^{\text{Av}}\right)^\dagger, \qquad S_\Lambda^{\text{Ret}} = -\frac{i}{2}\left(G_\Lambda^{\text{Ret}}\right)^2 = \left(S_\Lambda^{\text{Av}}\right)^\dagger$$

$$s_\Lambda^{\text{K}} = F(\omega)\left[s_\Lambda^{\text{Av}} - s_\Lambda^{\text{Ret}}\right], \qquad S_\Lambda^{\text{K}} = \ldots \overset{\text{equil.}}{=} F(\omega)\left[S_\Lambda^{\text{Av}} - S_\Lambda^{\text{Ret}}\right]$$

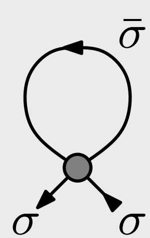
Initial conditions for 1PI-VFs:

$$\Sigma^{\text{Ret,Av}}(\Lambda = \infty) = U/2$$

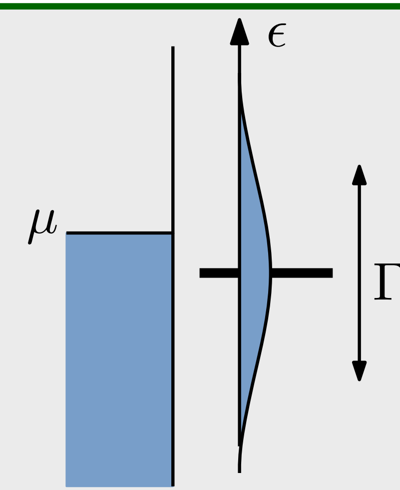
$$\Sigma^{\text{K}}(\Lambda = \infty) = 0$$

$$\gamma_2(\Lambda = \infty) = \bar{v} = \text{bare vertex}$$

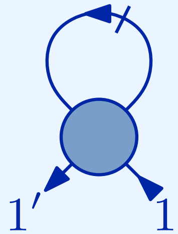
$$\gamma_n(\Lambda = \infty) = 0, \quad n \geq 3$$



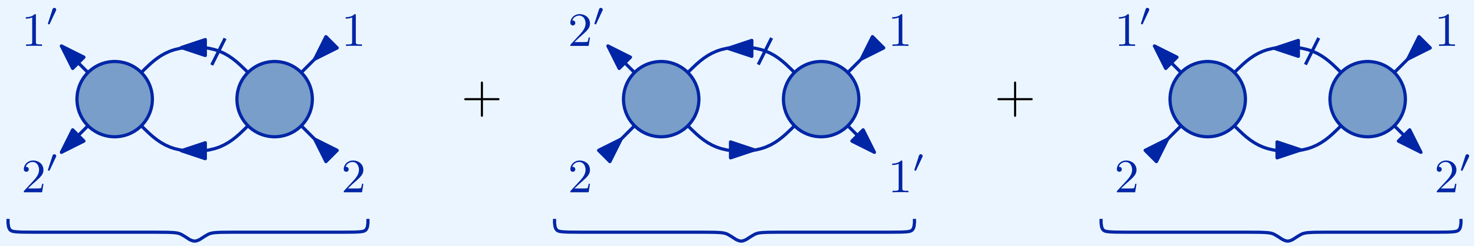
$$= U\langle n_{\bar{\sigma}}\rangle = U\tfrac{1}{2}$$



Flow equations and channels

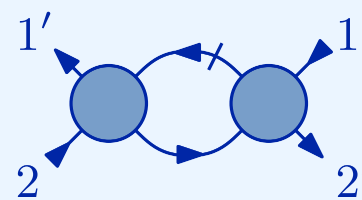
$\dot{\Sigma}(1'|1) \;=$ 

Second order truncation scheme

$\dot{\gamma}(1'2'|12) \;=$ 

pp-channel	dph-channel	xph-channel
$\Omega = \omega_1 + \omega_2 = \omega_1' + \omega_2'$	$\Delta = \omega_1' - \omega_1 = \omega_2 - \omega_2'$	$X = \omega_2' - \omega_1 = \omega_2 - \omega_1'$

Simplest freq. depend. approximation: Keep only one channel.

Example: only xph-channel $\quad \dot{\gamma}(1'2'|12) \;=$ 

$\rightarrow$ yields RPA:

$$\text{at } \Lambda = 0: \quad \gamma^{12|22}_{\sigma\bar{\sigma}|\sigma\bar{\sigma}}(X) \;=\; \frac{U}{2} - i\,\frac{U^2}{2\pi}\,\underbrace{\frac{1}{X - i\left(\frac{\Gamma}{2} - \frac{U}{\pi}\right)}}_{\text{singularity for } U=\frac{\pi}{2}\Gamma} + \mathcal{O}\Big[\Big(\frac{X}{\Gamma}\Big)^2\Big]$$

$(T = 0, B = 0, \text{ph-symm.})$

Minimal coupling of channels

- Each channel feeds back into own flow exactly
- Each channel feeds into flow of other channels as constant (renormalising the interaction)

