BOSE-EINSTEIN CONDENSATION OF MAGNONS IN SUPERFLUID ^{3}He-B

and its applications to vortex studies

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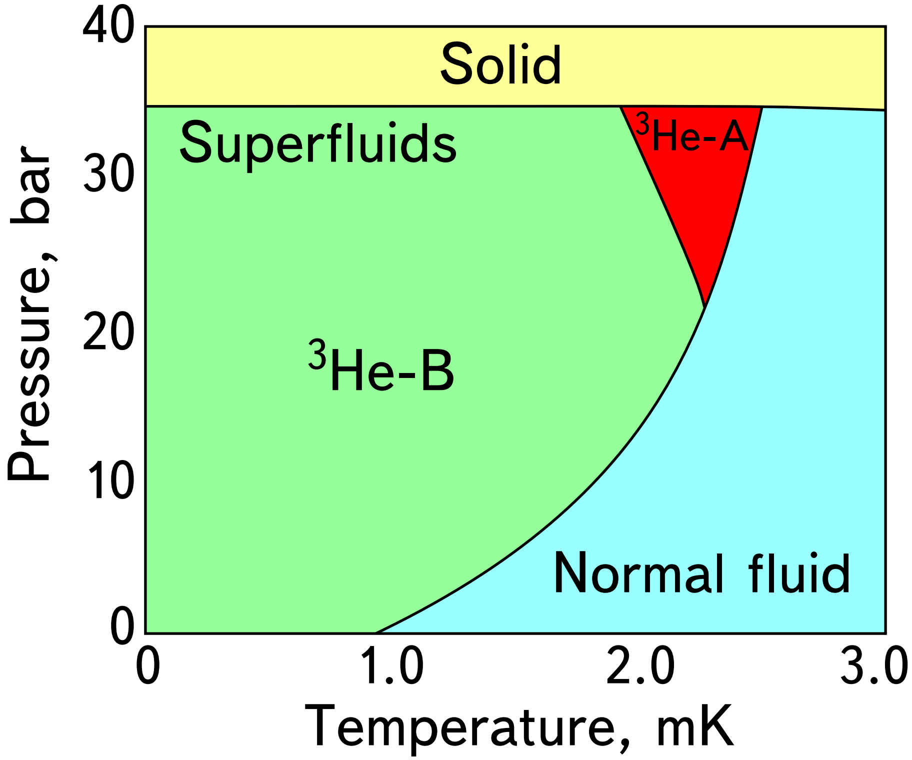
Low Temperature Laboratory
Aalto University

OVERVIEW

1. Superfluid ^{3}He-B and traps for magnon quasiparticles.
2. Filling the ground and excited levels in the trap with magnons and spectroscopy of the trap levels.
3. Coherent precession of the ground- and excited-level condensates.
4. Interaction of the magnon condensates with the trapping potential (self-trapping).
5. Measurements of relaxation of magnon condensates in rotating ^{3}He-B filled with vortex lines: A tool to observe vortex-core bound fermions.

SUPERFLUID ^{3}He



Fermi system, which goes superfluid through Cooper pairing with $S = 1$ and $L = 1$.

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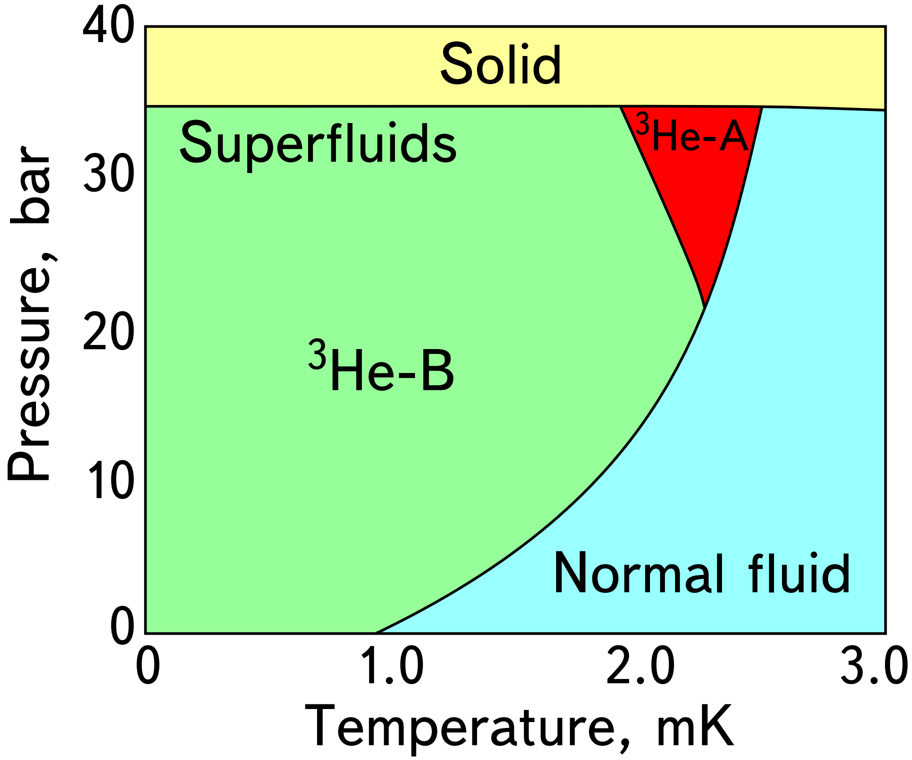

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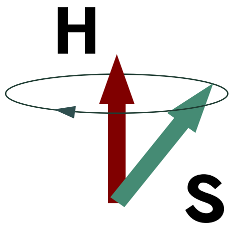
Spin-orbit interaction:

Orbital momentum ⇔ spin precession

$$\frac{\partial \mathbf{S}}{\partial t} = \gamma \mathbf{S} \times \mathbf{H} + \mathbf{R}_D$$

dipole torque



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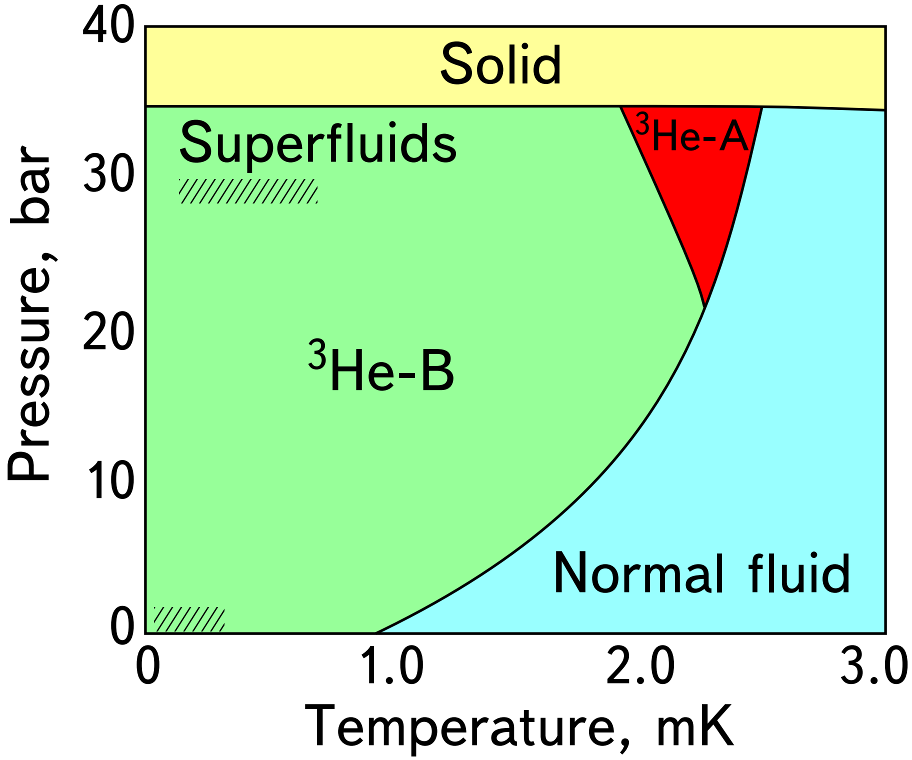

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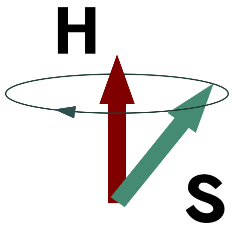
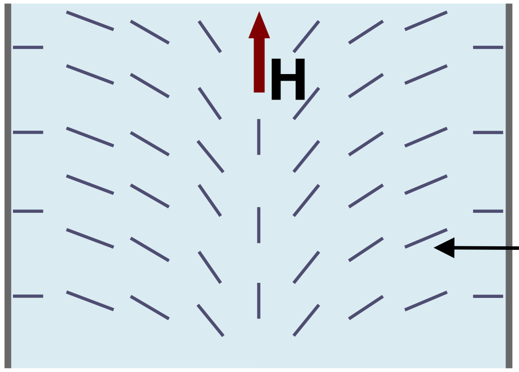


In ^{3}He-B in magnetic field net **L** and $\mathbf{S} = (\chi/\gamma)\mathbf{H}$ appear.

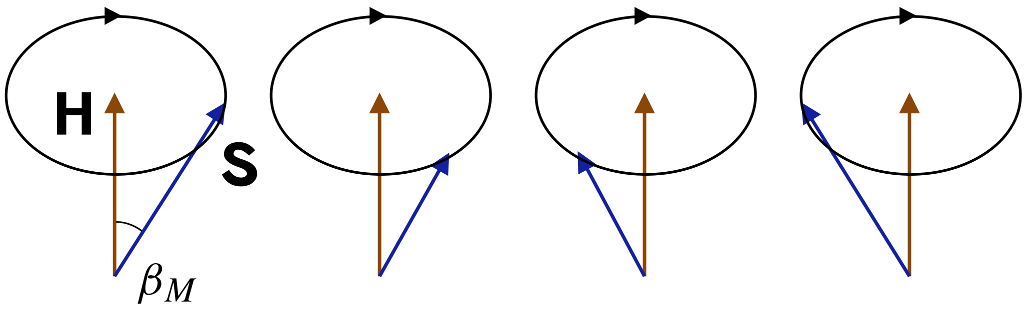
Connection **L** ⇔ **S** is given by the *order parameter* ⇒ gradient energy

⇒ equilibrium texture and waves.



Texture of $\hat{\mathbf{l}} = \mathbf{L}/L$ in a cylindrical sample results from competition of $\mathbf{L} \parallel \mathbf{H}$ and $\mathbf{L} \perp$ wall

TRAPPED MAGNON CONDENSATES IN ^{3}He-B



Spin waves: $S_\perp = S \sin\beta_M e^{i\omega t + i\alpha}$

$$\omega \approx \omega_{\rm L} + \hbar k^2/2m_M$$

$$S = \chi H/\gamma,\ \omega_{\rm L} = \gamma H,\ m_M \sim 10^{-4} m_{\rm He}$$

Magnons with spin $-\hbar$: $\hat{N}_{\rm m} = \dfrac{S - \hat{S}_z}{\hbar}, \quad N_{\rm m} \propto 1 - \cos\beta_M = 2\sin^2\dfrac{\beta_M}{2}$

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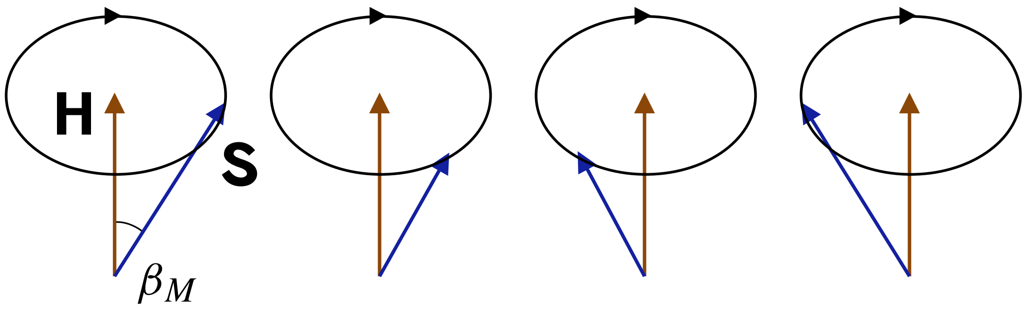

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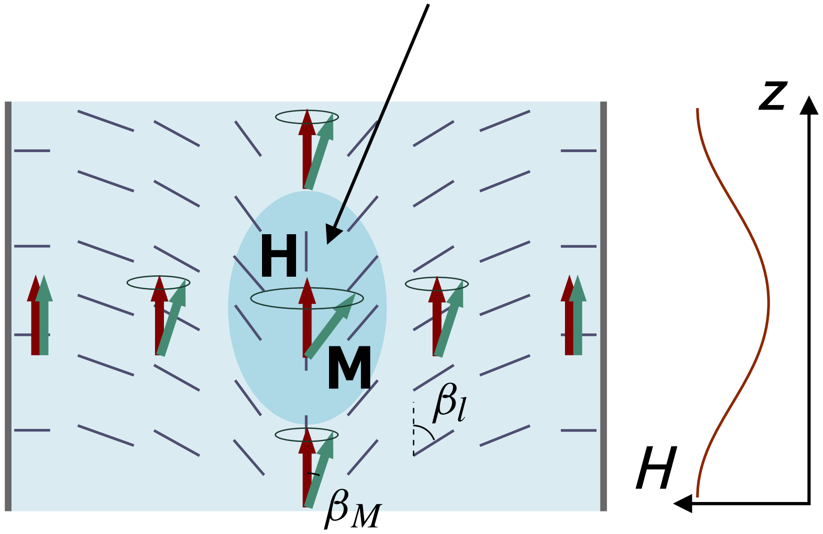
Magnon condensate in ^{3}He-B: *coherently precessing* magnetization

$$\Psi(\mathbf{r}) \propto \sin\frac{\beta_M(\mathbf{r})}{2} e^{i\omega t + i\alpha(\mathbf{r})}$$

$$N_\mathrm{m}^{1/2} \propto M_\perp$$

$\omega \equiv$ chemical potential

$\alpha \equiv$ phase of wave function



Review:

