Ultracold gases and Functional renormalization II

Michael Scherer (Jena)

Work in collaboration with

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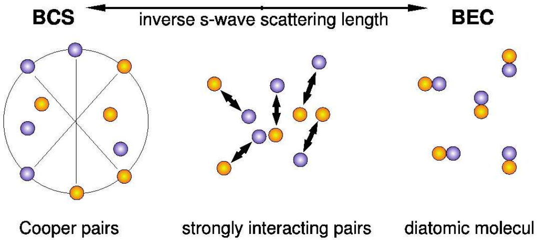
FOR 723 Retreat, LadenbuRG 2008

The BCS-BEC Crossover

Ultracold gases of fermionic atoms near Feshbach resonance:
Crossover between BCS superfluidity and BEC of molecules.



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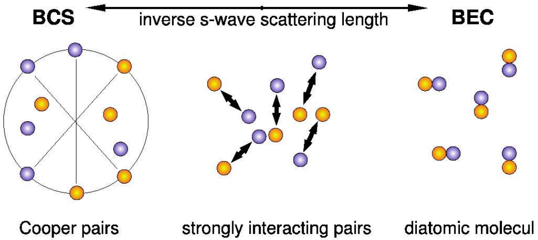

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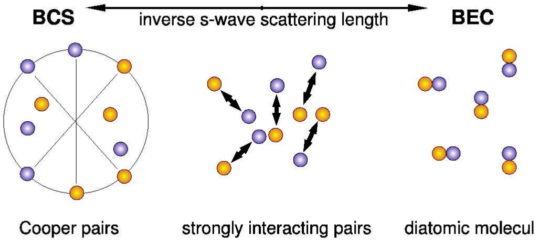
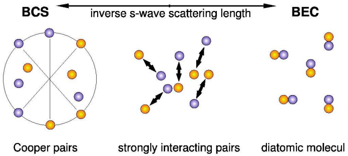


- Stefan: Basics of FRG for ultracold fermions, Thermodynamics, BEC-side
- Today: BCS-side (Particle-hole fluctuations, Rebosonization,...), Unitarity regime

Parametrization

Crossover can be parametrized by the dimensionless inverse s-wave scattering length $c^{-1} \rightarrow$. Experimental realization by the phenomenon of Fesbach resonances.



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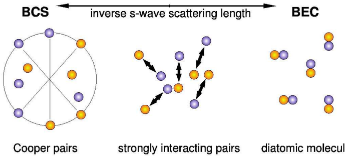

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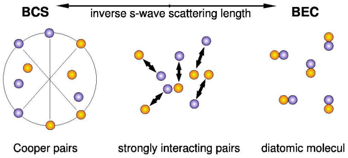

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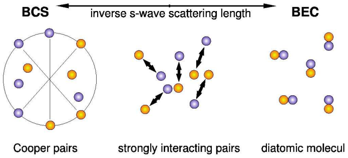

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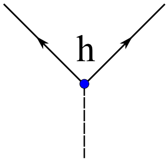
- $|c^{-1}| < 1$: Strongly correlated regime, Unitarity limit at $c^{-1} \rightarrow 0$

Universality

- Limit of broad Feshbach resonances (experiments, e.g. with ^{6}Li and ^{40}K)



Universality

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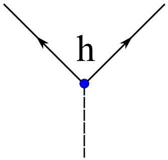
- Universality: Thermodynamic quantities are independent of the microscopic details and can be expressed in terms of only two dimensionless parameters: The concentration $c = ak_F$ and the temperature T/T_F. The units are set by the density $n = k_F^3/(3\pi^2)$.



Microscopic action

$$S = \int_0^{1/T} d\tau \int_{\vec{x}} \Big\{ \quad \psi^\dagger(\partial_\tau - \Delta - \mu)\psi + \phi^*(\partial_\tau - \frac{\Delta}{2} - 2\mu + \nu)\phi$$

$$- \quad h(\phi^* \psi_1 \psi_2 + h.c.) \Big\} \, .$$

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where $\lambda_{\psi,\text{eff}}$ is a momentum-dependent effective four-fermion vertex.

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In momentum space the effective four-fermion vertex reads

$$\lambda_{\psi,\mathrm{eff}} = -\frac{h^2}{P_\phi(q)}\,,$$

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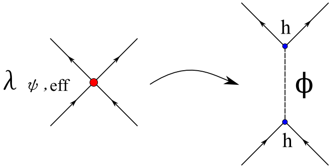
The inverse process, going from a purely fermionic theory to a theory of fermions and bosons, is called "bosonization".