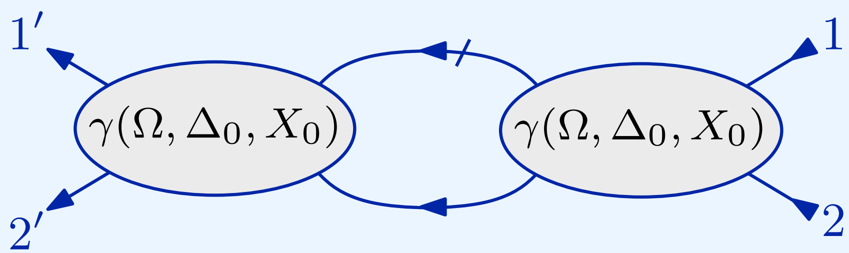
$$\gamma(\Omega,\Delta,X) \;=\; \bar{v} + \phi_{\mathsf{pp}}(\Omega) + \phi_{\mathsf{dph}}(\Delta) + \phi_{\mathsf{xph}}(X)$$

$$\dot{\gamma}(\Omega,\Delta,X) \;=\; \dot{\phi}_{\mathsf{pp}}(\Omega) + \dot{\phi}_{\mathsf{dph}}(\Delta) + \dot{\phi}_{\mathsf{xph}}(X)$$

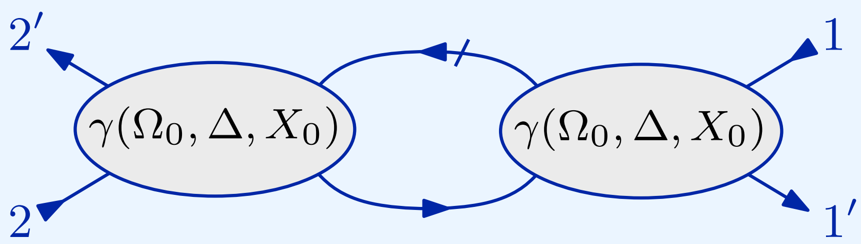
For real constants:

$$\Omega_0 = \mu_{\mathsf{L}} + \mu_{\mathsf{R}}$$
$$\Delta_0 = 0$$
$$X_0 = 0$$

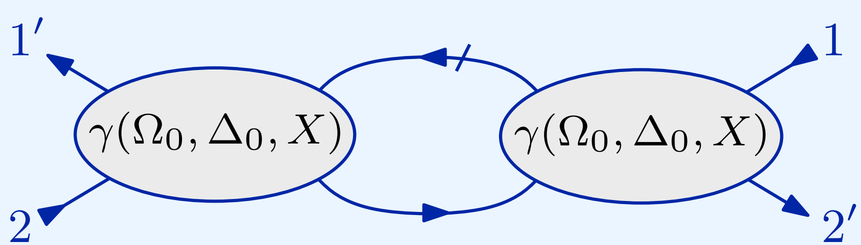
$\dot{\phi}_{\mathsf{pp}}(\Omega) =$



$\dot{\phi}_{\mathsf{dph}}(\Delta) =$



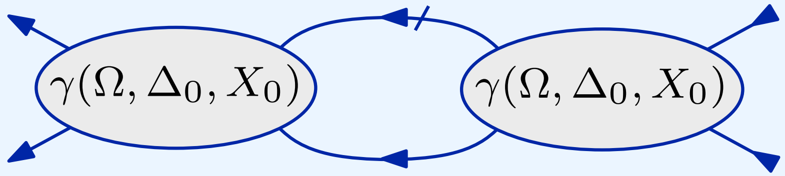
$\dot{\phi}_{\mathsf{xph}}(X) =$



Minimal coupling scheme respects causality relations and KMS

Example: $\phi_{\mathsf{pp}}^{12|22}(\Omega)$ analyt. in uhp of Ω

$\dot\phi^{12|22}_{\mathsf{pp},\,\sigma\bar\sigma|\sigma\bar\sigma}(\Omega) \;=$



$$= \;\dots$$

$$= \frac{i}{2\pi}\Big[\underbrace{\gamma^{12|22}_{\sigma\bar\sigma|\sigma\bar\sigma}(\Omega,\Delta_0,X_0)}_{\text{an. for } \Omega\in \text{uhp}}\Big]^2 \int d\omega\,\Big[\underbrace{G^{\mathsf{Ret}}_{\sigma}(\Omega+\omega)}_{\text{an. for } \Omega\in \text{uhp}} S^{\mathsf{K}}_{\bar\sigma}(-\omega) + \underbrace{S^{\mathsf{Ret}}_{\sigma}(\Omega+\omega)}_{\text{an. for } \Omega\in \text{uhp}} G^{\mathsf{K}}_{\bar\sigma}(-\omega) + (\sigma\leftrightarrow\bar\sigma)\Big]$$