Bunkov and Volovik, arXiv:1003.4889

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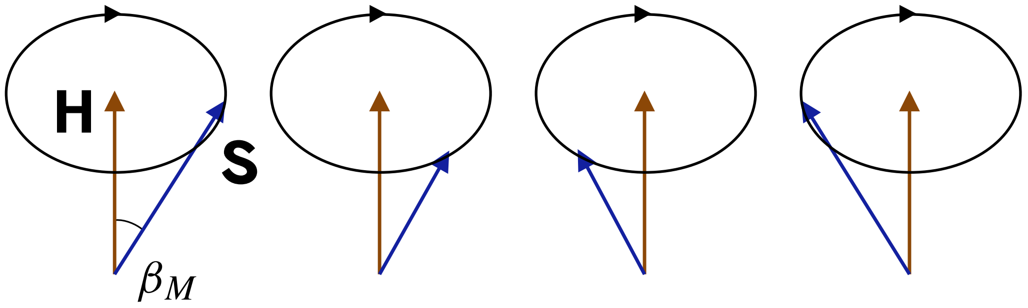

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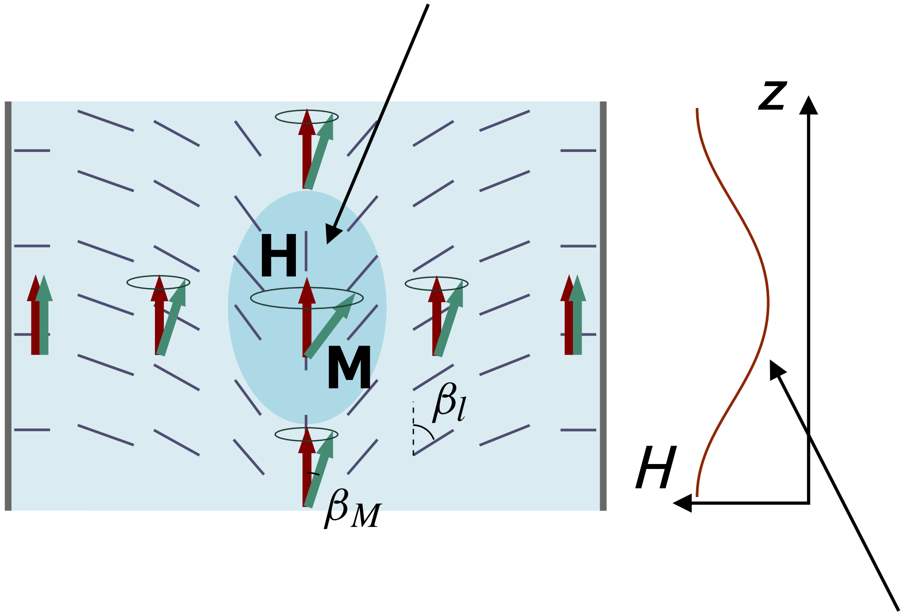

Axial trap

$$F_\mathrm{Z} = (\omega - \omega_\mathrm{L})|\Psi|^2$$

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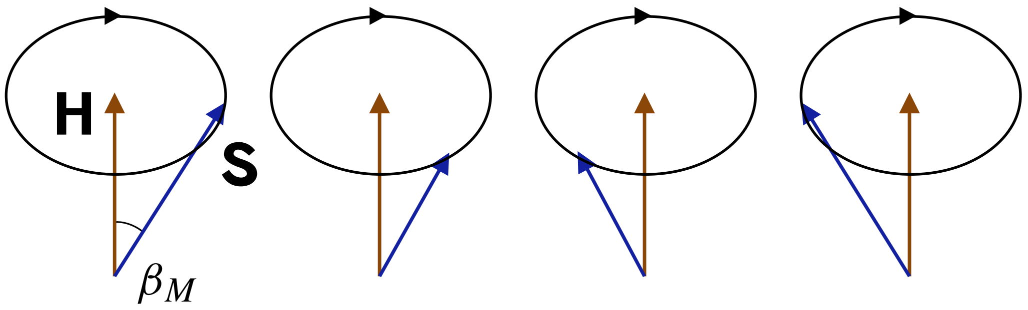

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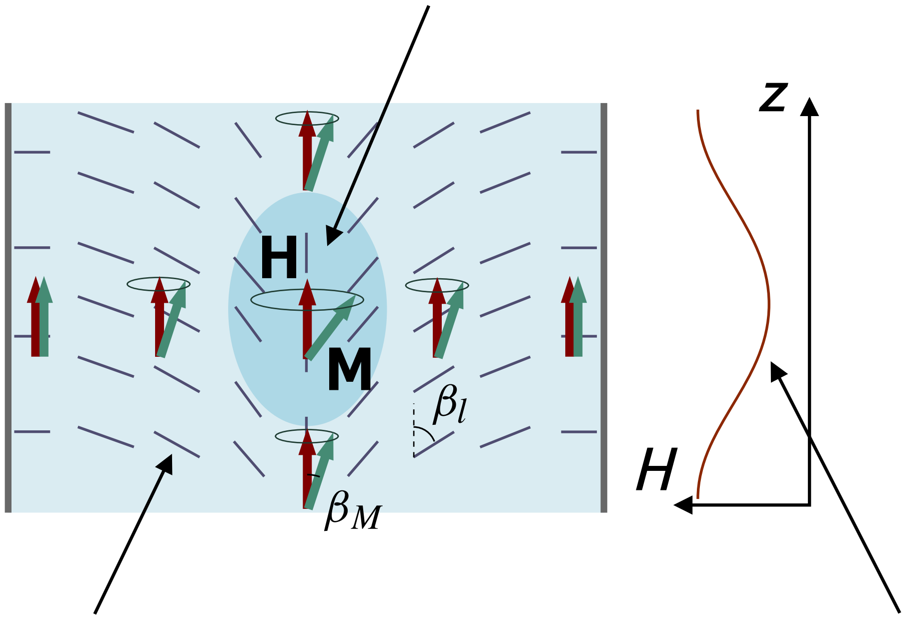

$\hat{\mathbf{l}}$ texture: radial trap

$$F_\mathrm{so} \propto \sin^2 \frac{\beta_l}{2} |\Psi|^2$$

Axial trap

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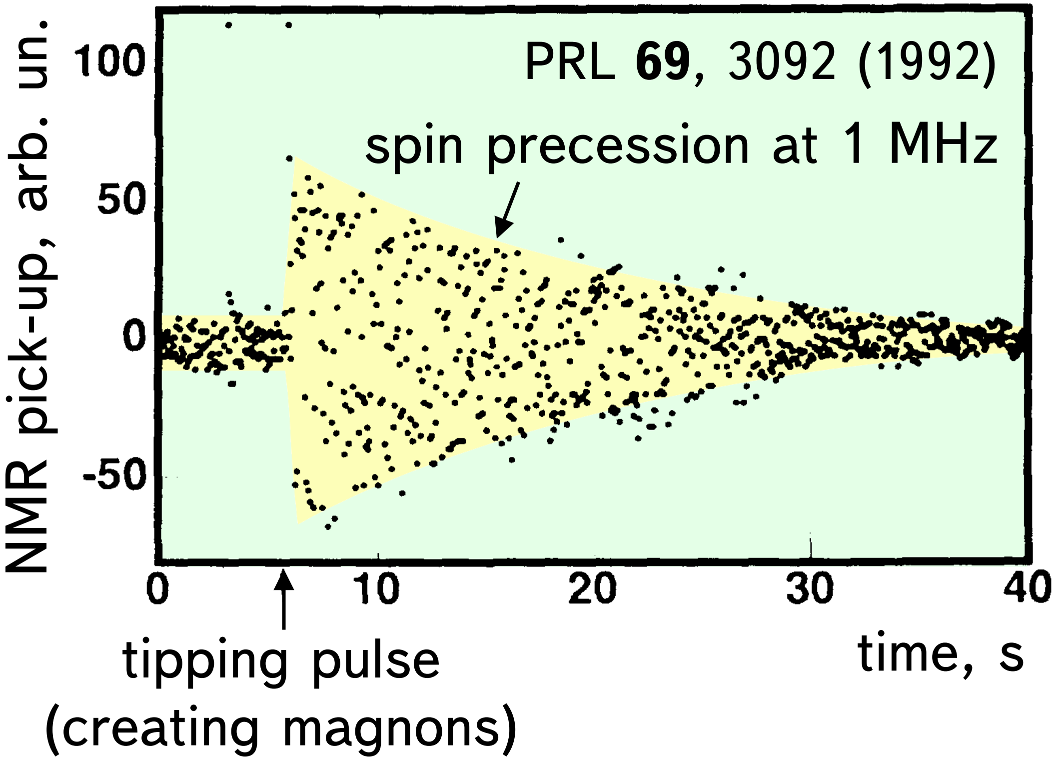
Discovered in Lancaster in pulsed NMR experiments at $T < 0.2T_c$


PRL **69**, 3092 (1992)

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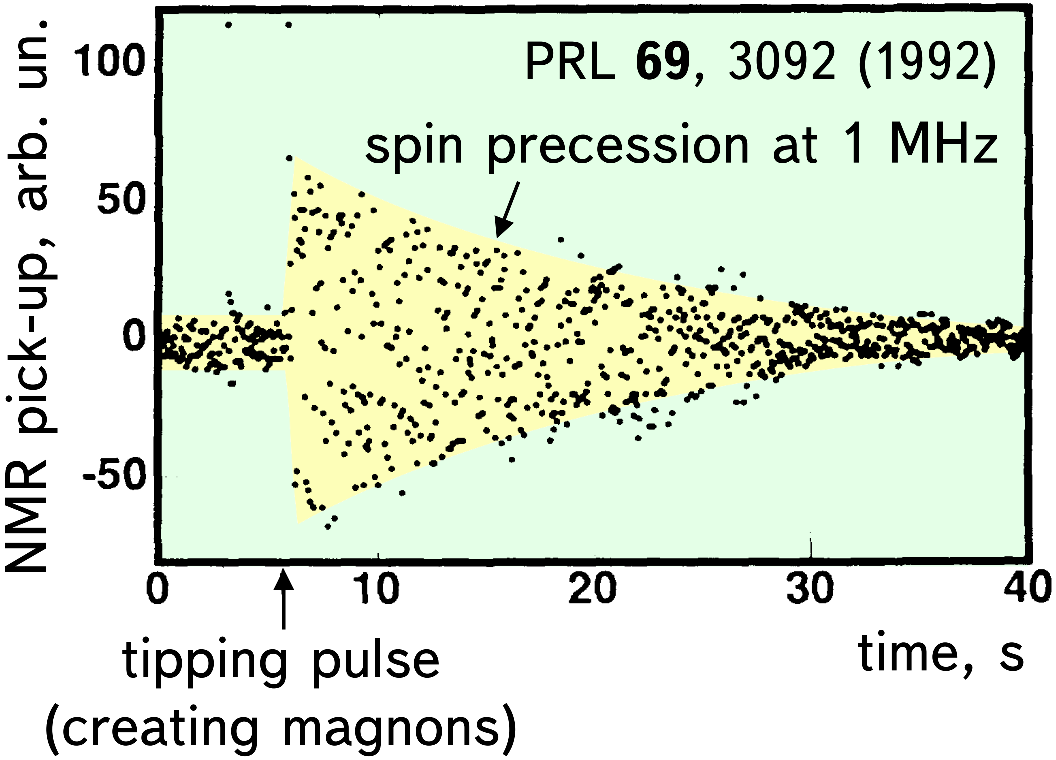

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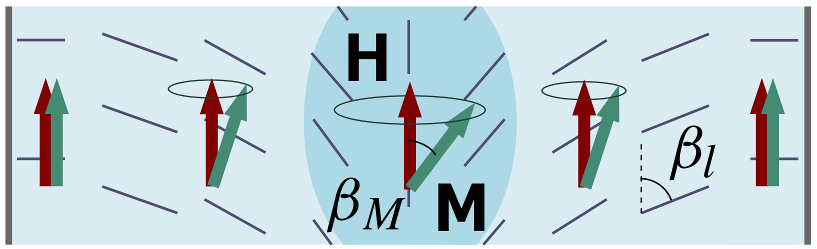
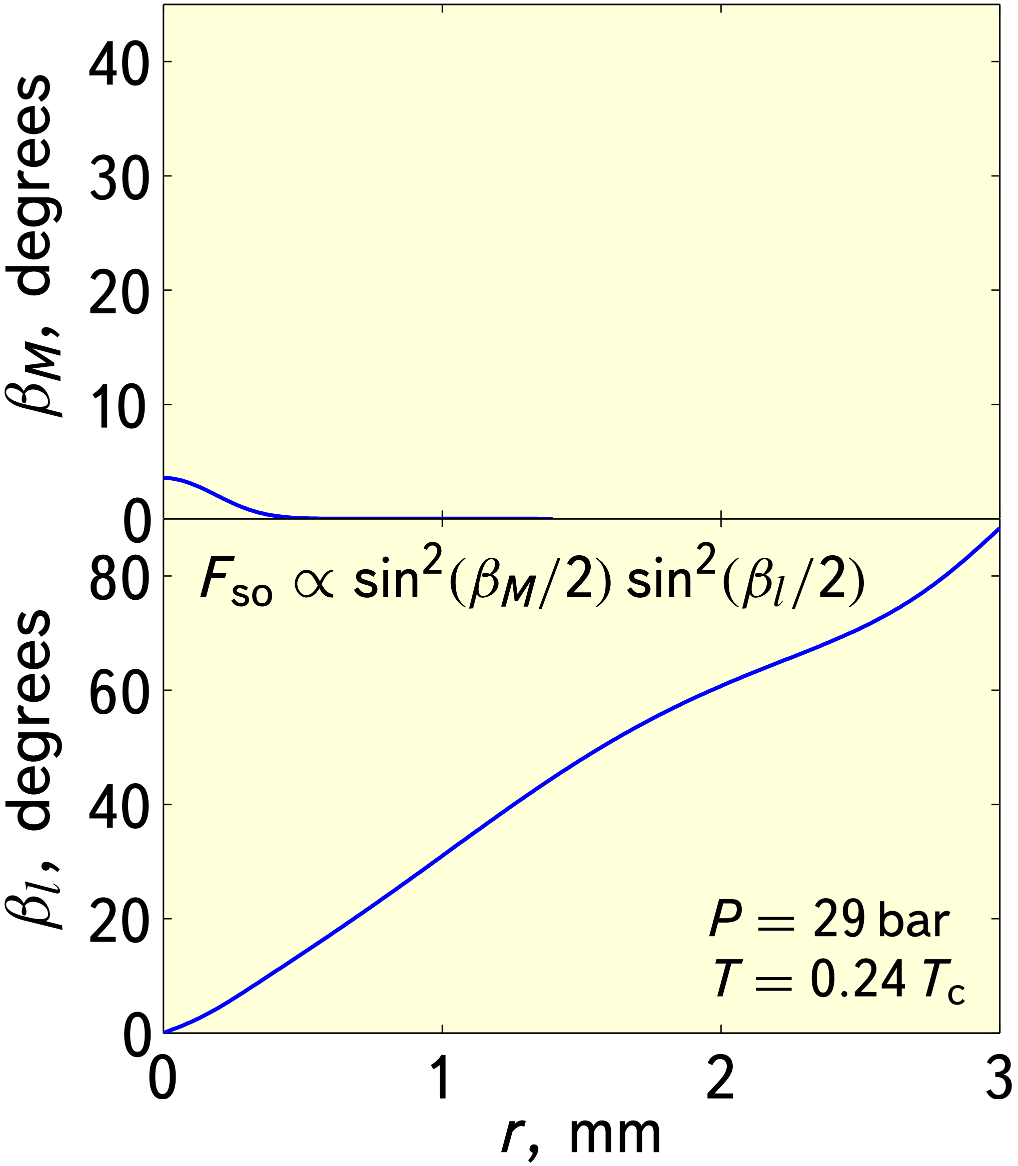
These features (and more) find explanations in the picture of the magnon BEC in the magneto-textural trap.