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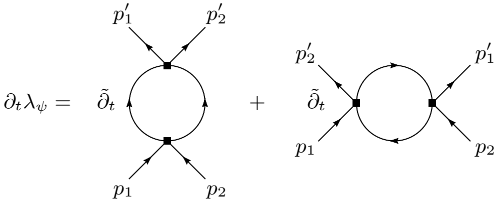
A generally momentum dependent four-fermion interaction is renormalized. The flow of λ_ψ has two contributions:



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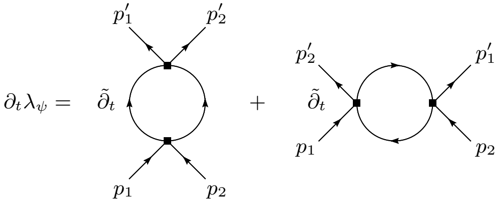

The first one is referred to as particle-particle loop, the second one as particle-hole loop.

BCS-theory in RG language I

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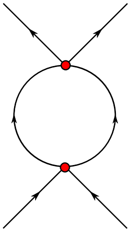

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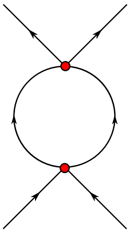

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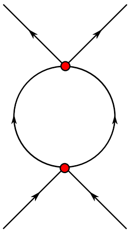

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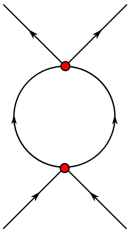

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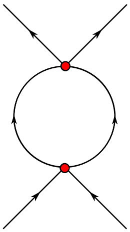

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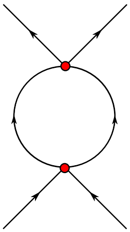
- The temperature at which $1/\lambda_\psi \to 0$ at the scale $k = 0$ is the BCS transition temperature $T_{c,\mathrm{BCS}}$



BCS-theory in RG language II

Within BCS theory the outer momenta are averaged over the Fermi surface and the critical temperature is found to be

$$\frac{T_{c,\mathsf{BCS}}}{T_F} \approx 0.61 e^{-\frac{\pi}{2k_F}|a^{-1}|} .$$

Here a is the (vacuum) s-wave scattering length.

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For $k \to 0$, $\mu \to 0$, $T \to 0$, $n \to 0$:

$$a = \frac{\lambda_\psi}{8\pi}$$

Gorkov's correction to BCS-theory I

- pp-loop diverges for $T \longrightarrow 0$ leading to a transition to superfluidity, ph-loop remains finite

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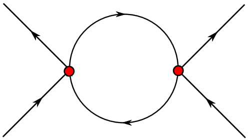
$\Rightarrow$ Any shift in $\left(\lambda^{\text{eff}}_{\psi,\Lambda}\right)^{-1}$ results in a multiplicative factor for T_c.

Gorkov's correction to BCS-theory II



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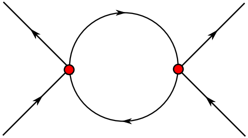

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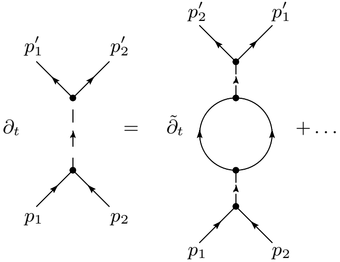
$$T_c = \frac{1}{(4e)^{1/3}} T_{c,\mathsf{BCS}} \approx \frac{1}{2.2} T_{c,\mathsf{BCS}} \,.$$

This is the Gorkov effect (1963).

Bosonization

In a bosonized language, the fermionic interaction is described by boson exchange



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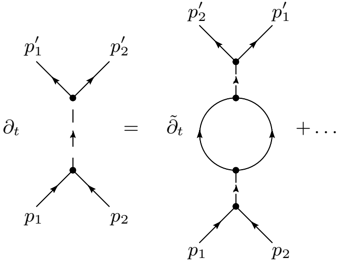

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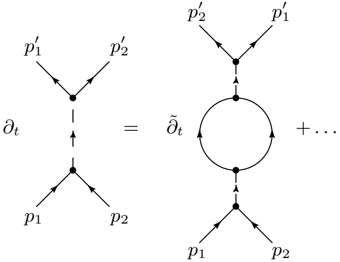

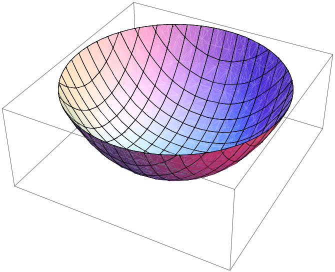

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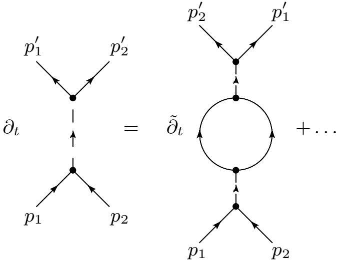

