Towards Bridging the Gap Between Quarks and Gluons and Baryonic Degrees of Freedom

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05/07/2008
From Microscopic Degrees to Macroscopic DoF

Renormalization Group
From Microscopic Degrees to Macroscopic DoF

Density Functional Renormalization Group flows
(JB, J. Polonyi, A. Schwenk)
From Microscopic Degrees to Macroscopic DoF

**Functional Renormalization Group flows:**
Today’s talk ...
(JB, H. Gies, J. M. Pawlowski)
Challenges in QCD

\[ \alpha_s(Q) \]

Data
- Deep Inelastic Scattering
- e+e- Annihilation
- Hadron Collisions
- Heavy Quarkonia

Theory
- NLO
- NNLO
- Lattice

\[ \Lambda_{MS}^{(5)} \]

QCD
- 245 MeV \( \cdot \) 0.1209
- 210 MeV \( \cdot \) 0.1182
- 180 MeV \( \cdot \) 0.1155

Bethke '04

Gluons
Quarks
Challenges in QCD

- **Asymptotic freedom** at high momenta (Gross & Wilczek ’73, Politzer ’73)

- Running coupling exhibits Landau pole at small momenta → pQCD fails

- Understanding of QCD in the mid-momentum regime is needed to study *confinement* & *chiral symmetry* breaking
Heavy-Ion Collision Experiments

measured relative abundances

fitted very well with Boltzmann-distribution

\[ (T, \mu_B) \text{ for given } \sqrt{s} \]

chemical freeze-out

\[ T_{\text{exp.}} \leq T_{\chi} \]

P. Braun-Munzinger, J. Stachel, C. Wetterich '03
measured relative abundances

fitted very well with Boltzmann-distribution $(T, \mu_B)$ for given $\sqrt{s}$

chemical freeze-out

Difference

$\Delta = T_\chi - T_{\text{exp.}}$

depends on the order of the phase transition
QCD phase diagram?

perturbation theory fails:
• not convergent even for very high temperatures:
  strongly interacting theory even at high $T$
• phase transitions: long-range fluctuations are important
QCD phase diagram? Many open questions!

Strong indications that there is no critical point at all (de Forcrand, Philipsen '07)

Low-energy models (QM/NJL): parameter dependence
QCD phase diagram? Many open questions!

curvature for vanishing $\mu_q$ from the Lattice:

$t_2 = -\pi^2 T_c(0) \frac{d^2 T_c(\mu_q)}{d\mu_q^2} \bigg|_{\mu_q=0}$

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QCD phase diagram? Many open questions!

$N_f = 2 + 1$
QCD phase diagram? Many open questions!

Aoki et al. ’06:
Deconfinement phase transition and chiral phase transition do not occur at the same temperature: $T_{\chi S B} < T_d$

in contradiction to experiments and (naive) expectations?! (and to Chen et al.)
QCD phase diagram? Many open questions!

Why is there difference even at $T=0$?

- different lattice sizes
- different implementation of fermions
QCD phase diagram from QCD RG flows?

- **pros**
  - continuum formulation
  - chiral fermions
  - no sign-problem

- **cons**
  - truncated eff. action

complementary to Lattice QCD
Outline

✓ Motivation

• Functional Renormalization Group

• Chiral Phase Boundary of QCD

• Polyakov-Loop and (De-)Confinement Phase Transition

• Conclusions and Outlook
Functional Renormalization Group

“Theory space”: spanned by all couplings

well-defined starting point: fix strong coupling at UV scale

χPT or NJL, Quark-Meson-model, PNJL

perturbative QCD

Γ_{k=Λ}
Functional Renormalization Group

\[ \partial_t \Gamma_k = \frac{1}{2} \text{Tr} \frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k} \]

(\text{C. Wetterich '93})

“Theory space”: spanned by all couplings
Functional Renormalization Group

\[ \partial_t \Gamma_k = \frac{1}{2} \text{Tr} \frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k} \]

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"Theory space": spanned by all couplings

RG flow
\[ \partial_t \Gamma_k = \frac{1}{2} \text{Tr} \frac{\partial_t R_k}{\Gamma^{(2)}_k + R_k} \]

"Theory space": spanned by all couplings

(C. Wetterich '93)
Functional Renormalization Group

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“Theory space”: spanned by all couplings
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• Chiral Phase Boundary of QCD
  • Quark-gluon dynamics and the chiral phase boundary
  • QCD with one quark flavor: from quarks and gluons and mesons