(Bunkov and Volovik, PRL **98**, 265302 (2007))

SELF-TRAPPING OF THE MAGNON BEC





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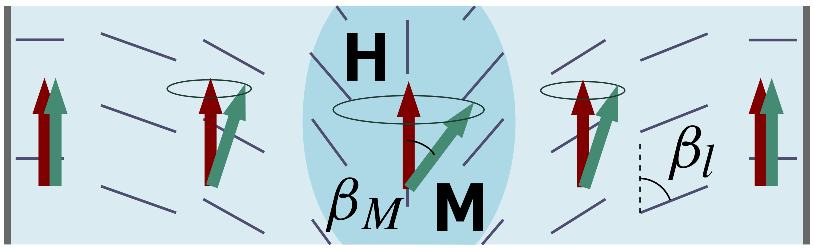
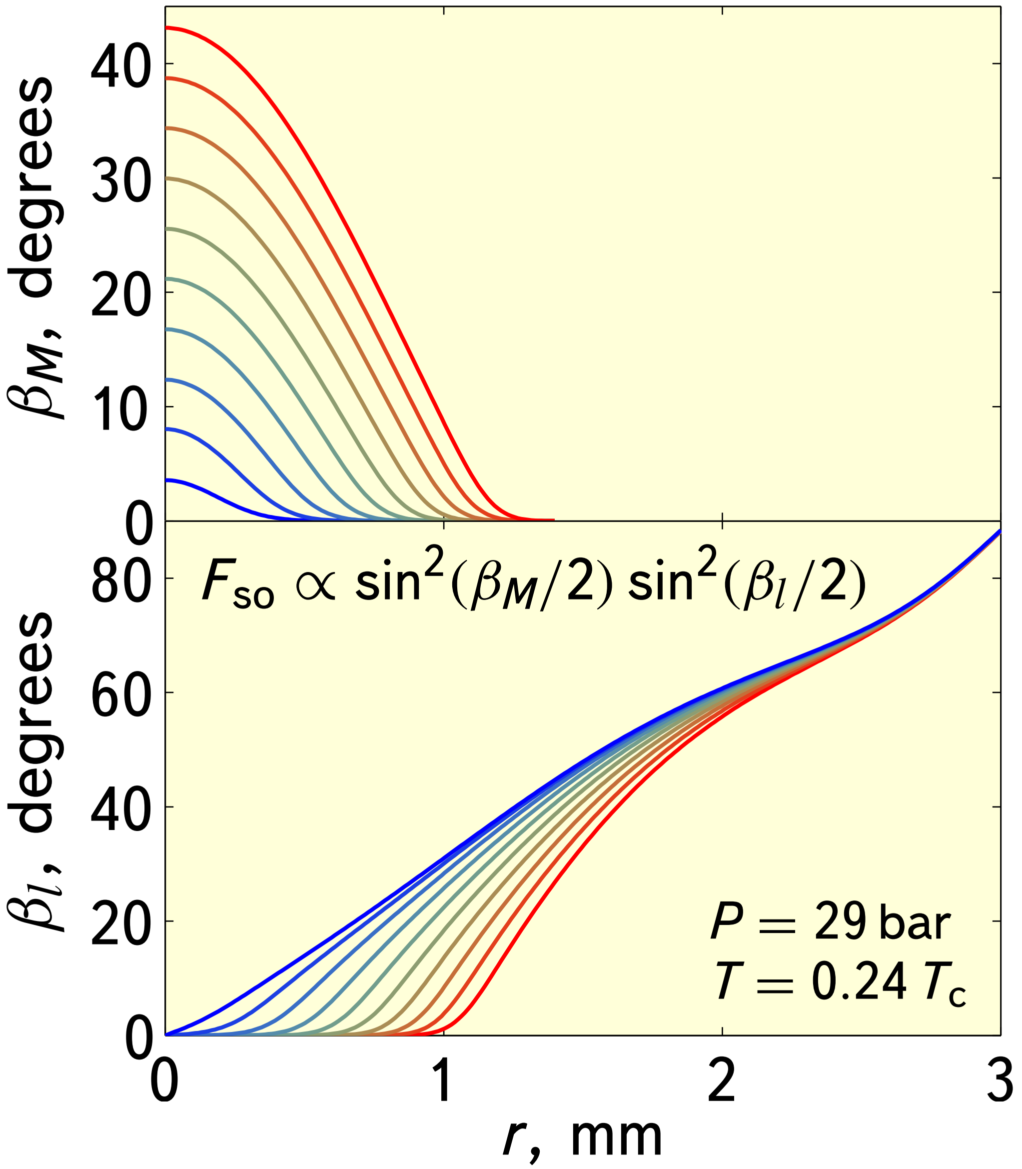


Texture is flexible, when β_M increases β_l tends to decrease: Magnon condensate forms a "bubble" with $\hat{\mathbf{l}} \parallel \mathbf{H}$.



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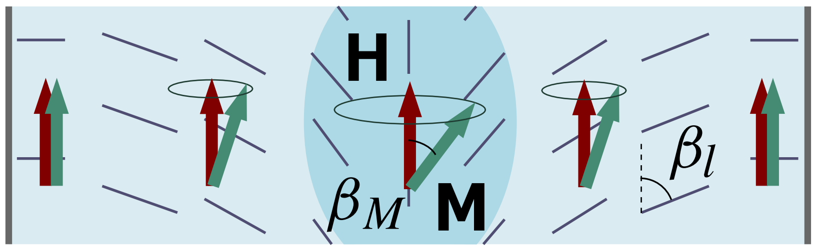

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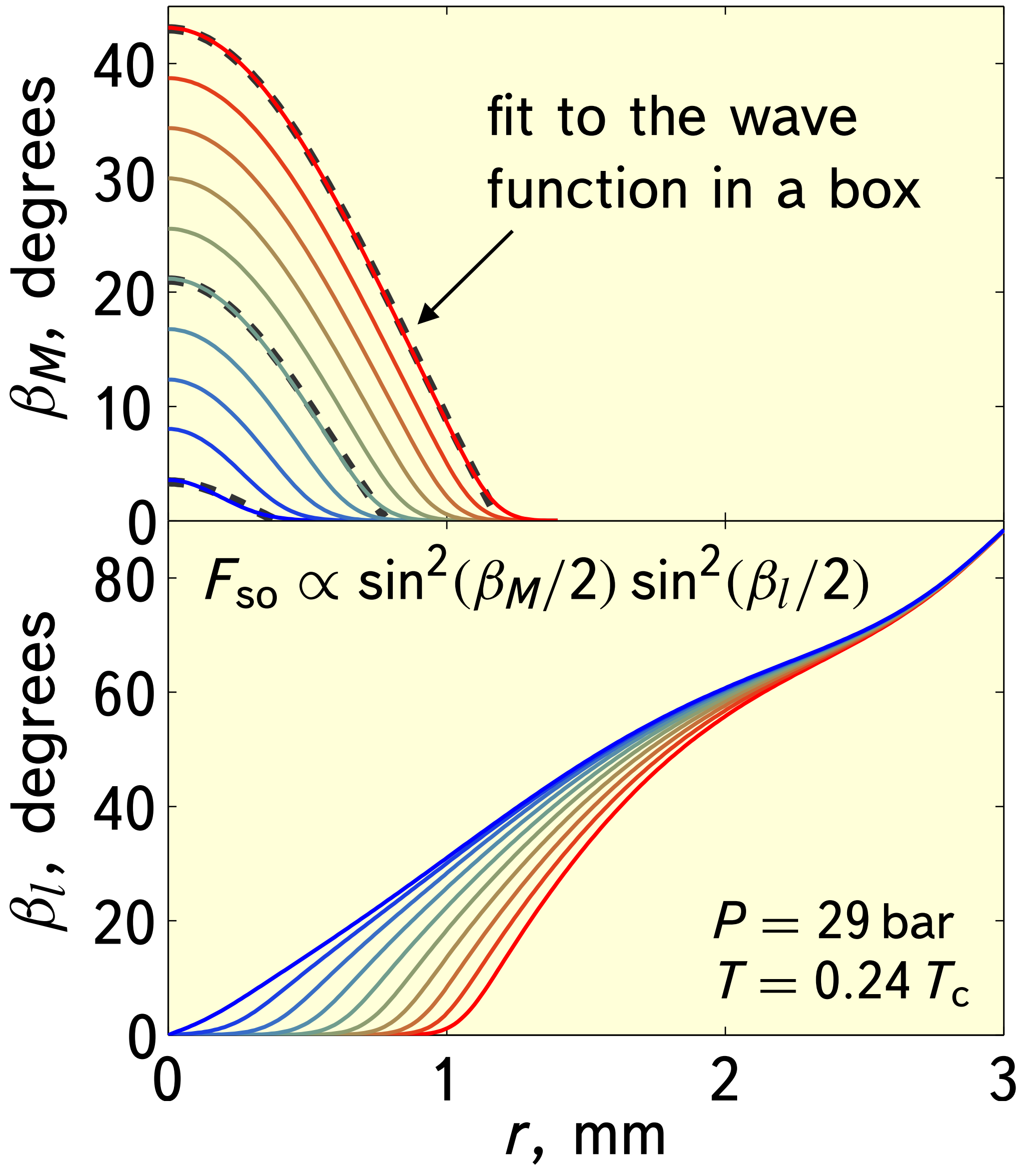


- Harmonic trap transforms to a box with impenetrable walls. First example of BEC in a box.
- Texture-mediated interaction results in $d\mu/dN_{\mathrm{m}} < 0$.
- Analog of the electron bubble in helium and of the MIT bag model of hadrons.

PRL **108**, 145303 (2012)

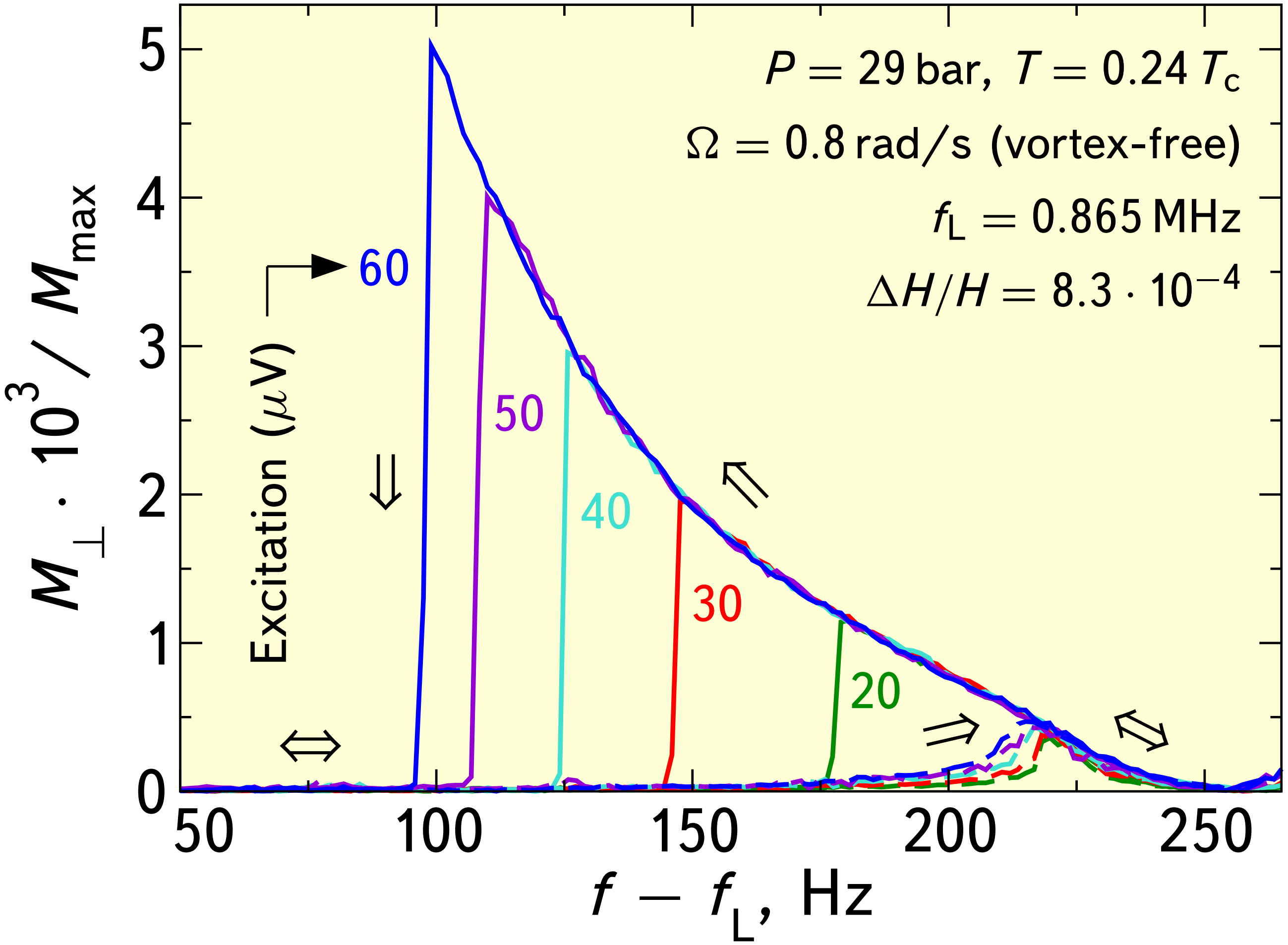
FILLING TRAP WITH MAGNONS AT THE GROUND LEVEL

CW NMR: downward frequency (upward field) sweep.