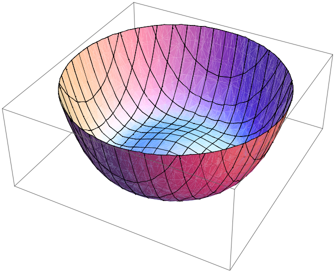

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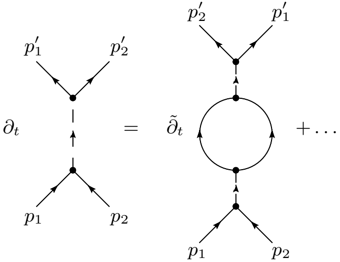

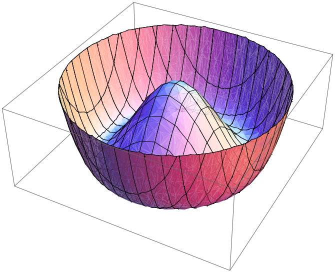

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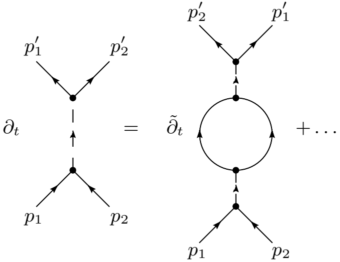

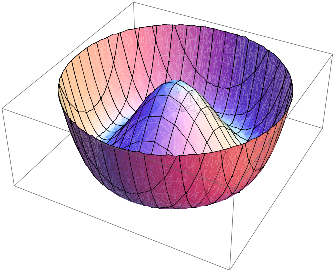

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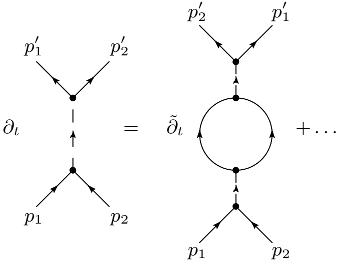

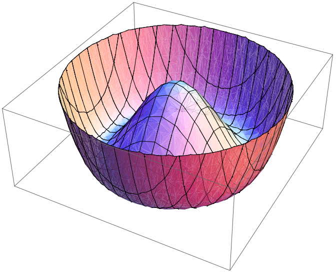

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In this setting, where the bosonization took place only on the microscopic scale, we do not account for particle-hole fluctuations.

Bosonization is destroyed by the RG flow

We neglected so far, that the term

$$\int_{\tau,\vec{x}} \lambda_\psi \psi_1^\dagger \psi_1 \psi_2^\dagger \psi_2 \, ,$$

is re-generated by the flow.

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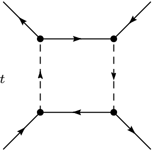

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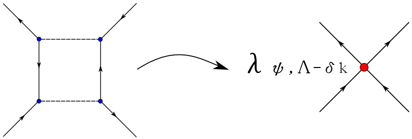

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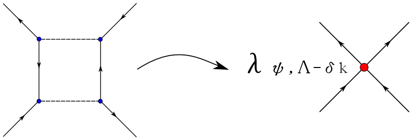

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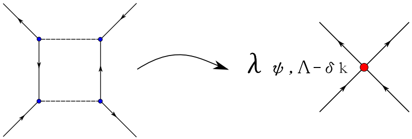

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$\rightarrow$ Connection between divergence of $\lambda_{\psi,\text{eff}}$ and SSB?

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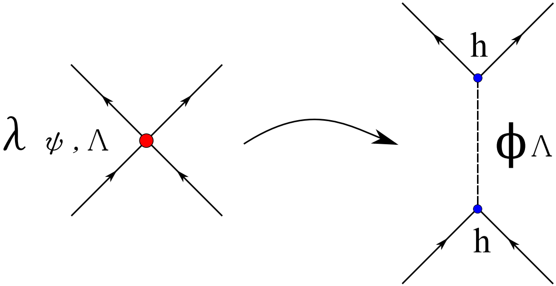

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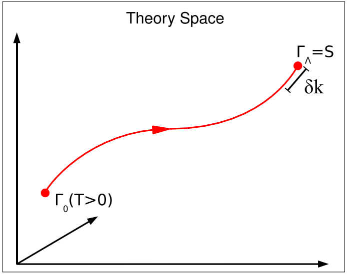