• Polyakov-Loop and (De-)Confinement Phase Transition

• Conclusions and Outlook
Aspects of the NJL model

• classical action of the NJL model:

\[ S = \int x \left\{ \bar{\psi} i \partial \psi + \bar{\lambda}_\sigma [ (\bar{\psi} \psi)^2 - (\bar{\psi} \gamma_5 \psi)^2 ] \right\} \]

• spontaneous symmetry breaking if quark condensate is non-vanishing: \( \langle \bar{\psi} \psi \rangle \neq 0 \)
Aspects of the NJL model

- Classical action of the NJL model:
  \[
  S = \int \left\{ \bar{\psi} i \partial_x \psi + \bar{\lambda}_\sigma \left[ (\bar{\psi} \psi)^2 - (\bar{\psi} \gamma_5 \psi)^2 \right] \right\}
  \]

- Bosonization of the NJL model yields \((\sigma = -2\bar{\lambda}_\sigma \bar{\psi} \psi, \pi = -2\bar{\lambda}_\sigma \bar{\psi} \gamma_5 \psi)\)
  \[
  S = \int \left\{ \bar{\psi} i \partial_x \psi + \bar{\psi} (\sigma + i \gamma_5 \pi) \psi - \frac{1}{\bar{\lambda}_\sigma} (\sigma^2 + \pi^2) \right\}
  \]

\(\bar{\lambda}_\sigma\) is inverse proportional to the scalar mass parameter, \(m^2 \propto \frac{1}{\bar{\lambda}_\sigma}\)
Four-Fermion Interactions in QCD

• at the UV scale \( k = \Lambda \gg \Lambda_{\text{QCD}} \):

\[
\Gamma_\Lambda = \int_x \left\{ \frac{1}{4} F^a_{\mu\nu} F^{a}_{\mu\nu} + \bar{\psi}(i\partial + \bar{g} A)\psi \right\}
\]
Four-Fermion Interactions in QCD

• at the UV scale \( k = \Lambda \gg \Lambda_{QCD} \):

\[
\Gamma_\Lambda = \int_x \left\{ \frac{1}{4} F^{a}_{\mu \nu} F^{a}_{\mu \nu} + \bar{\psi} (i \partial + gA) \psi \right\}
\]

\[
k = \Lambda - \delta k
\]

\[
\Gamma_{\Lambda-\delta k} = \int_x \left\{ \frac{1}{4} F^{a}_{\mu \nu} F^{a}_{\mu \nu} + \bar{\psi} (i \partial + gA) \psi + \frac{\lambda \sigma}{2k^2} [ (\bar{\psi}^a \psi^b)^2 - (\bar{\psi}^a \gamma_5 \psi^b)^2 ] + \ldots \right\}
\]

• quark-gluon dynamics generate four-fermion interactions
RG flow for the chiral QCD sector

- effective action:

\[ \Gamma_k = \int_x \left\{ \frac{g^2}{g^2} F_{\mu \nu}^a F_{\mu \nu}^a + w_2 (F_{\mu \nu}^a F_{\mu \nu}^a)^2 + w_3 (F_{\mu \nu}^a F_{\mu \nu}^a)^3 + \ldots \right\} \]

\[ + \int_x \left\{ \bar{\psi} (iZ \psi \partial + Z_1 \bar{g} A) \psi + \frac{1}{2} \left[ \frac{\lambda_-}{k^2} (V - A) + \frac{\lambda_+}{k^2} (V + A) \right] \]

\[ + \frac{\lambda_\sigma}{k^2} (S - P) + \frac{\lambda_{VA}}{k^2} [2 (V - A)^{\text{adj}} + (1/N_c) (V - A)] \right\} \}

- no Fierz-ambiguity

- four-fermion interactions \( \left( \lim_{\Lambda \to \infty} \lambda_i = 0 \right) \)

- truncation checks: momentum dependencies, regulator dependencies, higher order interactions (H. Gies, J. Jaeckel, C. Wetterich '04)
“Criticality” at zero and finite temperature

- flow of four-fermion couplings:

\[ \partial_t \lambda = 2\lambda - \lambda A \left( \frac{T}{k} \right) \lambda - b \left( \frac{T}{k} \right) \lambda \alpha_s - c \left( \frac{T}{k} \right) \alpha_s^2 \]
"Criticality" at zero and finite temperature

