Quantum Field Theory I, 8th problem sheet

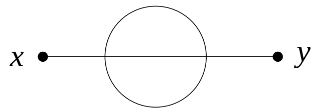
Discussion of solutions in the tutorials of Dec 20th

Problem 1: Position and momentum space Feynman rules

Consider a real scalar field with a ϕ^4 interaction term:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4.$$

1. Draw all connected Feynman graphs that contribute to the 2-point function, up to 2-loop order.
2. Write down the position-space expression represented by the graph



and perform the Fourier transforms to momentum space to see the momentum-space Feynman rules emerge.

Problem 2: The generating functional for connected correlation functions

In the lecture the generating functional Z for correlation functions of a real scalar field theory was introduced:

$$Z[J] = \langle 0|T \exp\left(i\int d^4x\,\phi(x)J(x)\right)|0\rangle,$$

$$\frac{1}{i^n}\frac{\delta}{\delta J(x_1)}\cdots\frac{\delta}{\delta J(x_n)}Z[J]\bigg|_{J=0} = \langle 0|T\phi(x_1)\ldots\phi(x_n)|0\rangle.$$

We define the generating functional W for *connected* correlation functions by $W[J] = i\log Z[J]$. Inspect the connected 4-point function (i.e. the contributions to the 4-point function from connected Feynman diagrams) and show that it is indeed generated by $W[J]$.

Problem 3: Feynman rules for the complex scalar field

Consider a complex scalar field with a $|\phi|^4$ interaction term:

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi)^* - m^2 |\phi|^2 - \frac{\lambda}{4} |\phi|^4 .$$

Work out the Feynman rules for this theory. In particular, discuss what changes in Wick's theorem with respect to the case of real scalar fields, to show that lines in Feynman graphs must now carry an orientation.