Number of magnons $N_\mathrm{m} \propto M_\perp^2$

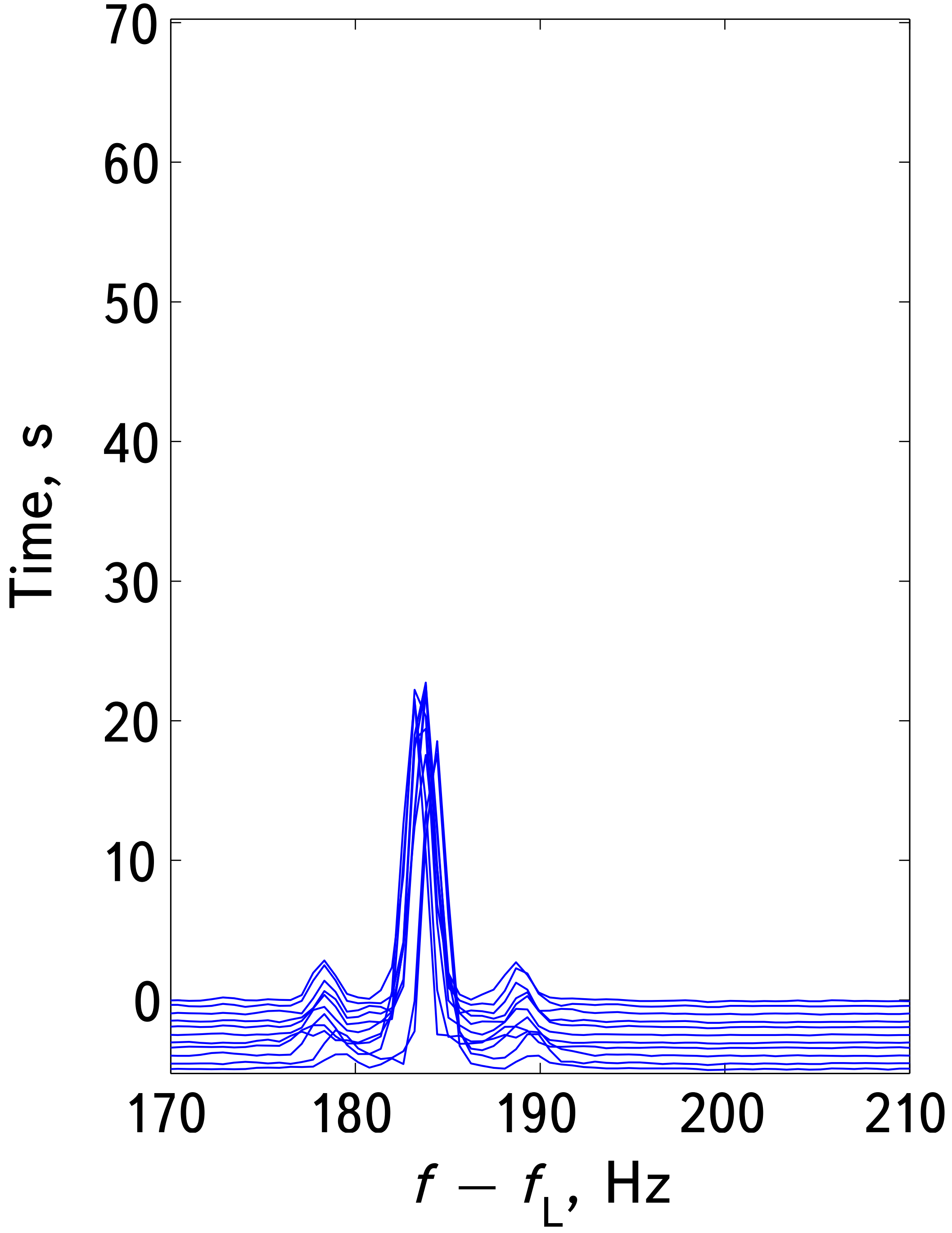
Chemical potential $\mu \propto f - f_\mathrm{L}$

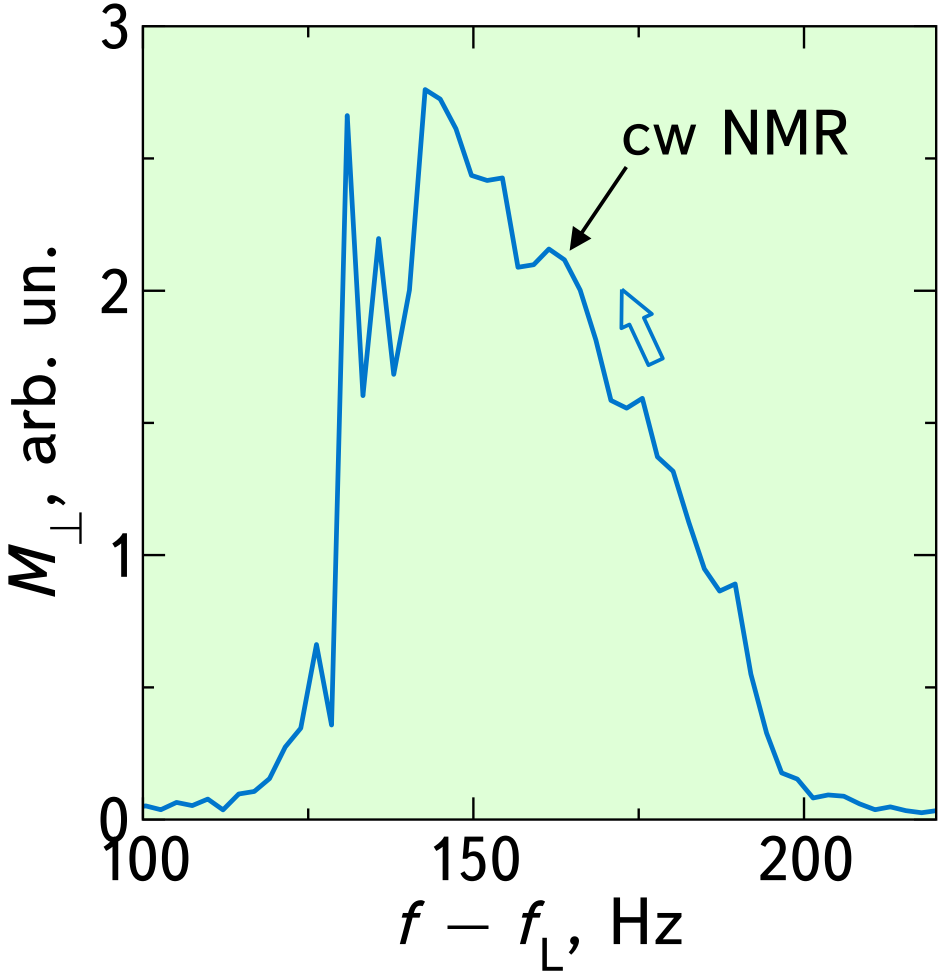
$$d\mu/dN_\mathrm{m} < 0$$



COHERENT PRECESSION OF THE MAGNON CONDENSATE

Condensation is demonstrated by long decay times of free precession.





$P = 0.5$ bar, $T = 0.14\,T_c$

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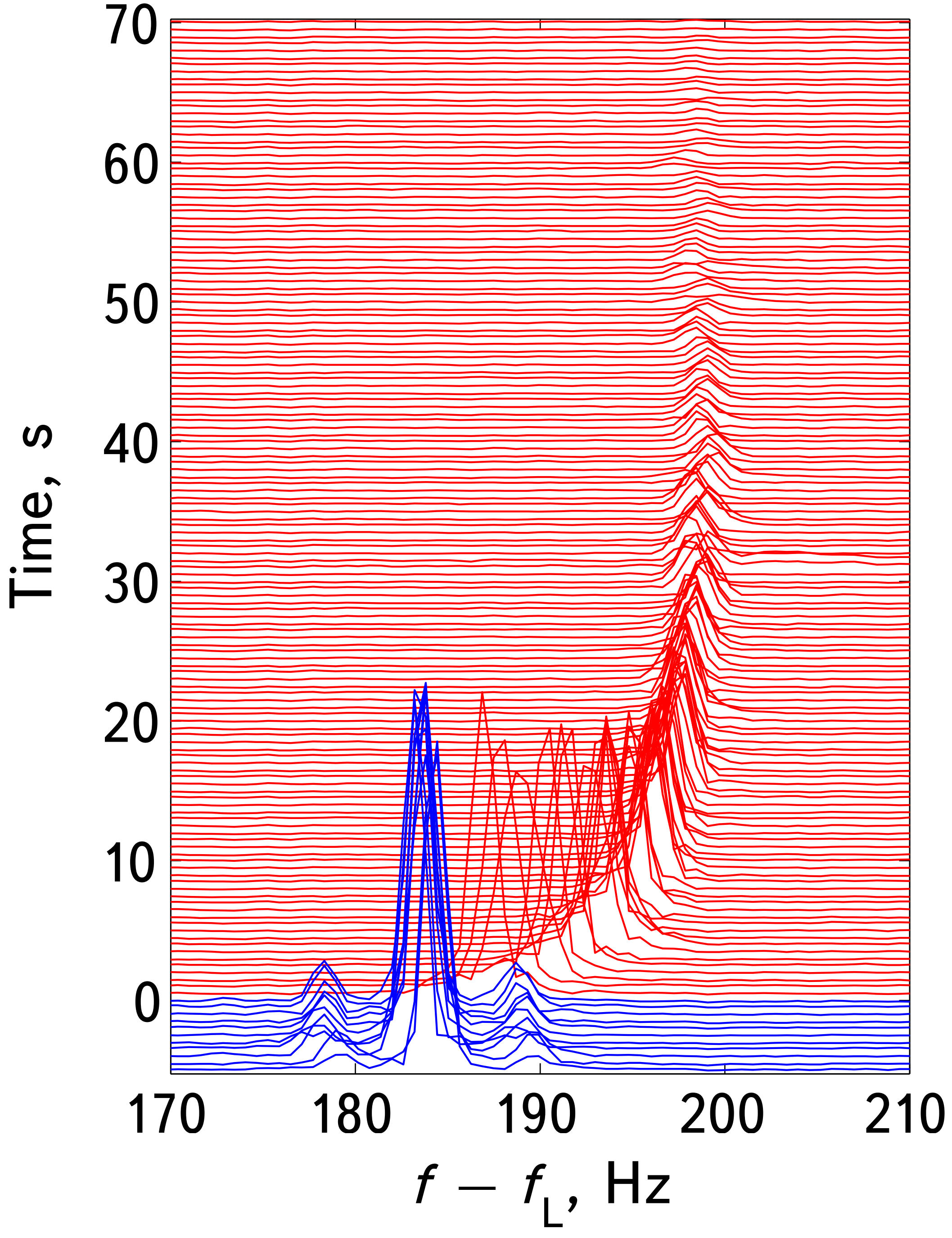