• Critical gauge coupling $\alpha_{cr}$:
  
  if $\alpha_s > \alpha_{cr}$ \quad no fixed points \quad $\chi_{SB}$

• At zero temperature: (H. Gies, J. Jaeckel '05)
  
  $N_c = N_f = 3$  
  $\alpha_{cr} \approx 0.85$
"Criticality" at zero and finite temperature

- Critical gauge coupling $\alpha_{cr}$:
  
  \[
  \text{if } \alpha_s > \alpha_{cr} \rightarrow \text{no fixed points} \rightarrow \chi_{SB}
  \]

- At finite temperature: (JB, H. Gies '05)
  
  \[
  \alpha_{cr}(T/k) > \alpha_{cr}(T = 0)
  \]
  
  quarks acquire a thermal mass
RG flow of gluodynamics

\[ p^2_{g,0} \equiv \omega_n^2 = 4n^2\pi^2T^2 \quad \rightarrow \quad \omega_0^2 = 0 \]

• \( k_{max} \propto T \) decoupling of hard gluonic modes \( \rightarrow \) “finite-size” effect:

\[ \alpha_{4D} \approx \alpha_{3D}^* \frac{k}{T} + \mathcal{O} \left( (k/T)^2 \right) \quad \text{with} \quad \alpha_{3D}^* \approx 2.7; \ \eta_{3d} \rightarrow 1 \]

SDE: v. Smekal et al. '97, Fischer et al. '02;
RG: Pawlowski et al. '04, Fischer&Gies '04;
Gies '02; Gies&Braun '05/
Lattice: e. g. Sternbeck et al. '05; ...

cf. vertex expansion in Landau-gauge QCD:

Yang-Mills theory: strong interactions at high temperatures (JB, H. Gies '06; Lattice: Cucchieri et al. '07)
Chiral Phase Transition in QCD

- study: $\alpha_{cr}(T/k)$ vs. $\alpha_s(T/k)$
- intersection point of $\alpha_{cr}$ and $\alpha_s$ indicates onset of $\chi^SB$

• single input parameter: $\alpha_s(m_\tau) = 0.322$

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Chiral Phase Transition in QCD: Error estimate

- study: $\alpha_{cr}(T/k)$ vs. $\alpha_s(T/k)$
- intersection point of $\alpha_{cr}$ and $\alpha_s$ indicates onset of $\chi^{SB}$

- single input parameter: $\alpha_s(m_\tau) = 0.322 \pm 0.03$

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(JB, H. Gies ’06)
What to expect for QCD with many flavors?

- one-loop $\beta$-function

\[
\partial_t \alpha \equiv \beta(\alpha) = -\frac{1}{6\pi} (11N_c - 2N_f) \alpha^2
\]

- $b_1 < 0 \implies N_f > \frac{11}{2} N_c^{N_c=3} 16.5$ (QCD is NOT asymptotically free)
What to expect for QCD with many flavors?

• one-loop $\beta$-function

\[ \partial_t \alpha \equiv \beta(\alpha) = -\frac{1}{6\pi} (11N_c - 2N_f) \alpha^2 \]

• $b_1 < 0 \implies N_f > \frac{11}{2} N_c \quad N_c=3 \quad 16.5 \quad \text{(QCD is NOT asymptotically free)}$

• $b_1 > 0$: QCD is asymptotically free
Many-flavor QCD

- small $N_f$ : fermionic screening
- critical number of quark flavors: $N_{f,cr} = 12$ (cf. e.g. Appelquist '07 & '96)
- "conformal phase" for $N_{f,cr} < N_f < 16.5$: asymptotic freedom but no $\chi_{SB}$
Many-flavor scaling regime

- **fixed-point regime** for large $N_f$: critical exponent $|\Theta|

\[ \partial_t g^2 \approx |\Theta| (g^2 - g_*^2) \]

- **shape of the phase boundary** for $N_f \approx N_{f,cr}$ (JB, H. Gies '06)

\[ T_{cr} \propto |N_f - N_{f,cr}| \frac{1}{|\Theta|} \]  with  $|\Theta| \approx 0.71$

(currently under investigation on the lattice, Deuzeman et al. '08)
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• Conclusions and Outlook
Challenge:
How to penetrate the phase boundary in order to get access to the low-energy observables?
From microscopic to macroscopic DoFs

\[ \partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \frac{\partial_t R_k}{\Gamma_k^{(2)}[\phi] + R_k} \]