Example: KMS for self energy

$$e^{\beta\Delta^{i'|i}(\omega'|\omega)}\,\dot\Sigma^{i'|i}(\omega'|\omega) = e^{\beta\Delta^{i'|i}(\omega'|\omega)}\frac{-i}{2\pi}\int d\nu d\nu'\,\gamma^{i'j'|ij}(\omega'\nu'|\omega\nu)S^{j|j'}(\nu|\nu')$$

$$\Delta^{i'|i}(\omega'|\omega) = \Delta^{i'j'|ij}(\omega'\nu'|\omega\nu) + \Delta^{j|j'}(\nu|\nu')$$

$$\underset{\text{KMS for } \gamma, S}{=} -\frac{i}{2\pi}\int d\nu d\nu'\,\zeta^{m^{i'j'|ij}}\gamma^{\bar i'\bar j'|\bar i\bar j}(\omega'\nu'|\omega\nu)^*\zeta^{m^{j|j'}}S^{\bar j|\bar j'}(\nu|\nu')^*$$

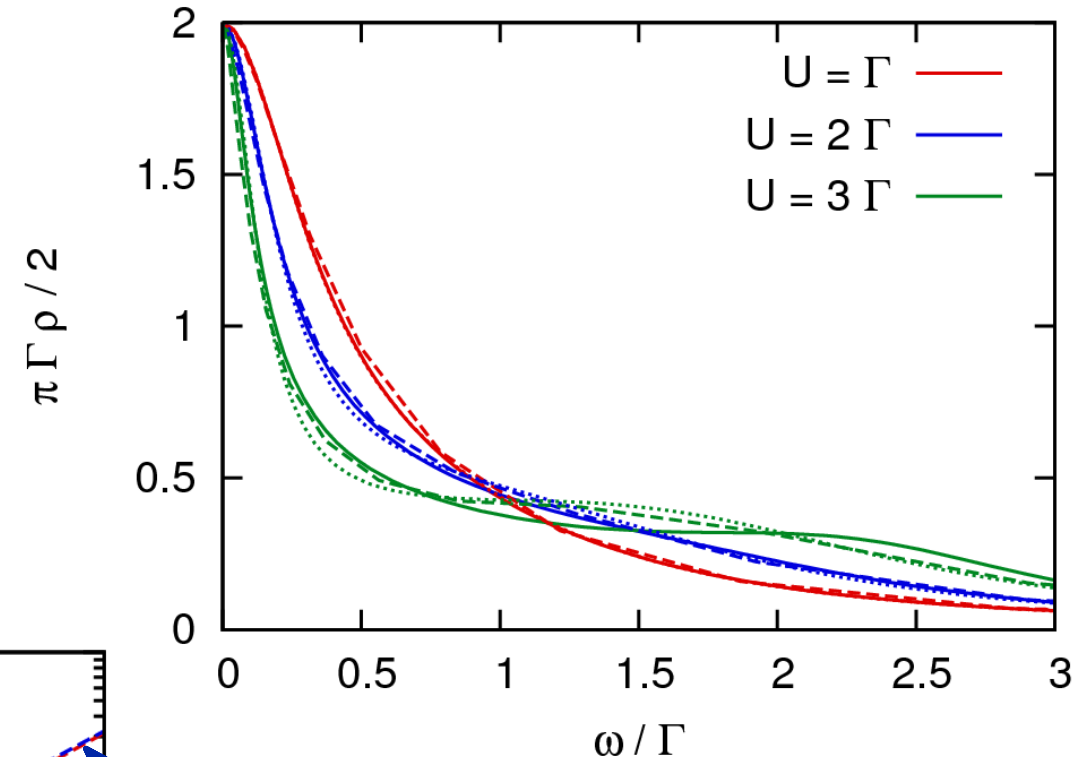
$$m^{i'|i} = m^{i'j'|ij} + m^{j|j'}$$

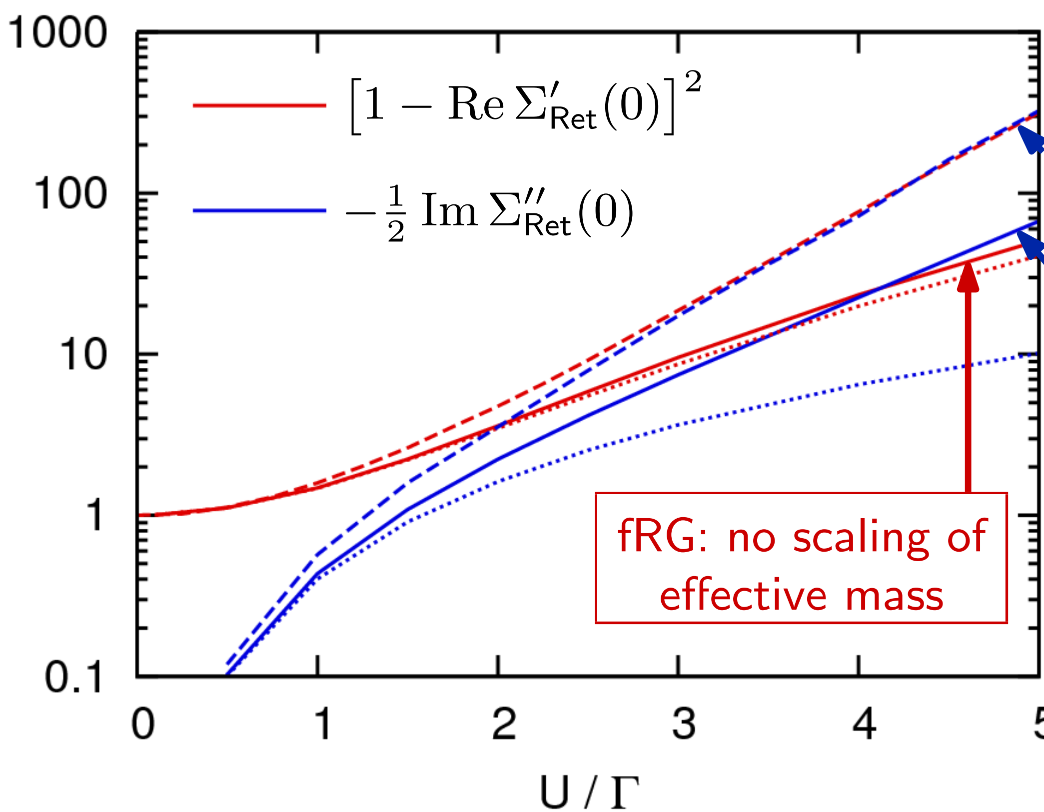
$$= -\zeta^{m^{i'|i}}\dot\Sigma^{\bar i'|\bar i}(\omega'|\omega)^*$$

