ml^2 übernimmt Rolle von M in obiger Betrachtung

erhalte:

$$t - t_0 = \pm \int_{\varphi_0}^{\varphi} \frac{d\bar{\varphi}}{\left(2\frac{(E - mgl(1-\cos\bar{\varphi}))}{ml^2}\right)^{1/2}}$$

$E \leftrightarrow \varphi_0$ bei maximalem Ausschlag ($\dot{\varphi} = 0$)

o.B.d.A. dort $t_0 = 0$ $\quad (\dot{\varphi}(0) = 0)$

$\varphi(0) = \varphi_0$

$E = mgl(1 - \cos\varphi_0)$

$$\Rightarrow t = \pm \int_{\varphi_0}^{\varphi} \frac{d\bar{\varphi}}{\left(\frac{2mgl(\cos\varphi_0 + \cos\varphi)}{ml^2}\right)^{1/2}}$$

$$= \pm\left(\frac{l}{2g}\right)^{1/2} \int_{\varphi_0}^{\varphi} \frac{d\bar{\varphi}}{(\cos\bar{\varphi} - \cos\varphi_0)^{1/2}}$$

$\cos\varphi - \cos\varphi_0 = 2\left(\sin^2\left(\frac{\varphi_0}{2}\right) - \sin^2\left(\frac{\varphi}{2}\right)\right)$!

Etwas trickreicher als harmon. Oszillator!

$$= \pm \frac{1}{2}\left(\frac{l}{g}\right)^{1/2} \int_{\varphi_0}^{\varphi} \frac{d\bar{\varphi}}{\left(\sin^2\frac{\varphi_0}{2} - \sin^2\frac{\varphi}{2}\right)^{1/2}}$$

Substitution: $\sin\varphi = \frac{\sin\varphi/2}{\sin\varphi_0/2}$

$\varphi = \varphi_0 \rightsquigarrow \varphi = \frac{\pi}{2}$

$\cos\varphi\, d\varphi = \frac{\cos\varphi/2}{\sin\varphi_0/2}\, d\varphi/2 = \frac{\left(1 - \sin^2\varphi \sin^2\frac{\varphi_0}{2}\right)^{1/2}}{\sin\varphi_0/2}\, d\varphi/2$

$$= \pm \frac{1}{2}\sqrt{\frac{l}{g}} \int_0^{\varphi} \frac{2\, d\bar{\varphi}\cos\bar{\varphi}}{(1 - \sin^2\bar{\varphi})^{1/2}} \cdot \left(1 - \sin^2\varphi \sin^2\frac{\varphi_0}{2}\right)^{-1/2}$$

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