- Macroscopic DoF
- Microscopic DoF

RG flow

large length scales  \( k \rightarrow 0 \)

small length scales  \( k \rightarrow \Lambda \)
From microscopic to macroscopic DoFs: Do it by hand

macroscopic DoF

For example:
(constituent-quark-) meson-model

\[ h \bar{\psi} (\sigma + i \gamma_5 \pi) \psi + m^2 (\sigma^2 + \pi^2) \]

RG flow

bosonization at fixed scale

Hubbard-Stratonovich transformation

microscopic DoF

quark-gluon dynamics

\[ \bar{\lambda}_\sigma [ (\bar{\psi} \psi)^2 - (\bar{\psi} \gamma_5 \psi)^2 ] \]
From microscopic to macroscopic DoFs

- problem:

\[ m^2 \sim \frac{1}{\lambda_\sigma} \]

bosonization at fixed scale \( k \)

at scale \( k - \delta k \): \( h \bar{\psi} \psi, g \bar{\psi} A \psi \)

'comeback'

generate four-fermion Interaction
From microscopic to macroscopic DoFs

\[ \partial_t \phi_k \sim \bar{\psi}_L \psi_R \]

solution: scale-dependent degrees of freedom

\[ \partial_t \Gamma_k [\phi_k] = \frac{1}{2} \text{Tr} \left( \Gamma_k^{(2)} [\phi_k] + R_k \right) - \int_x \frac{\delta \Gamma_k [\phi_k]}{\delta \phi_k} \partial_t \phi_k \]

(H. Gies, C. Wetterich '01, '02; J. M. Pawlowski '05)
QCD with one quark flavor

• ansatz:
\[ \Gamma_k = \int_x \left\{ \bar{\psi}(i\slashed{D} + i\gamma_0\mu_q)\psi + \frac{\bar{\lambda}_\sigma}{2} [(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2] + Z_\phi \partial_\mu \phi^* \partial_\mu \phi + U(\phi^2) + \bar{h}[(\bar{\psi}_R\psi_L)\phi - (\bar{\psi}_L\psi_R)\phi^*] \right\} + \Gamma_{gauge} \]

• initial conditions: $\bar{\lambda}_\sigma|_\Lambda = 0$, $\bar{\lambda}_\phi|_\Lambda = 0$, $\bar{h}|_\Lambda = 0$, $Z_\phi|_\Lambda = 0$, $\alpha_s(M_Z) = 0.117$

• allows to include (momentum-dependent) four-fermion interactions to arbitrary order can be easily included

• serves as a check for the approach incorporating “only” quark-gluon dynamics

\[
\begin{align*}
\frac{m_q(T)}{m_q(0)}
\end{align*}
\]
QCD with one quark flavor: phase boundary

\[
\frac{T_c(\mu_q)}{T_c(0)} = 1 - t_2 \left( \frac{\mu_q}{\pi T_c(0)} \right)^2 + \ldots
\]

- large \( N_c \) expansion: \( t_2 \sim \frac{N_f}{N_c} \)

- results from different approaches:

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<td>FRG: QCD flow</td>
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<td>(J.B. '08, in prep.)</td>
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*red*: obtained from extrapolation
QCD with one quark flavor: phase boundary

\[ \frac{T_c(\mu_q)}{T_c(0)} = 1 - t_2 \left( \frac{\mu_q}{\pi T_c(0)} \right)^2 + \ldots \]

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red: obtained from extrapolation

• only one single input parameter: \( \alpha_s(M_Z) \)
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Confinement at zero temperature

Potential of a quark-antiquark pair: \( \mathcal{F}_{q\bar{q}}(r) \propto \sigma r \)
confinement at finite temperature

- Hamilton operator for an electron in an EM-field:

\[ H_{ED} = \frac{1}{2m} \left( -i \vec{\nabla} - e \vec{A} \right)^2 + e\Phi \]
confinement at finite temperature

• **infinitely heavy quark** moving in Euclidean time direction:

\[
\frac{\partial \Psi_q}{\partial \tau} = i \bar{g} A_0 \Psi_q \quad \Rightarrow \quad \Psi_q(\vec{x}, \tau) = \left[ \text{P} \exp \left( i \bar{g} \int_0^\tau dt A_0 \right) \right] \Psi_q(\vec{x}, 0)
\]

infinitely heavy quark propagating in (Euclidean) time direction

• **Polyakov-Loop:** \( \tau = \beta = 1/T \) \cite{Polyakov78, Susskind79}

\[
P(\vec{x}) = \frac{1}{N_c} \text{P} \exp \left( i \bar{g} \int_0^\beta dt A_0(t, \vec{x}) \right)
\]
confinement at finite temperature