Results – spectrum and effective mass

Particle hole symmetry,
$T = 0, \quad B = 0, \quad V = 0$

solid: fRG
dashed: NRG (Theo Costi)
dotted: 2nd order PT





$$\mathrm{Im}\,\Sigma_{\mathrm{Ret}}(\omega) \overset{\omega \to 0}{\sim} \frac{\omega^2}{T_K^2}$$

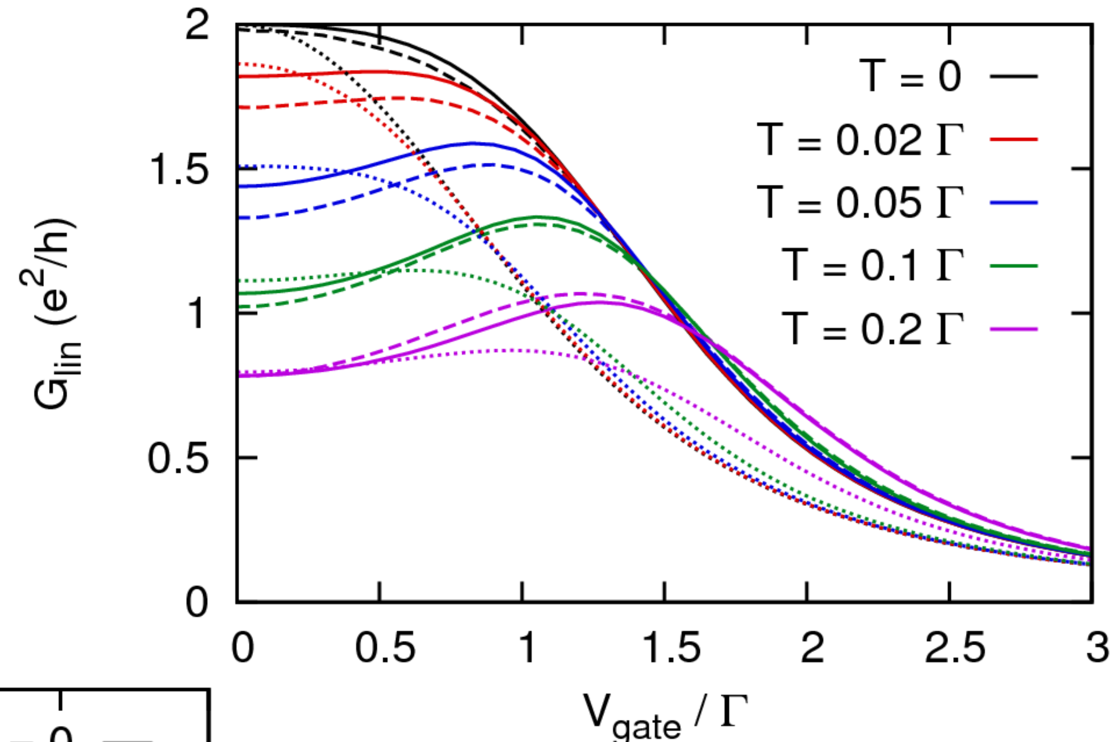
$$T_K(\text{NRG}) \sim \exp\left(-\frac{\pi}{4}\frac{U}{\Gamma}\right)$$

