z.B.* harmonischer Oszillator $\quad m\ddot{x} = -kx$

$$V(x) = \frac{1}{2}kx^2 \quad , \quad x(t_0=0) = x_0 = 0$$

zur Einfachheit.

$$\rightarrow \pm\sqrt{\frac{m}{2}}\int_0^x \frac{d\bar{x}}{\left(E-\frac{1}{2}k\bar{x}^2\right)^{1/2}} = t$$

geht ebenfalls mit direkter Lsg. von Newtonschaukel ?!

$$\bar{x} = \left(\frac{2E}{k}\right)^{1/2}\bar{y}$$

$$\rightarrow \left(\frac{m}{\cancel{2E}}\right)^{1/2}\left(\frac{\cancel{2E}}{k}\right)^{1/2}\underbrace{\int_0^y \frac{d\bar{y}}{(1-\bar{y}^2)^{1/2}}} = t$$

$$\bar{y} = \sin\bar{\varphi} \quad , \quad d\bar{y} = \cos\bar{\varphi}\, d\bar{\varphi}$$

$$\int_0^\varphi d\bar{\varphi}\,\frac{\cancel{\cos\bar{\varphi}}}{\cancel{(1-\sin^2\bar{\varphi})^{1/2}}} = \varphi \;!$$

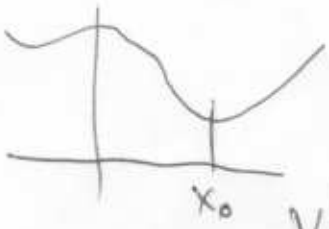
$$x = \left(\frac{2E}{k}\right)^{1/2}\sin\varphi = \left(\frac{2E}{k}\right)^{1/2}\sin\underbrace{\left(\frac{k}{m}\right)^{1/2}}_{\omega} t \quad \checkmark$$

o.k.

$$E = \frac{k}{2}x_{max}^2 \;! \quad \checkmark \text{ check}$$

wichtige Bem.

beliebiges Potential $V(x)$ mit Minimum bei $x = x_0$:



entwickle dort nach Taylor

$$V(x) = V(x_0) + V'(x_0)(x-x_0) + \frac{V''(x_0)(x-x_0)^2}{2!} + \dots$$

$V' = -F = 0$ am Minimum $\Rightarrow$ $V(x) \approx V(x_0) + \frac{V''(x_0)(x-x_0)^2}{2!}$

(harm. Oszillator!