- expectation value of Polyakov-loop is related to the quark free energy $\mathcal{F}_q$:

$$\langle \text{Tr}_F \mathcal{P}(\vec{x}) \rangle \sim \int \mathcal{D}A \text{Tr}_F \mathcal{P}(\vec{x}) e^{-S} \sim e^{-\beta \mathcal{F}_q}$$

- deconfinement:
  $$\mathcal{F}_q \text{ finite } \iff \langle \text{Tr}_F \mathcal{P}(\vec{x}) \rangle \neq 0$$

- confinement:
  $$\mathcal{F}_q \to \infty \iff \langle \text{Tr}_F \mathcal{P}(\vec{x}) \rangle = 0$$
confinement at finite temperature

• quark-antiquark correlator:

\[
\lim_{|\vec{x} - \vec{y}| \to \infty} e^{-\beta \mathcal{F}_{q\bar{q}}} \sim \lim_{|\vec{x} - \vec{y}| \to \infty} \langle \text{Tr} \mathcal{P}(\vec{x}) \cdot \text{Tr} \mathcal{P}^\dagger(\vec{y}) \rangle \leq |e^{-\beta \mathcal{F}_q}|^2
\]

\[
\mathcal{F}_{q\bar{q}} \sim \sigma |\vec{x} - \vec{y}|
\]

\[
\mathcal{F}_q \to \infty \iff \langle \text{Tr}_F \mathcal{P}(\vec{x}) \rangle = 0
\]

\[
\mathcal{F}_q \text{ finite} \iff \langle \text{Tr}_F \mathcal{P}(\vec{x}) \rangle \neq 0
\]

deconfinement
confinement criterion at vanishing temperature

(JB, H. Gies, J. M. Pawlowski '07)

• (RG) Polyakov-loop potential in Landau-background-field-gauge

\[ V(\beta\langle A_0 \rangle) = \frac{1}{\Omega T^4} \left( \frac{1}{2} \text{Tr} \ln \Gamma^{(2)}_A [\beta \langle A_0 \rangle] - \text{Tr} \ln \Gamma^{(2)}_{gh} [\beta \langle A_0 \rangle] \right) + \mathcal{O}(\partial_t \Gamma^{(2)}) \]

• (very) high-temperature: potential is dominated by modes \( k \sim p \sim T \)

\[ (\Gamma^{(2)}_A) \sim D^2[\langle A_0 \rangle], \quad (\Gamma^{(2)}_{gh}) \sim D^2[\langle A_0 \rangle] \]
perturbative Polyakov-Loop potential

\[ V(\beta\langle A_0 \rangle) \]

- perturbative Polyakov-loop potential in background-field gauge, \( A_\mu = \delta_{\mu 0} \langle A_0 \rangle \)

minimum at \( \beta\langle A_0 \rangle = 0 \):
- deconfinement (broken \( Z_3 \)-symmetry)

\[ \mathcal{F}_q \text{ finite} \iff \langle \text{Tr}_F \mathcal{P}(\vec{x}) \rangle \neq 0 \]
confinement criterion at vanishing temperature

(RG) Polyakov-loop potential in Landau-background-field-gauge

\[ V(\beta\langle A_0 \rangle) = \frac{1}{\Omega T^4} \left( \frac{1}{2} \text{Tr} \ln \Gamma^{(2)}_A[\beta\langle A_0 \rangle] - \text{Tr} \ln \Gamma^{(2)}_{gh}[\beta\langle A_0 \rangle] \right) + \mathcal{O}(\partial_t \Gamma^{(2)}) \]

low-temperature: \( k \sim p \sim T \lesssim \Lambda_{QCD} \)

\[ (\Gamma^{(2)}_A) \sim (D^2[\langle A_0 \rangle])^{1+\kappa_A}, \quad (\Gamma^{(2)}_{gh}) \sim (D^2[\langle A_0 \rangle])^{1+\kappa_{gh}} \]

what if ... \[ 3\kappa_A - 2\kappa_{gh} < -2 \]

\[ \kappa_{gh} > \frac{d - 3}{4} \]
perturbative Polyakov-Loop potential

- perturbative Polyakov-loop potential in background-field gauge, $A_\mu = \delta_\mu 0 \langle A_0 \rangle$