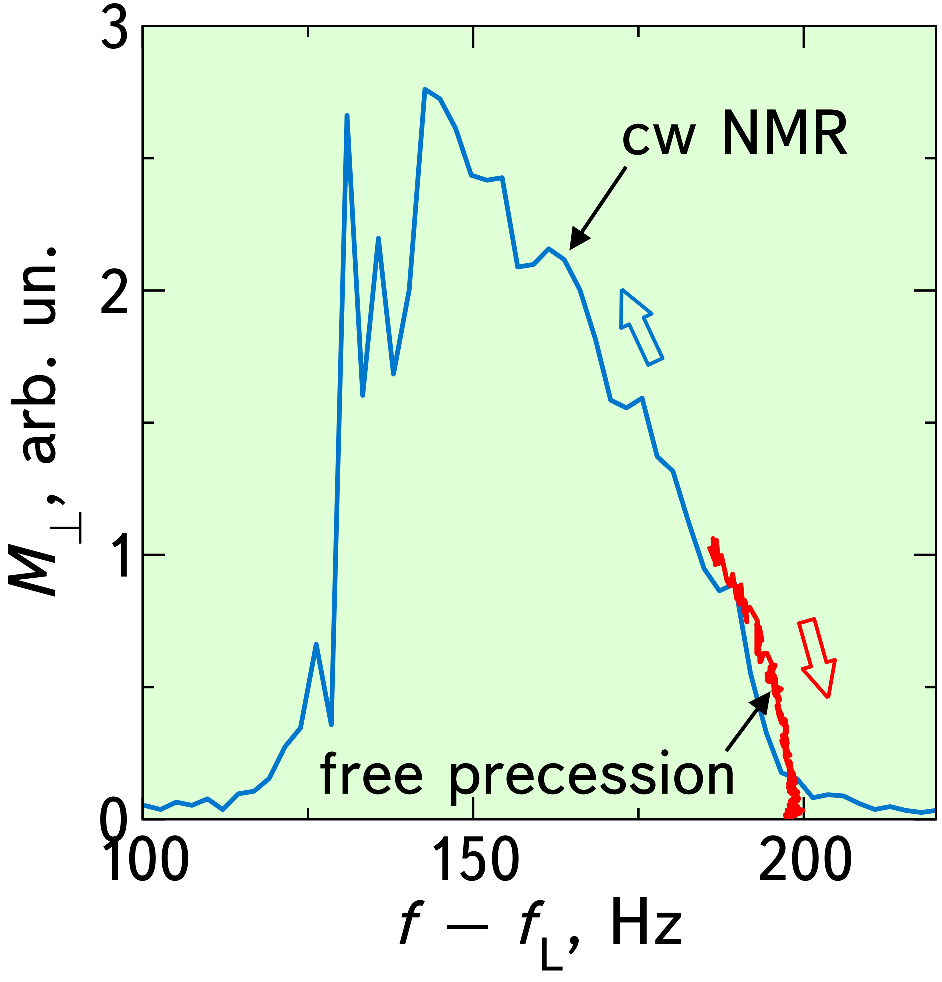

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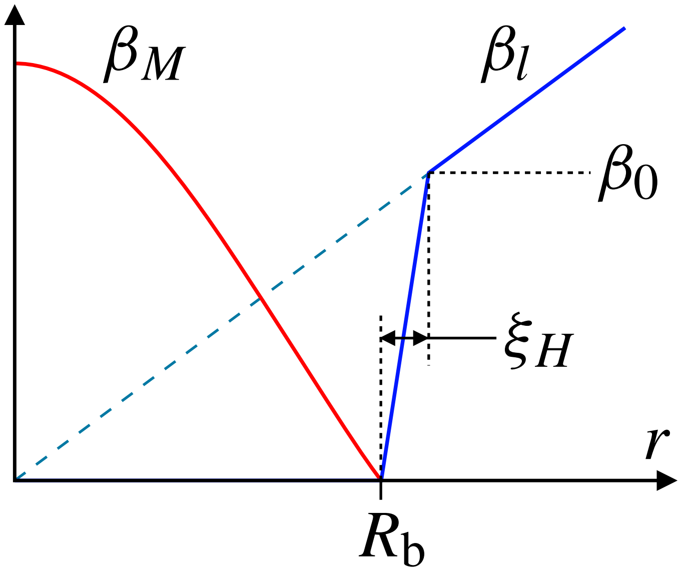
SCALING IN THE CYLINDRICAL BOX

Similar to electron bubble: $E(R_\mathrm{b}) = N_\mathrm{m}\dfrac{\hbar^2\lambda_\mathrm{m}^2}{2m_\mathrm{M}R_\mathrm{b}^2} + 2\pi R_\mathrm{b}\sigma(R_\mathrm{b}) \;\to\;$ min



kinetic energy of magnons

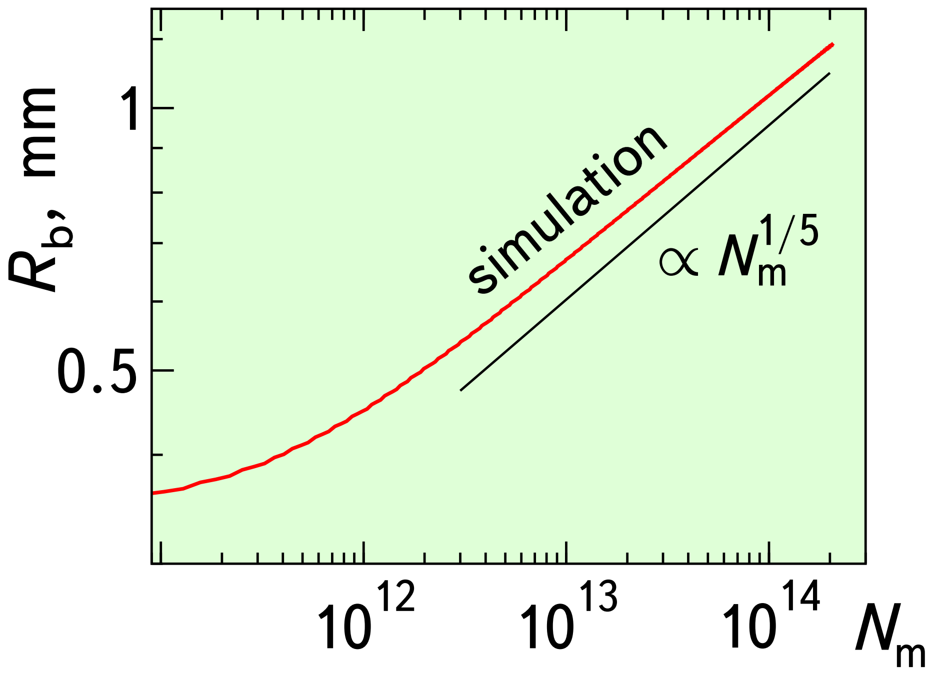
m_M - magnon mass

λ_m - root of the Bessel function

surface energy $\equiv$ orbital gradient energy:

$$2\pi R_\mathrm{b}\,\xi_H\left(\frac{\beta_0}{\xi_H}\right)^2$$

Scaling: $\sigma \propto \beta_0^2 \propto R_\mathrm{b}^2 \;\Rightarrow\; R_\mathrm{b} \propto N_\mathrm{m}^{1/5}$



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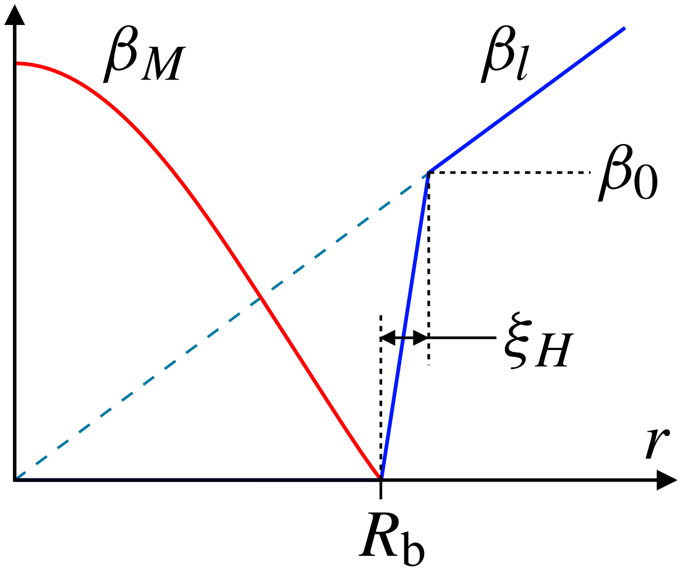

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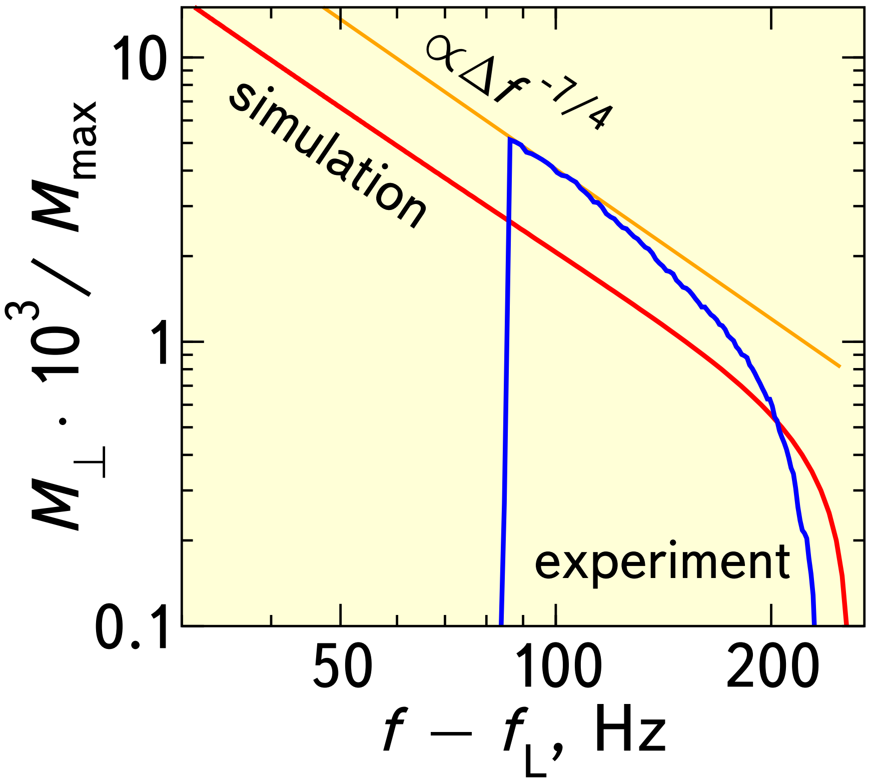
For the magnetization:

$$f - f_\mathrm{L} \propto R_\mathrm{b}^{-2}$$

$$M_\perp \propto \int \sin\beta_M dV,\;\; N_\mathrm{m} \propto \int (1-\cos\beta_M) dV$$

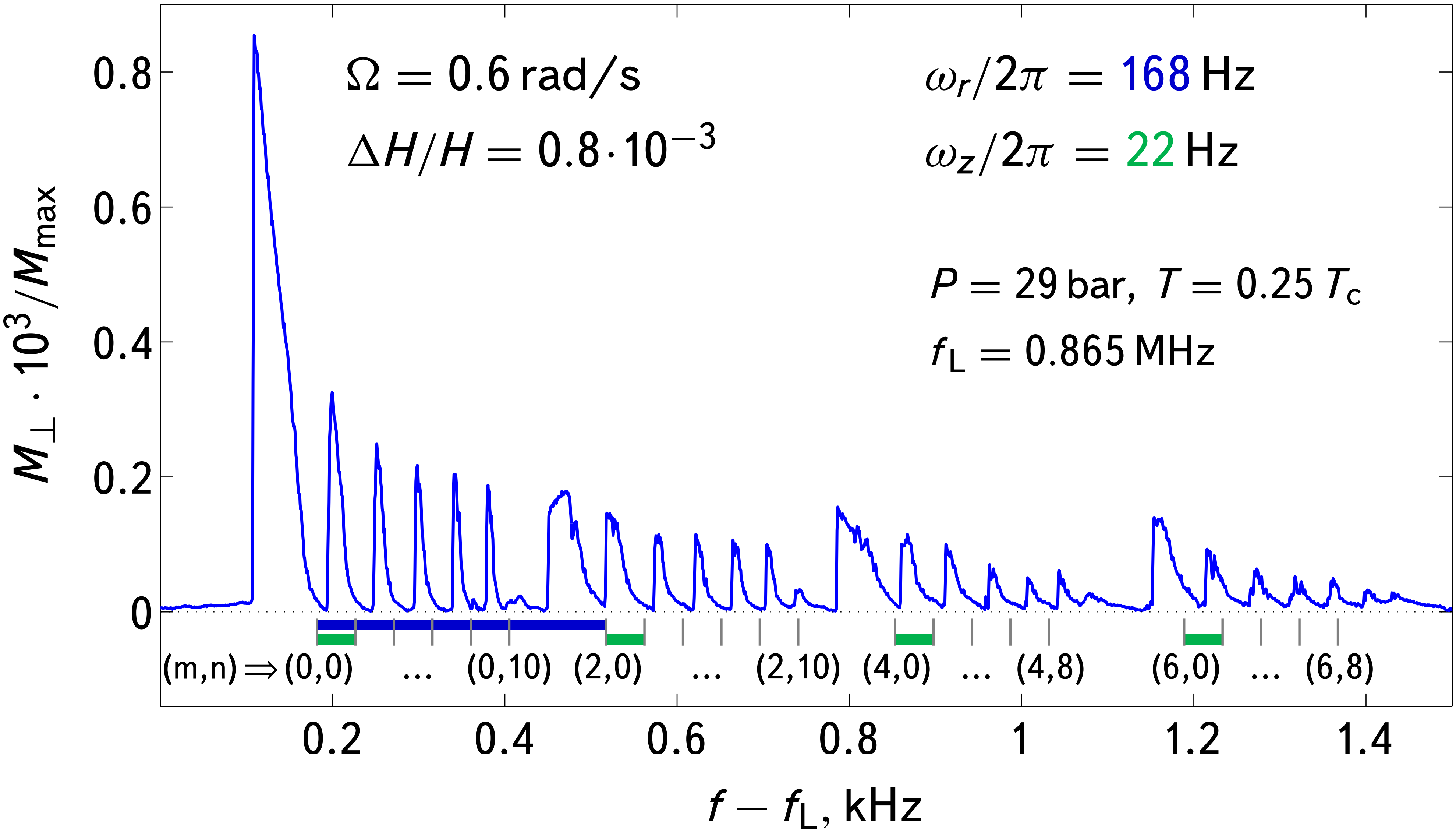
$$\Rightarrow\; M_\perp \propto N_\mathrm{m}^{1/2} R_\mathrm{b} \propto R_\mathrm{b}^{7/2} \propto (f - f_\mathrm{L})^{-7/4}$$



PROBING EXCITED MAGNON LEVELS

Since $d\mu/dN_\mathrm{m} < 0$, in cw NMR sweep excited levels in the trap are encountered first and macroscopic population of an excited level can be built.