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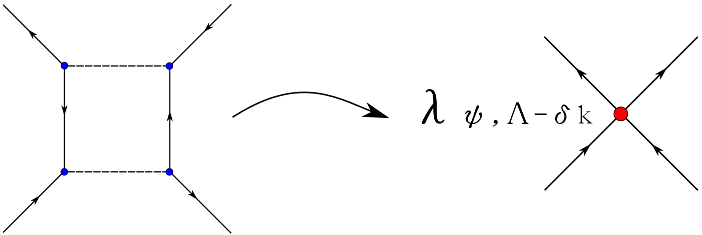

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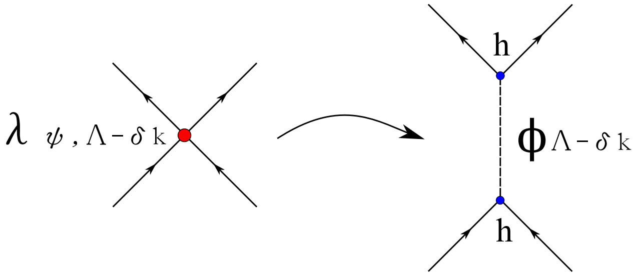

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- (Re-)appearance of a λ_ψ by the flow of the box diagrams can be absorbed by the introduction of scale dependent fields ϕ_k

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For scale dependent fields we obtain a modified flow equation (Gies & Wetterich, 2001)

$$\partial_k \Gamma_k[\chi_k] = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1}\partial_k R_k\right] + \int \frac{\delta\Gamma_k}{\delta\chi_k}\partial_k\chi_k\,.$$

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We choose the following scale dependence for the bosonic fields.

$$\partial_k \bar{\phi}_k(q) \;\; = \;\; (\psi_1 \psi_2)(q) \partial_k \upsilon \,.$$

With $\partial_k \upsilon$ to be determined.

Rebosonization III

In consequence the flow equations in SYM get modified

$$\begin{aligned}\partial_k \bar{h} &= \partial_k \bar{h}\big|_{\bar{\phi}_k} - \bar{m}^2 \partial_k \upsilon\,,\\ \partial_k \lambda_\psi &= \partial_k \lambda_\psi\big|_{\bar{\phi}_k} - 2\bar{h}\partial_k \upsilon.\end{aligned}$$

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Now, the four-fermion interaction is purely given by the boson exchange and ph-fluctuations are incorporated via the second term in the latter equation.

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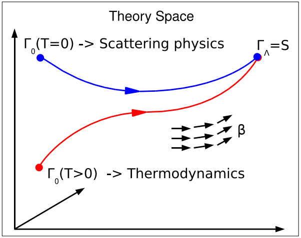

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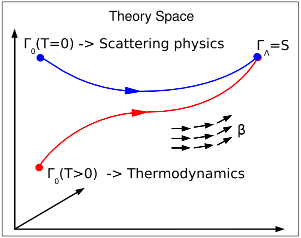

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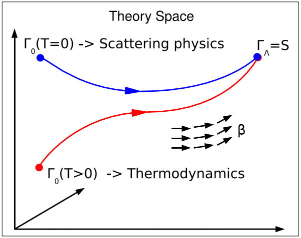

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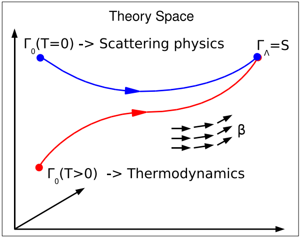
- The temperature for which $m_0^2 \to 0$ as $k \to 0$ is T_c. This is directly related to the divergence of the effective four-fermion coupling $\lambda_{\psi,eff} \propto \frac{-h^2}{m^2} \to \infty$.



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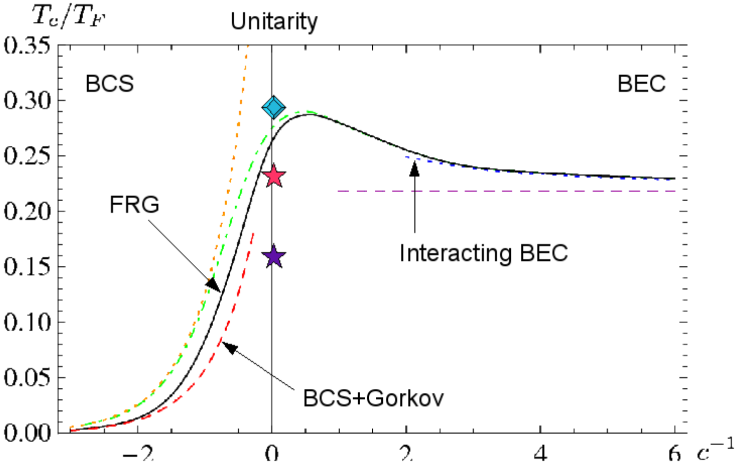

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Questions?