V(β⟨A0⟩)

minimum at $β\langle A_0 \rangle = 0$:
derconfinement (broken $Z_3$-symmetry)

$\mathcal{F}_q$ finite $\iff$ $\langle \text{Tr}_F \mathcal{P}(\vec{x}) \rangle \neq 0$

- for $T < T_c$

V(β⟨A0⟩)

minimum at $β\langle A_0 \rangle = (2/3)2\pi$:
derconfinement (broken $Z_3$-symmetry)

$\mathcal{F}_q \rightarrow \infty$ $\iff$ $\langle \text{Tr}_F \mathcal{P}(\vec{x}) \rangle = 0$
confinement criterion at vanishing temperature

\[ V(\beta\langle A_0 \rangle) = \frac{1}{\Omega T^4} \left( \frac{1}{2} \text{Tr} \ln \Gamma_A^{(2)}[\beta\langle A_0 \rangle] - \text{Tr} \ln \Gamma_{gh}^{(2)}[\beta\langle A_0 \rangle] \right) + O(\partial_t \Gamma^{(2)}) \]

\textbf{low-temperature:} \quad k \sim p \sim T \lesssim \Lambda_{QCD}

\begin{align*}
(\Gamma_A^{(2)}) & \sim (D^2[\langle A_0 \rangle])^{1+\kappa_A}, \\
(\Gamma_{gh}^{(2)}) & \sim (D^2[\langle A_0 \rangle])^{1+\kappa_{gh}}
\end{align*}

\textbf{quark confinement criterion}  \quad \text{(Landau gauge):}

\[ \langle \text{Tr}_F \mathcal{P}(\vec{x}) \rangle = 0 : \quad 3\kappa_A - 2\kappa_{gh} < -2 \quad \Rightarrow \quad \kappa_{gh} > \frac{d-3}{4} \]

\textbf{quark confinement induced by IR gluon suppression}

\textbf{confer:} \quad \text{Kugo-Ojima criterion: } \kappa_{gh} > 0 \quad \text{Gribov-Zwanziger condition: } \kappa_{gh} > \frac{1}{2}

(Kugo, Ojima '79) \quad (Gribov '78; Zwanziger '94,'03)
Landau-gauge propagators & color confinement

\[(\Gamma_A^{(2)})^{-1}_{\text{IR}} \rightarrow \frac{1}{(p^2)^{1-2\kappa}}\]

\(0.539 \leq \kappa \leq 0.595\)

- **Method**
  - DSE/SQ
  - FRG
  - Lattice

<table>
<thead>
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*(Pawlowski, Litim, Nedelko, v. Smekal '03; Fischer, Gies '04)

*(Lerche, v. Smekal '02; Zwanziger '02)

*(Pawlowski, Litim, Nedelko, v. Smekal '03; Fischer, Gies '04)

*(Sternbeck et al.'05; Olivera, Silva '06; Cucchieri, Mendes '06; Cucchieri, Mendes '07; Sternbeck et al.'07)*
Polyakov-Loop Potential in Landau-gauge

- order parameter \( L[\langle A_0 \rangle] = \frac{1}{N_c} \text{Tr}_F \exp \left( i \int_0^\beta dt \langle A_0 \rangle \right) \)

- first order phase transition for SU(3) (and second order for SU(2))

- SU(3): \( T_c = 284 \text{ MeV} \left( = 0.646 \sqrt{\sigma} \right) \)

Lattice QCD: \( T_c = 0.646 \sqrt{\sigma} \) (Kaczmarek et al.)
Outline

✓ Motivation

✓ Functional Renormalization Group

✓ Chiral Phase Boundary of QCD

✓ Polyakov-Loop and (De-)Confinement Phase Transition

• Conclusions and Outlook
Conclusions

• FRG allows to bridge the gap between regimes with different DoF

• good agreement with Lattice QCD studies for chiral as well as deconfinement phase transition

• critical number of quark flavors for SU(3): $N_{f,cr} = 12$

• shape of the phase boundary near $N_{f,cr}$ is determined by the underlying IR fixed point scenario (testable prediction!)

• promising results for finite chemical potential

• criterion for quark confinement
Outlook

• study finite chemical potential for $N_f = 2$ and $N_f = 3$
  ➞ first ‘shot’: phase diagram from bosonization at a fixed scale
    (with H. Gies, J. M. Pawlowski, B. J. Schaefer)

• order of the phase transition?
  (from finite-volume scaling, with B. Klein)

• deconfinement and chiral phase transition at the same temperature?

• ...