In the harmonic trap (small N_m): $2\pi(f - f_\mathrm{L}) = \omega_r(\mathrm{m}+1) + \omega_z(\mathrm{n}+1/2)$



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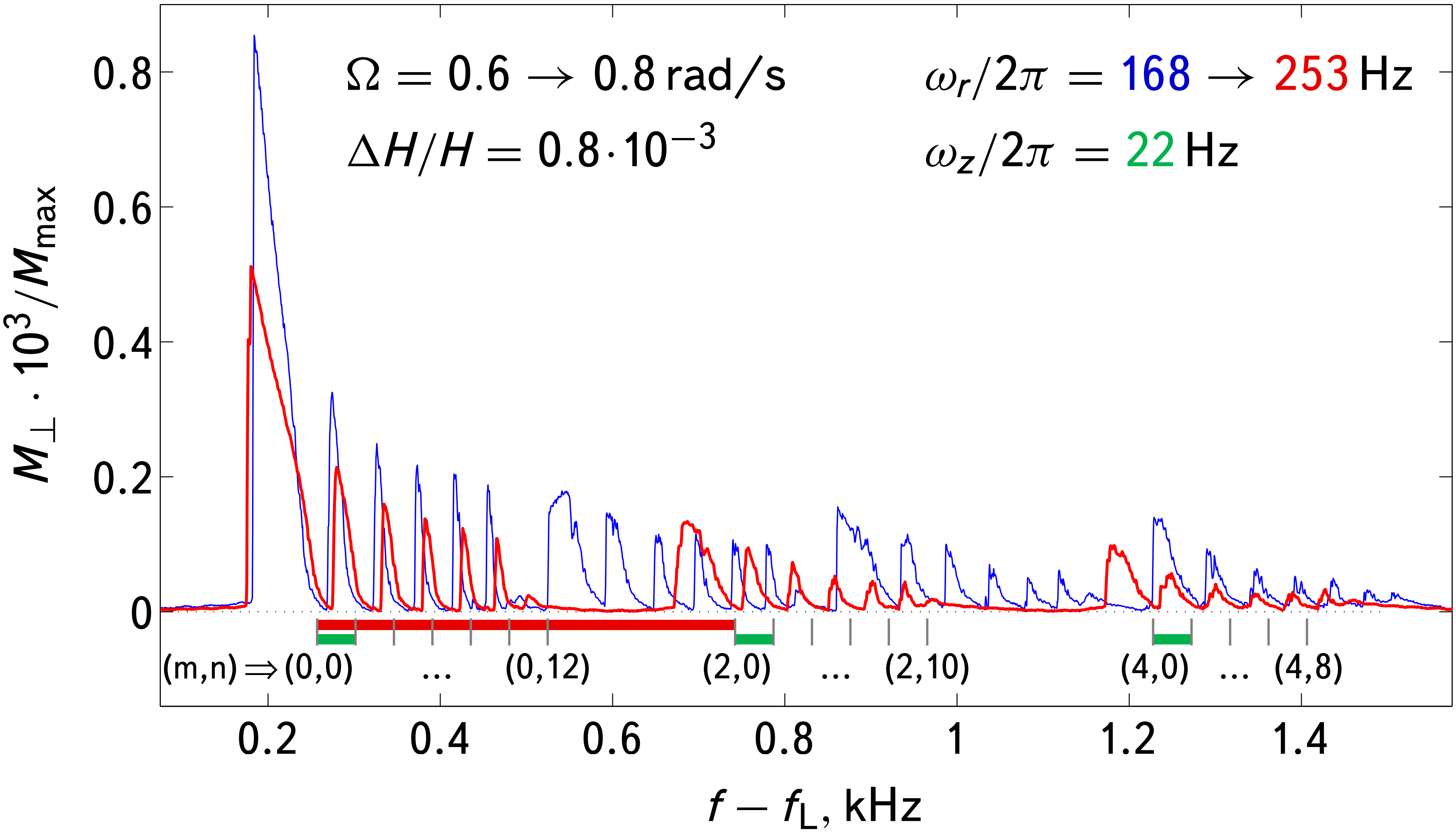

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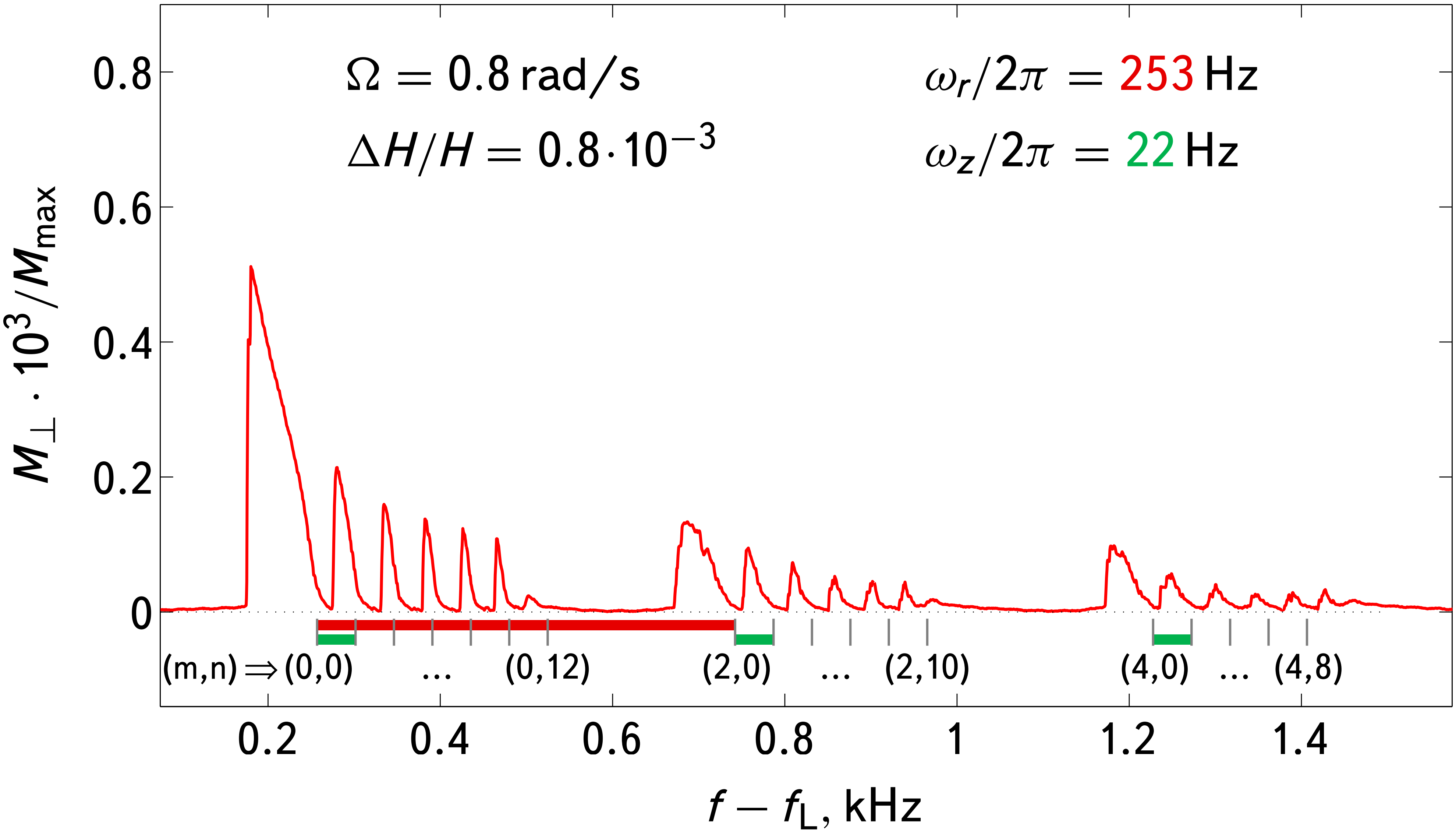

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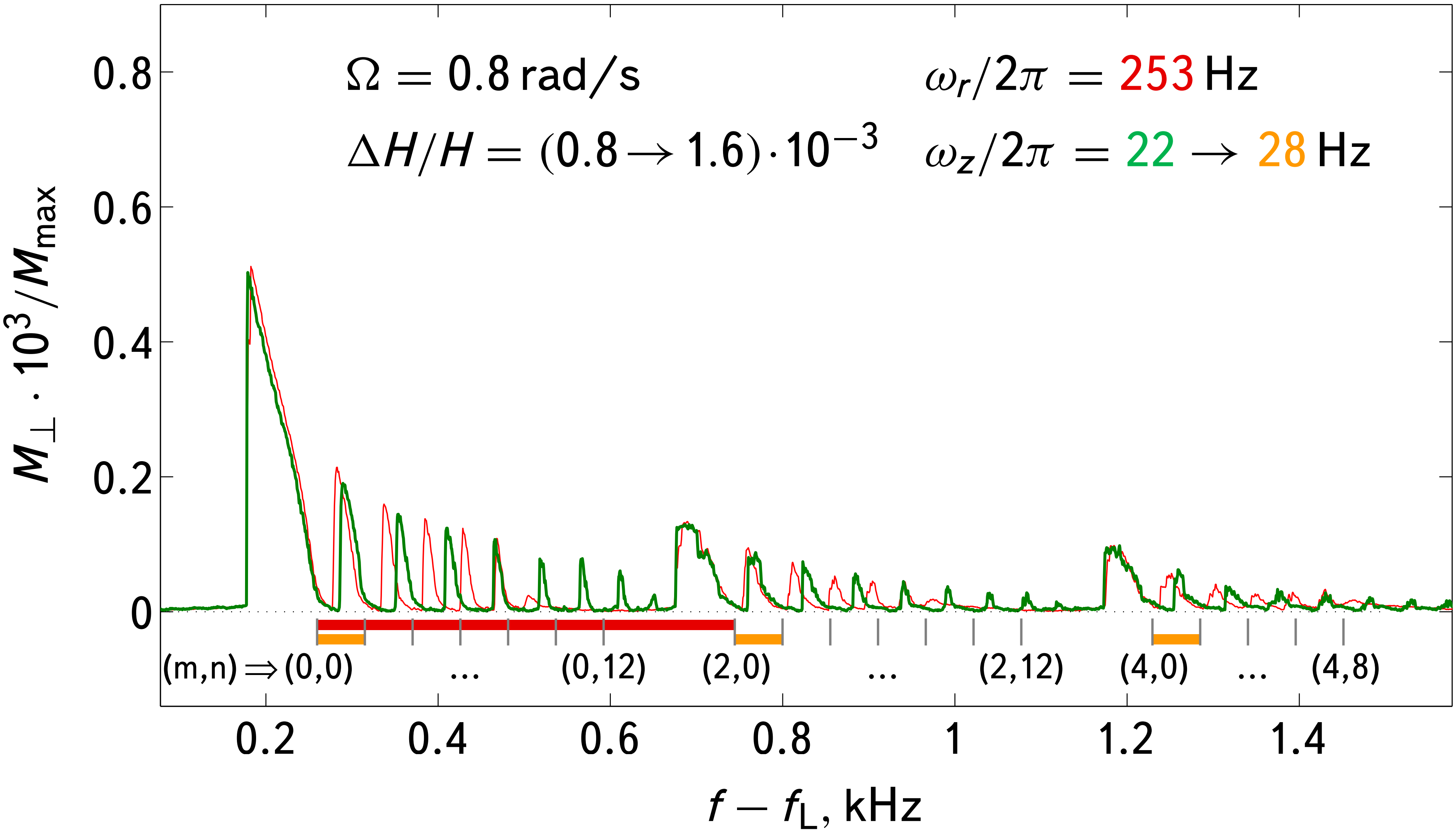

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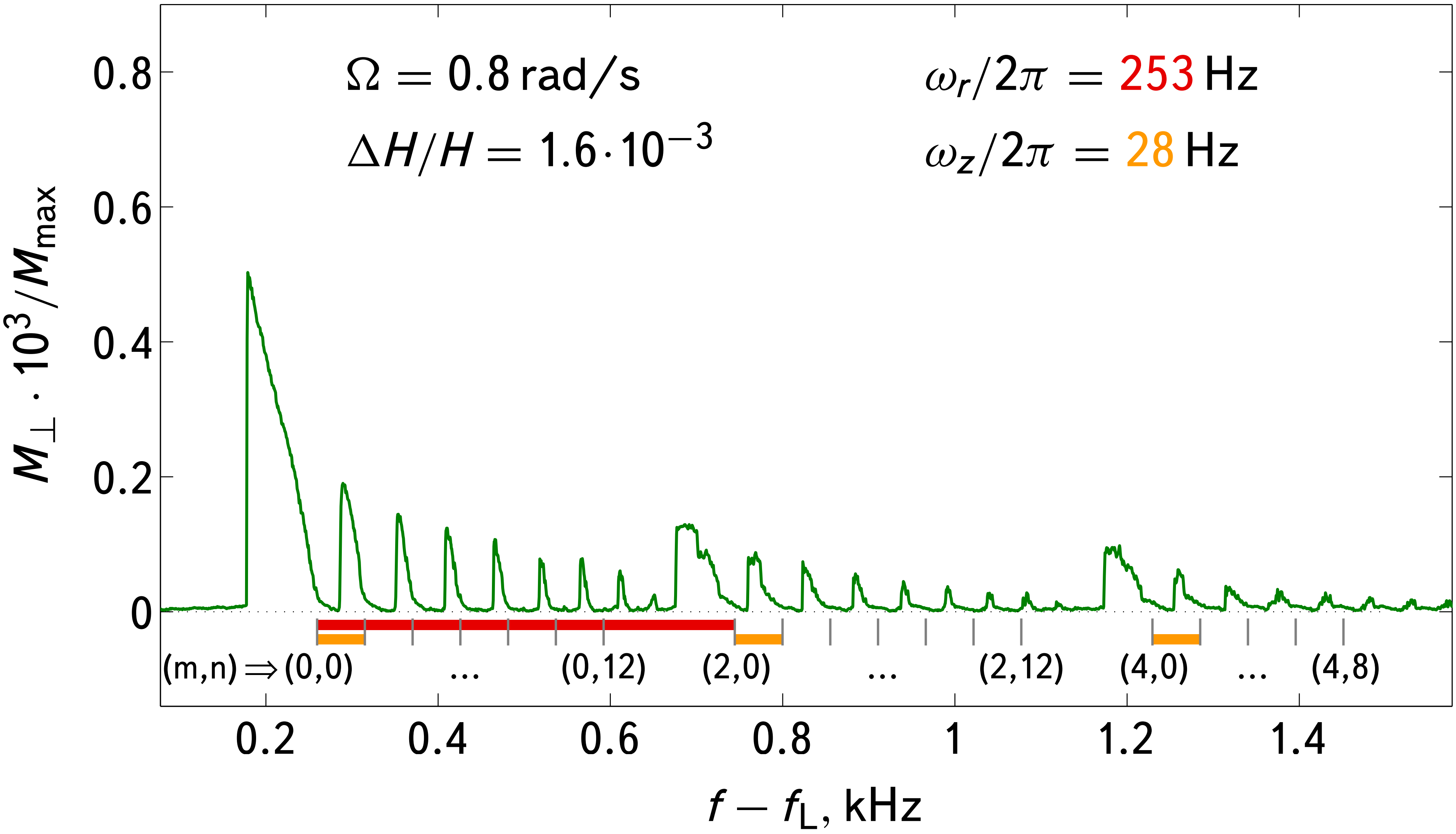