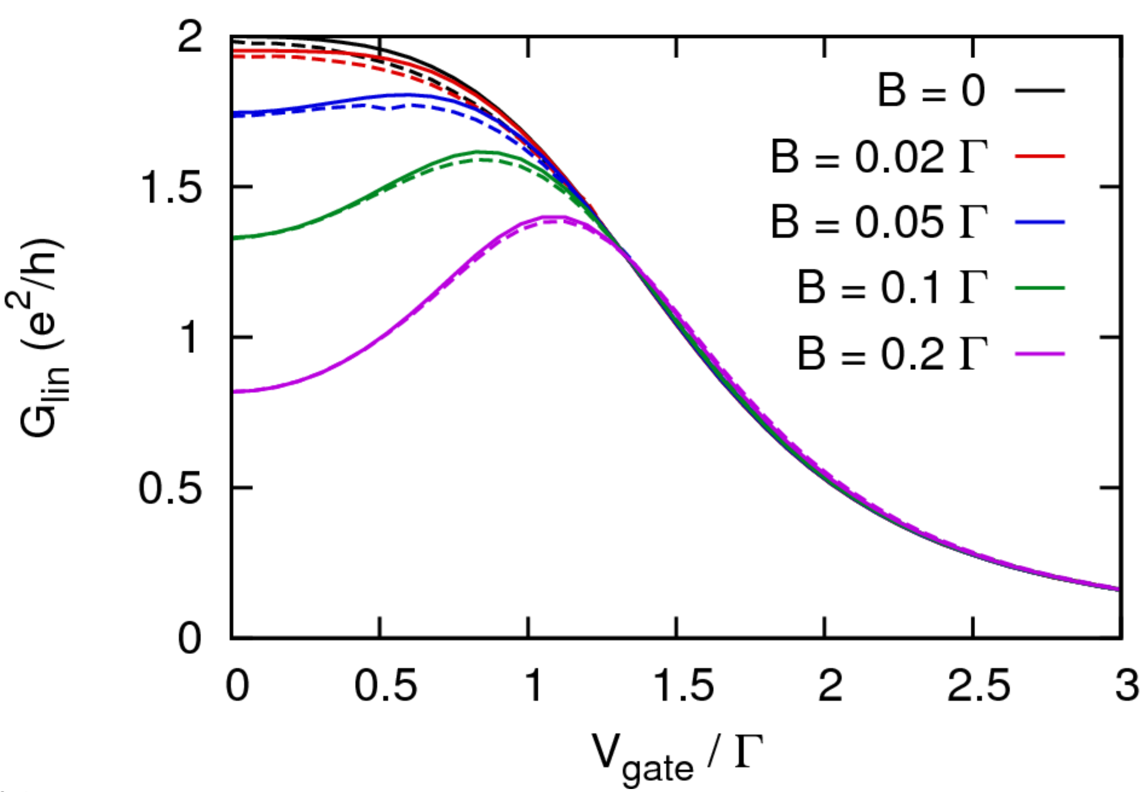
$$T_K(\text{fRG}) \sim \exp\left(-\frac{2}{\pi}\frac{U}{\Gamma}\right)$$

Results – linear conductance

$B = 0, \quad V = 0, \quad U = 3\Gamma$

solid: fRG
dashed: NRG (Theo Costi)
dotted: 2nd order PT
(using restr. HF prop.)