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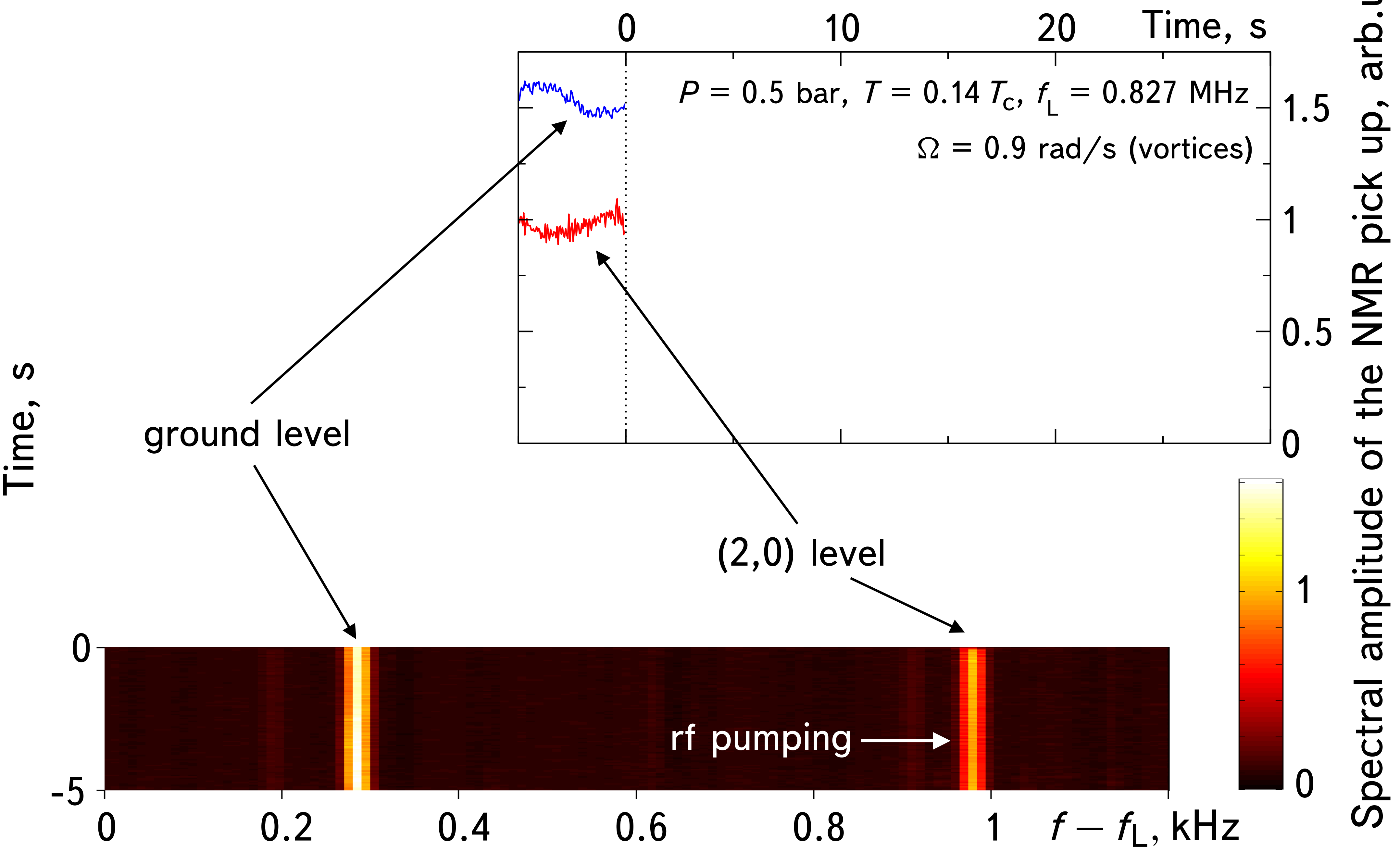

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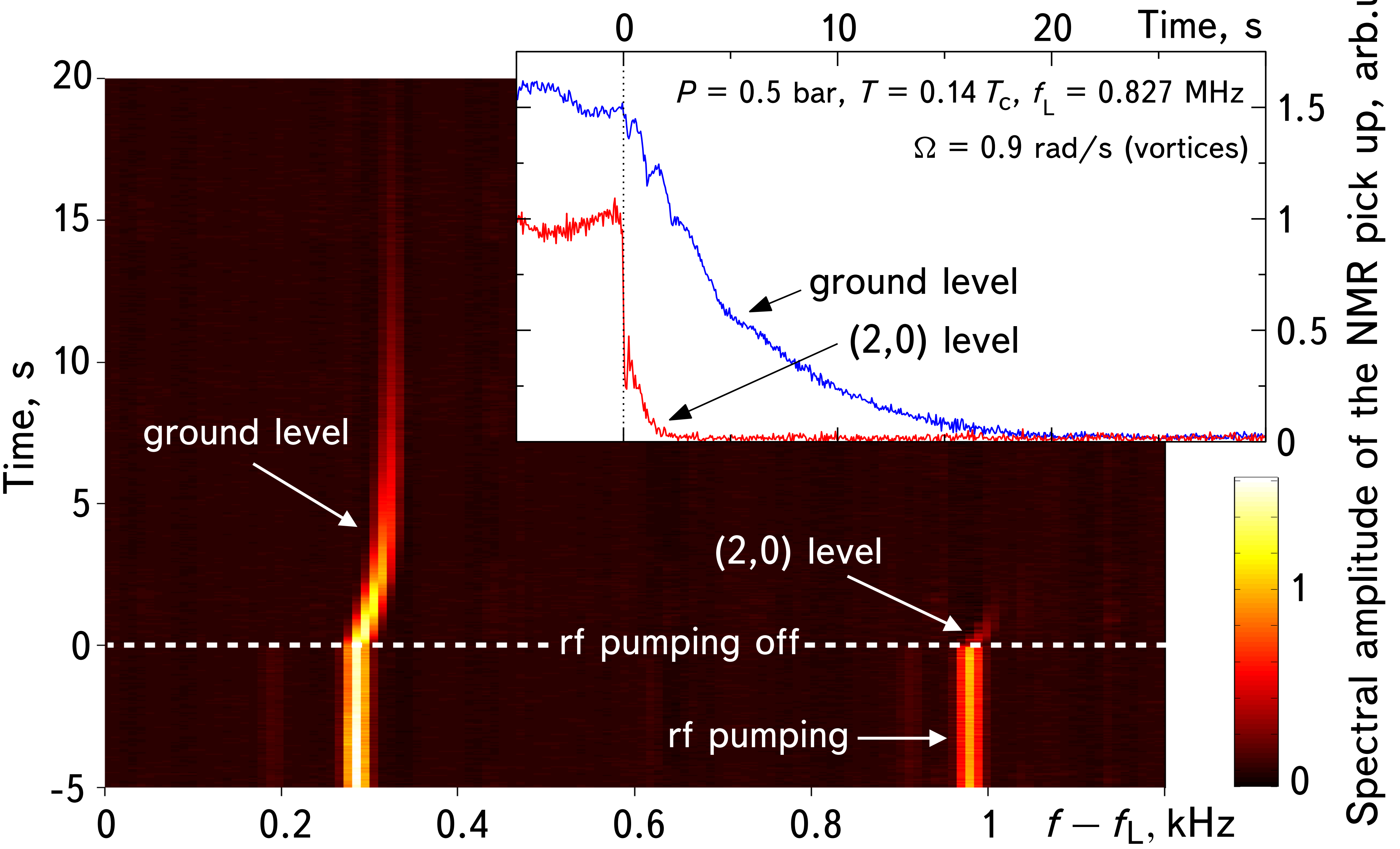

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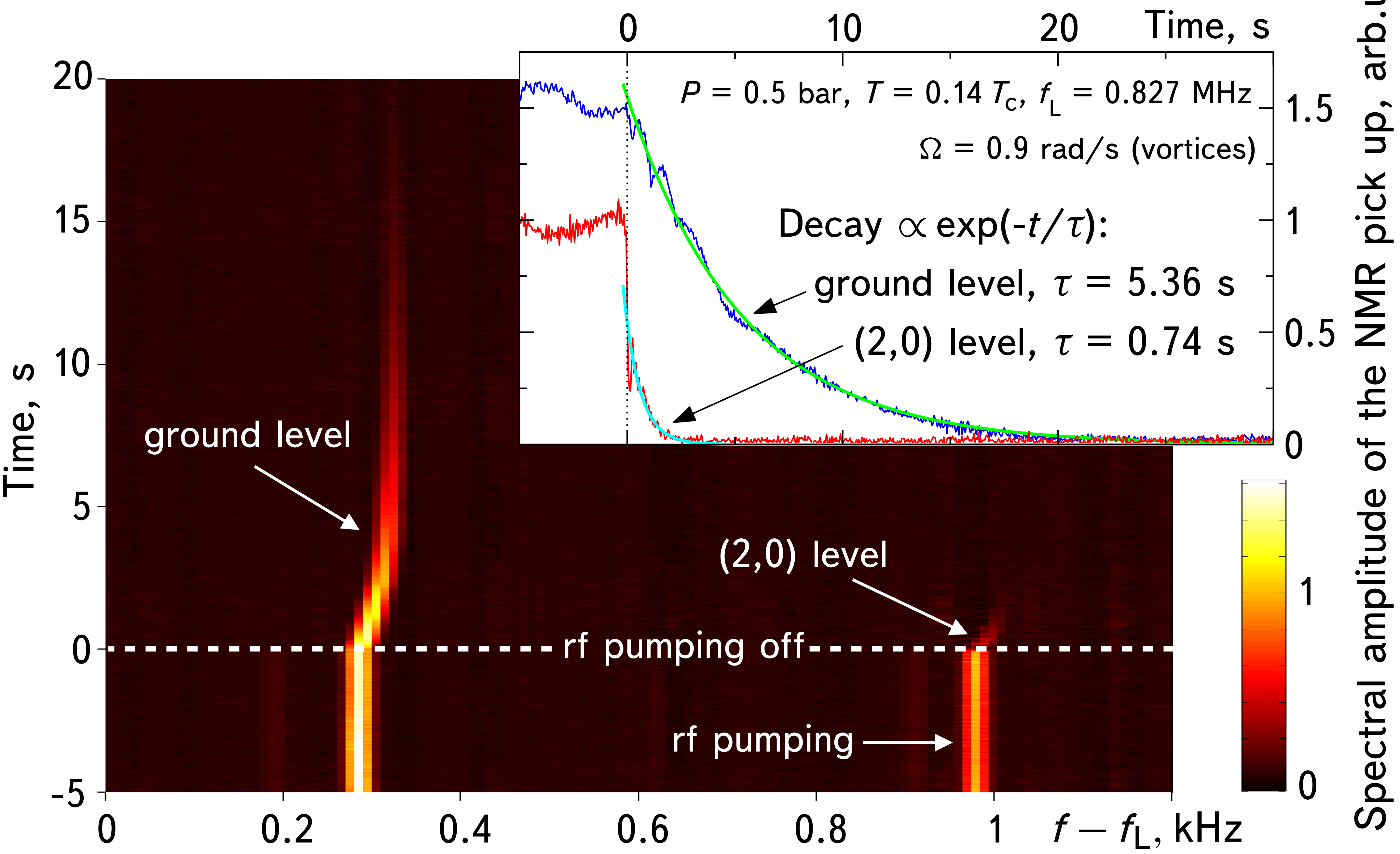
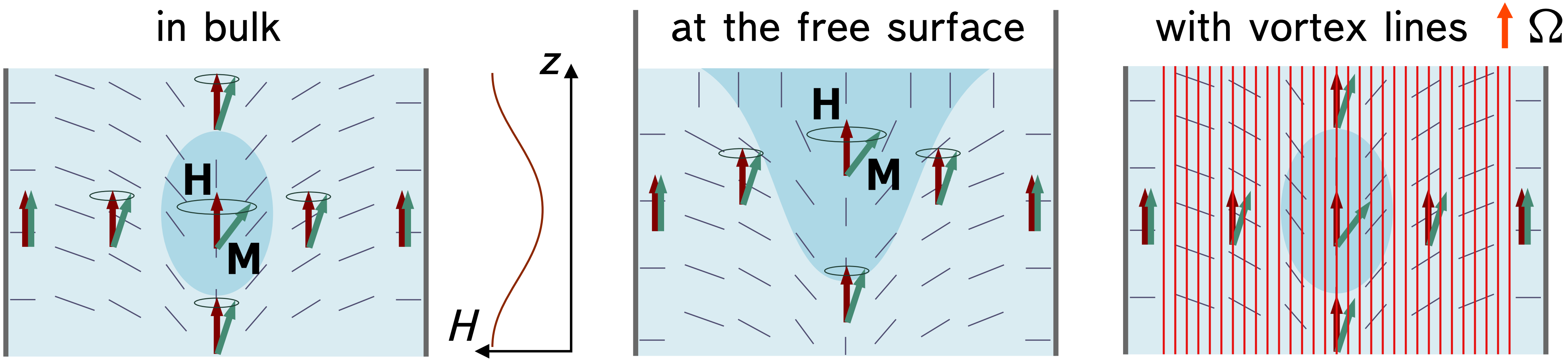


- Relaxation time in the excited state is longer than in linear NMR (∼ 10 ms).
- Ground state is filled simultaneously – two coexsisting condensates.