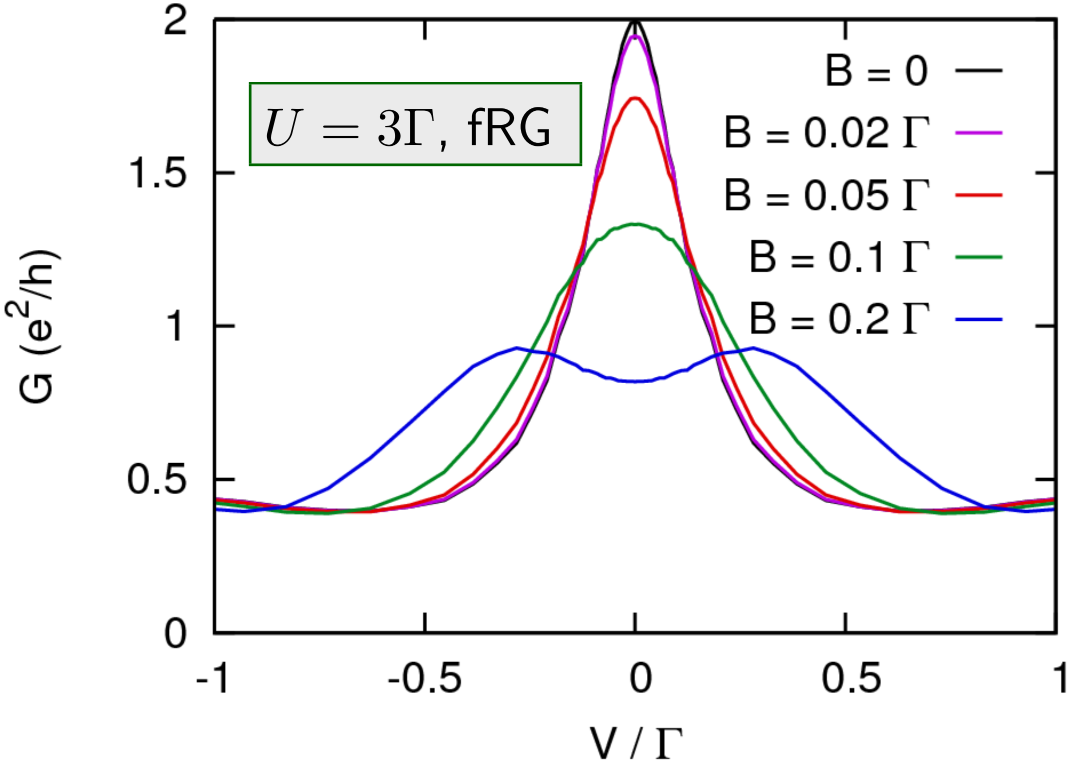


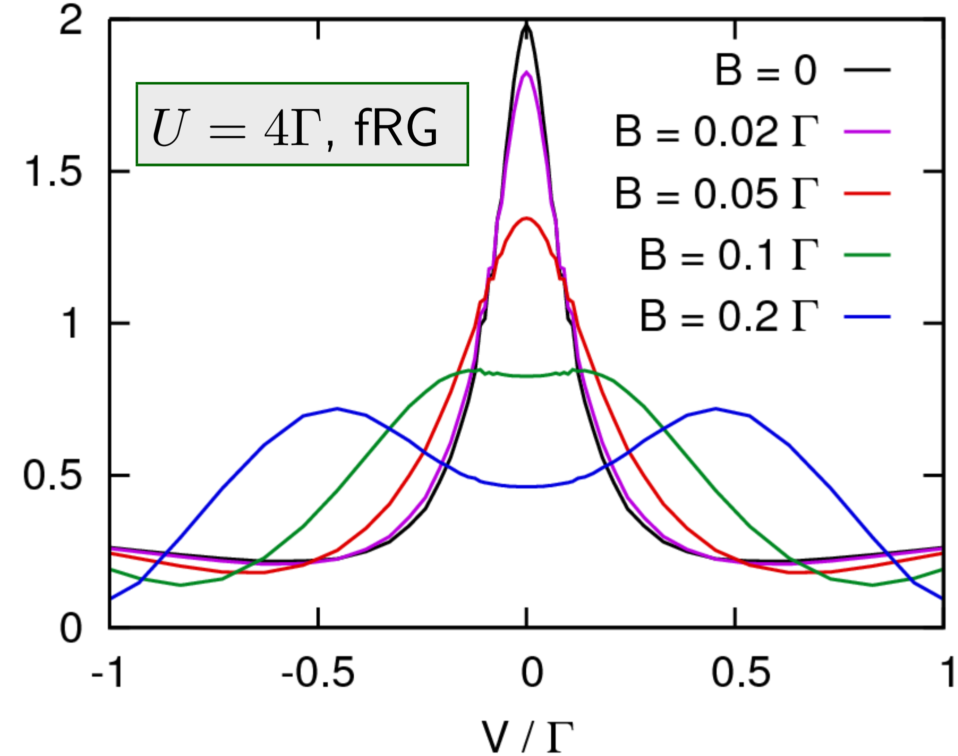
$T = 0, \quad V = 0, \quad U = 3\Gamma$

solid: fRG
dashed: NRG (Theo Costi)

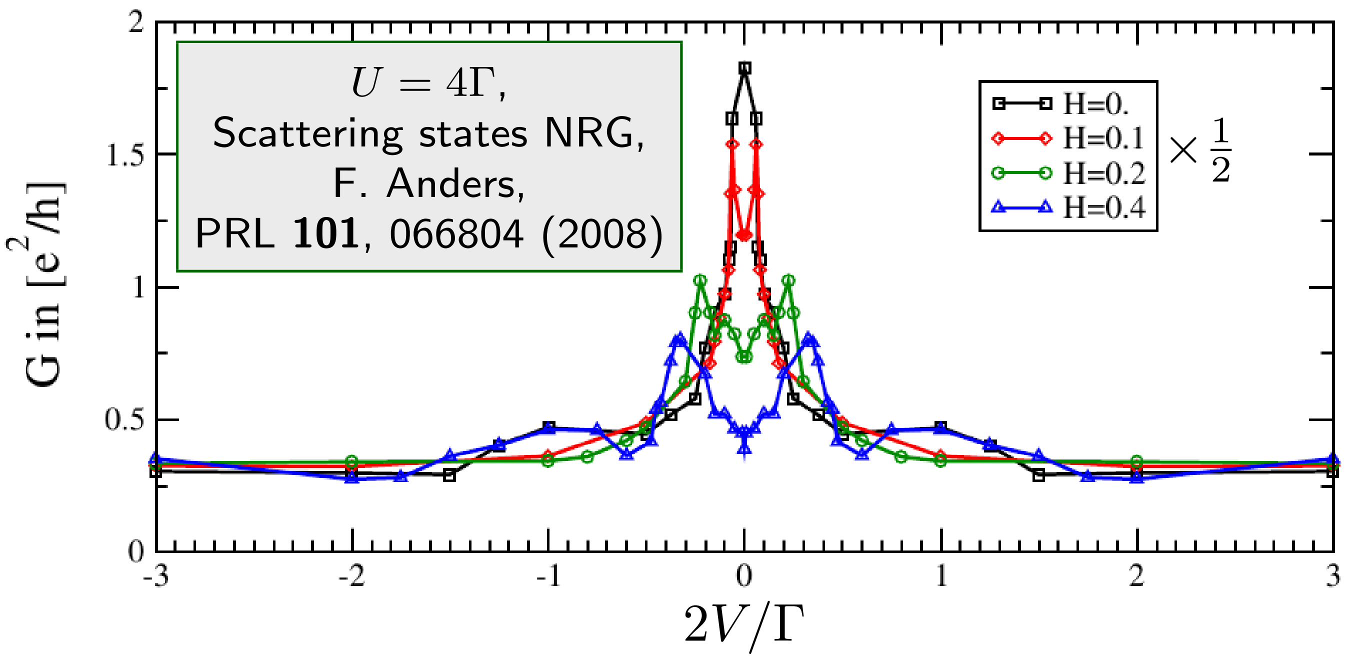
agreement already for static fRG, compare Andergassen et. al., cond-mat/0612229 (Les Houches proceedings)

Results – differential conductance

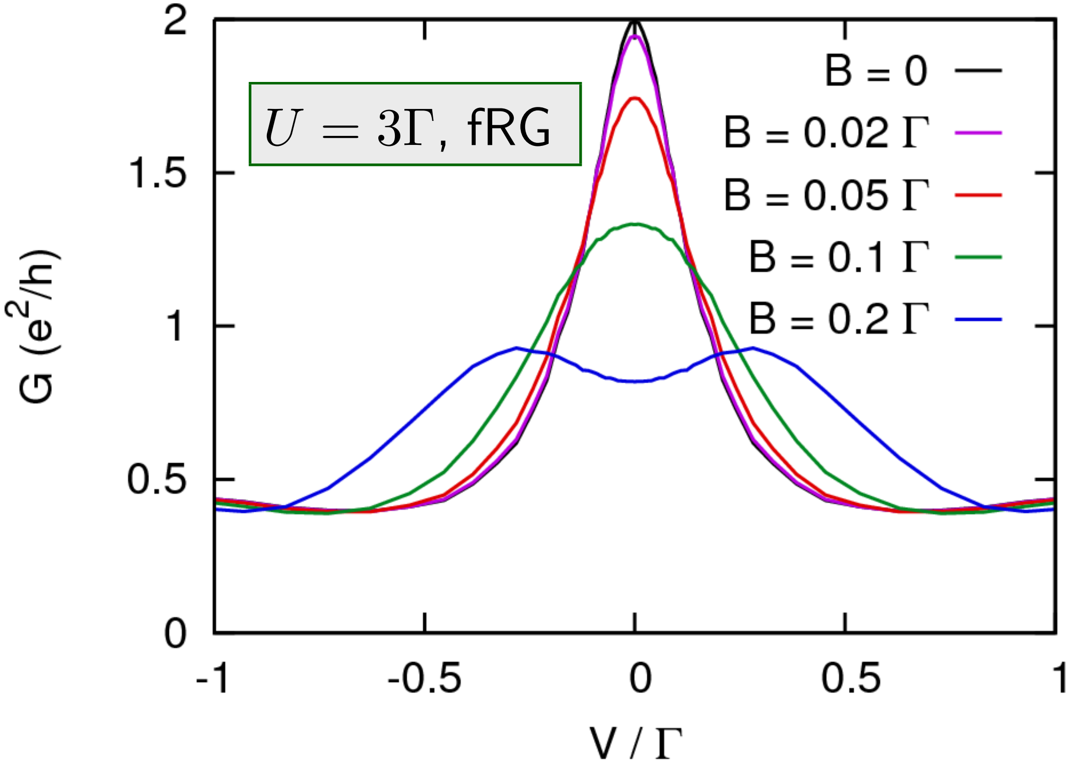


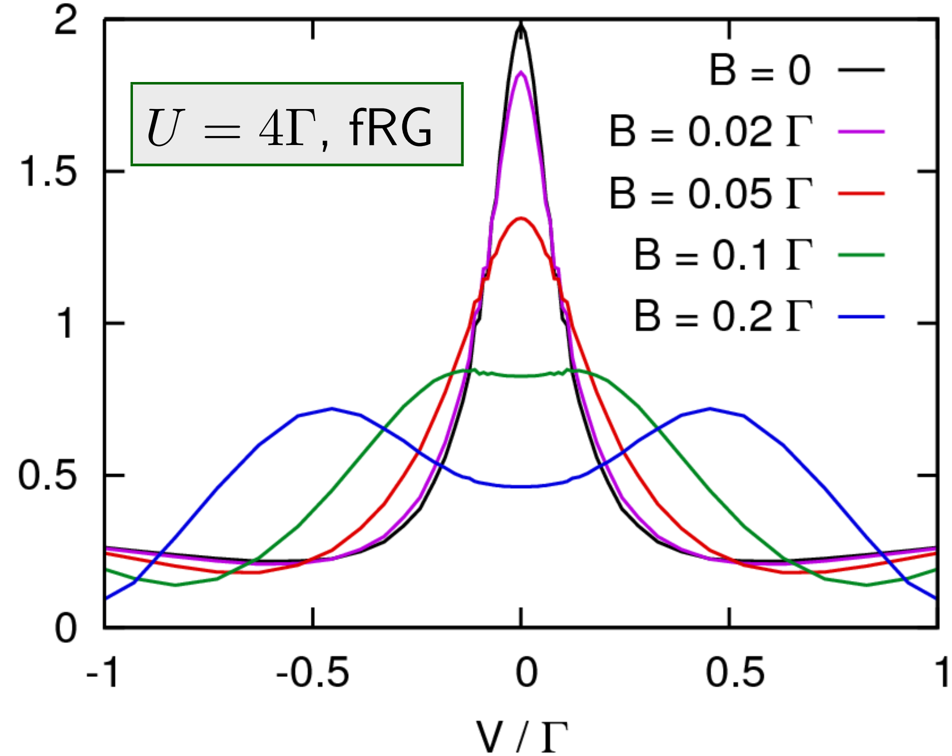


Particle hole symmetry
$T = 0$

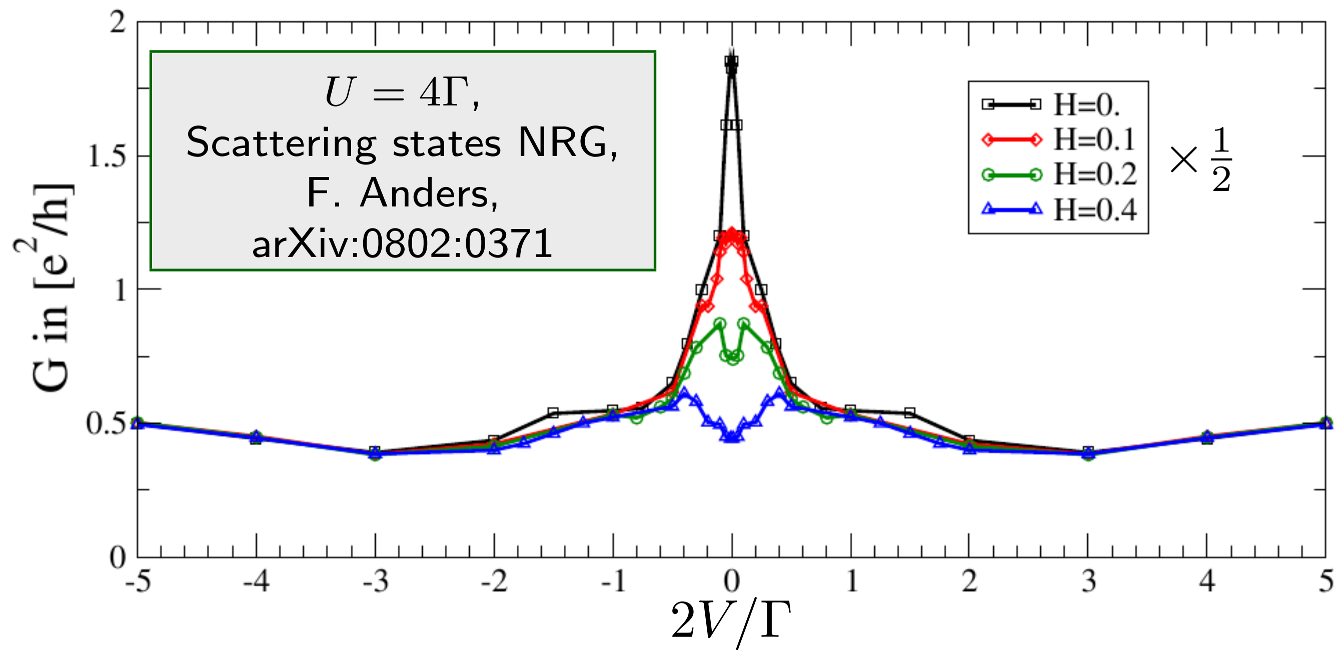


Results – differential conductance





Particle hole symmetry
$T = 0$



Conclusion

- Approximations to frequency dependent vertex functions should respect
  - causal properties (analyticity)
  - KMS-relations (in equilibrium)
- Γ-flow can be designed to do so →
- SIAM can be treated by our Keldysh-fRG for $U \lesssim 3\Gamma$
- Justification for truncation scheme unclear

Properties:

- Easy initial conditions
- Does not manipulate particle distribution
  → compatible with KMS, respects sum rule (SIAM)
- for static fRG in equilibrium (flowing $\epsilon_\Lambda, U_\Lambda$): identical flow equations as Matsubara fRG (SIAM)
- In case of log-divergencies: regularises

$$\sum_{\boldsymbol{k}} \frac{f(\epsilon_{\boldsymbol{k}})}{\epsilon_{\boldsymbol{k}} - \omega + i\eta} \sim \log \frac{\max\{T, |\omega - \mu|, \eta\}}{D}$$

- Flow equation does not replace one summation/integration
  → higher numerical effort

Collaborators

Mikhail Pletyukhov, Herbert Schoeller

Thanks for discussions:

Volker Meden, Christoph Karrasch,
Frank Reininghaus, Sabine Andergassen,
Tilman Enss, Enke Wang, Ulrich Heinz

Thanks for data:

Theo Costi, Frithjof Anders