MOTIVATION FOR RELAXATION STUDIES

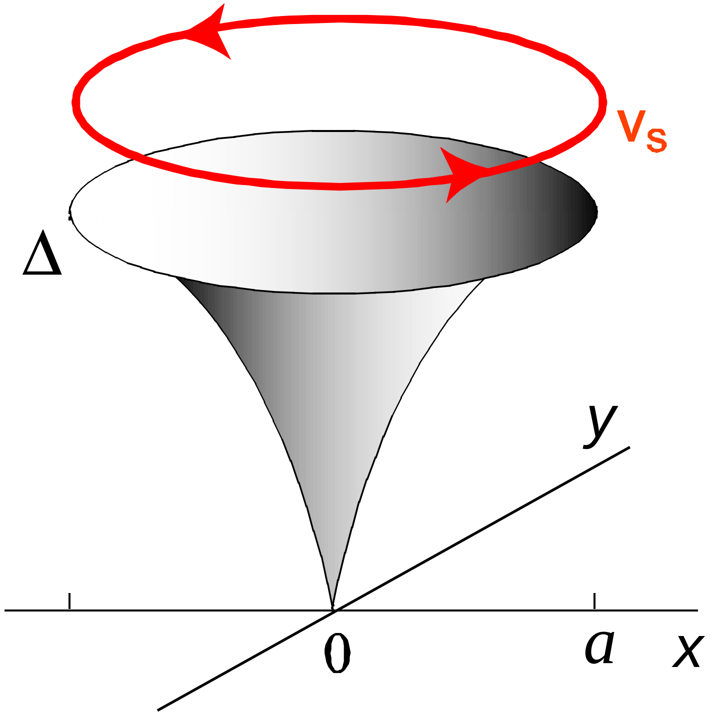
Long life time of the magnon BEC in the $T \to 0$ limit (exceeding the life time of atomic condenstates) makes them a sensitive probe for extra relaxation sources.



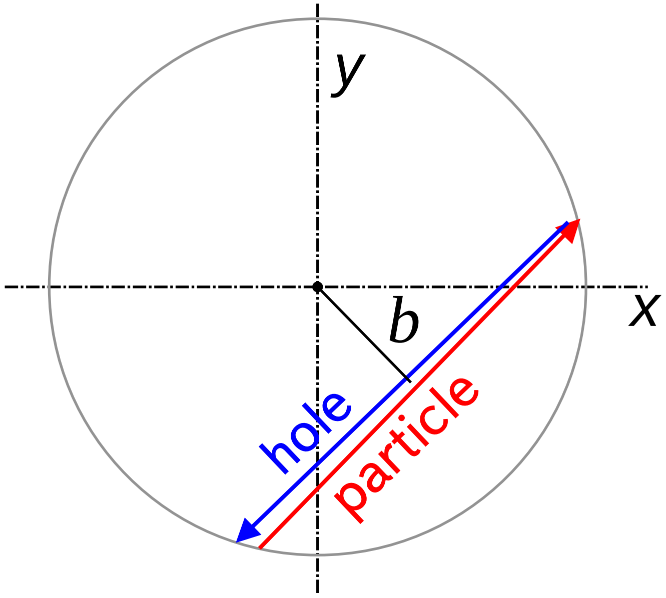
We hope to find the contribution from the Majorana fermion zero modes bound to the surface of cores of quatized vortices by comparing relaxation of magnon condensates in different trap configurations.

BOUND FERMION STATES IN THE VORTEX CORE

Caroli, de Gennes, Matricon 1964

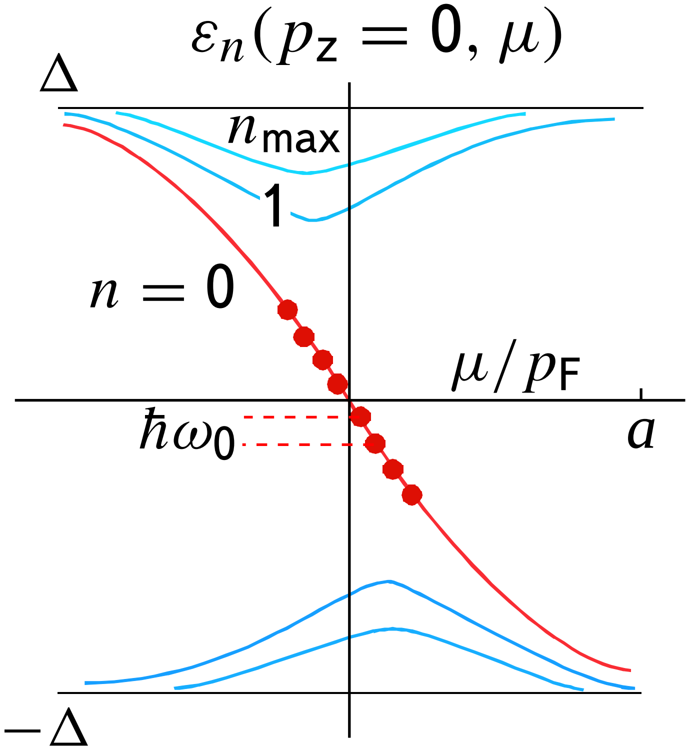


Andreev reflection



Radial quantum number n $(n_{\text{max}} \sim a/\xi)$.

Anomalous (crossing zero) branch $n = 0$.

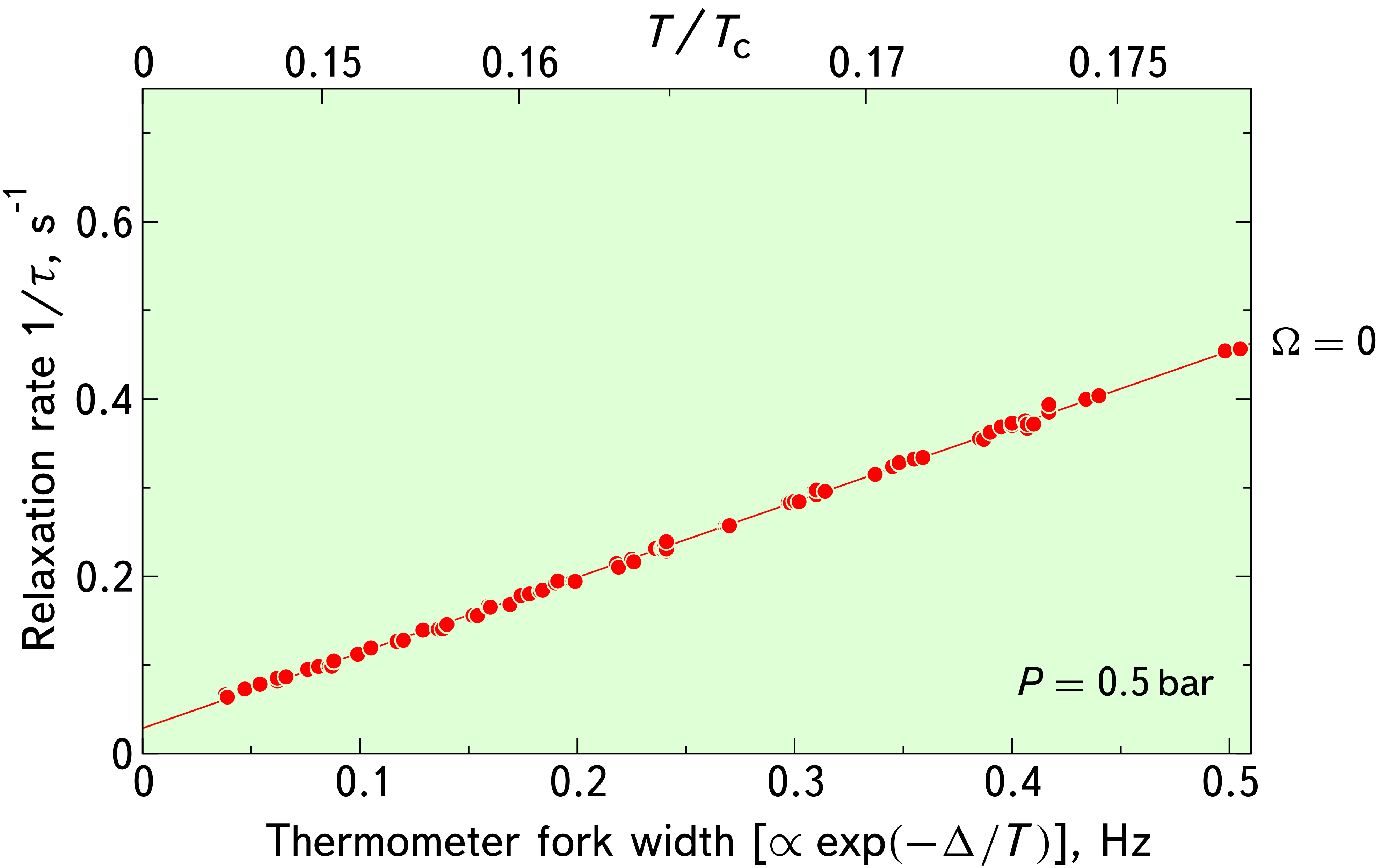


Angular momentum $\mu = b\, p_\perp$, quantized.

$$\mu/\hbar = \begin{cases} m + 1/2, & \text{s-wave superconductors} \\ m, & \text{superfluid } {}^3\text{He} \end{cases}$$

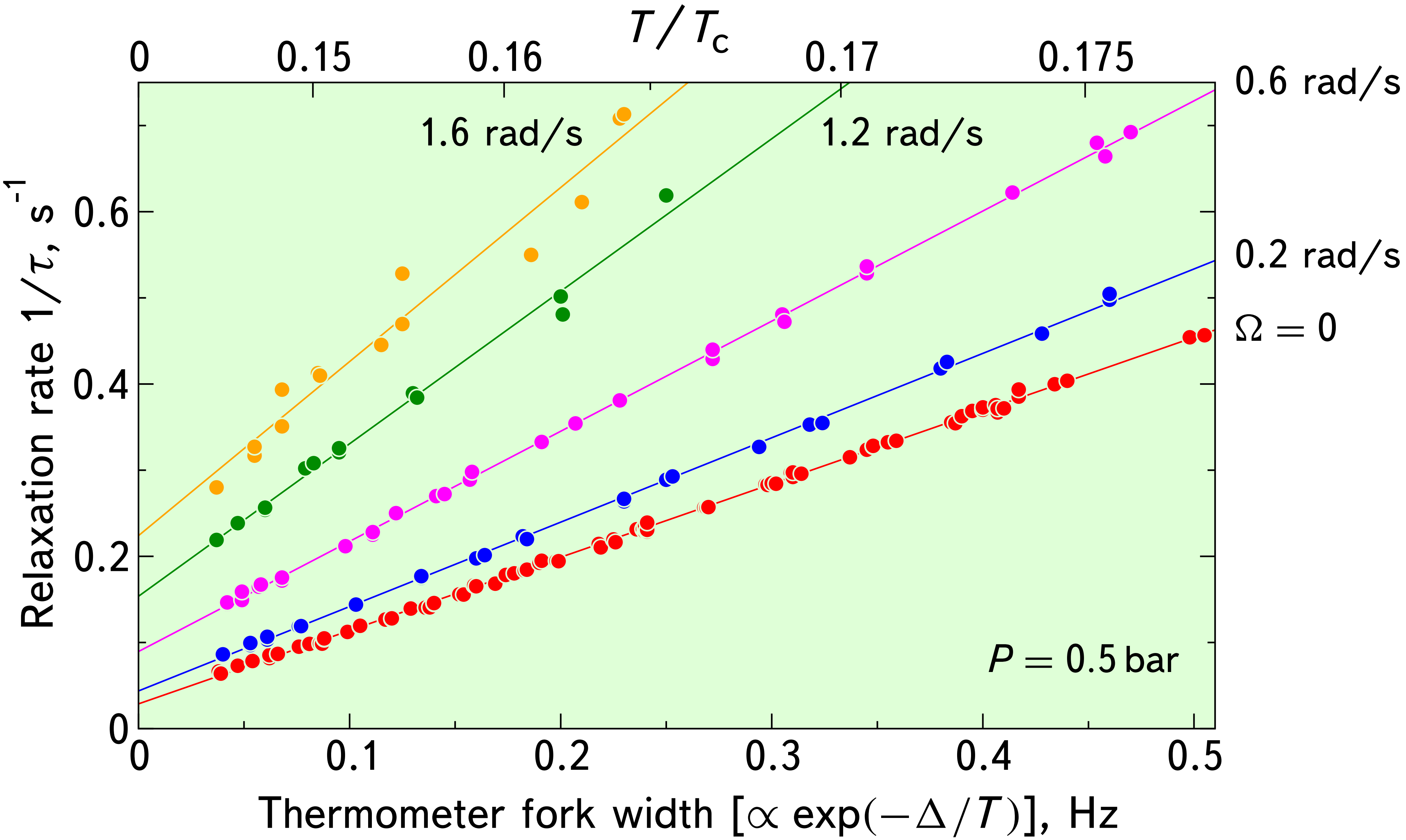
Minigap $\omega_0 \sim \dfrac{\Delta}{a\, p_F} \sim \dfrac{1}{\hbar}\dfrac{\Delta^2}{E_F} \ll \dfrac{\Delta}{\hbar}$.

RELAXATION IN THE VORTEX STATE



Relaxation rate: $1/\tau = 1/\tau_0 + C\exp(-\Delta/T)$

RELAXATION IN THE VORTEX STATE

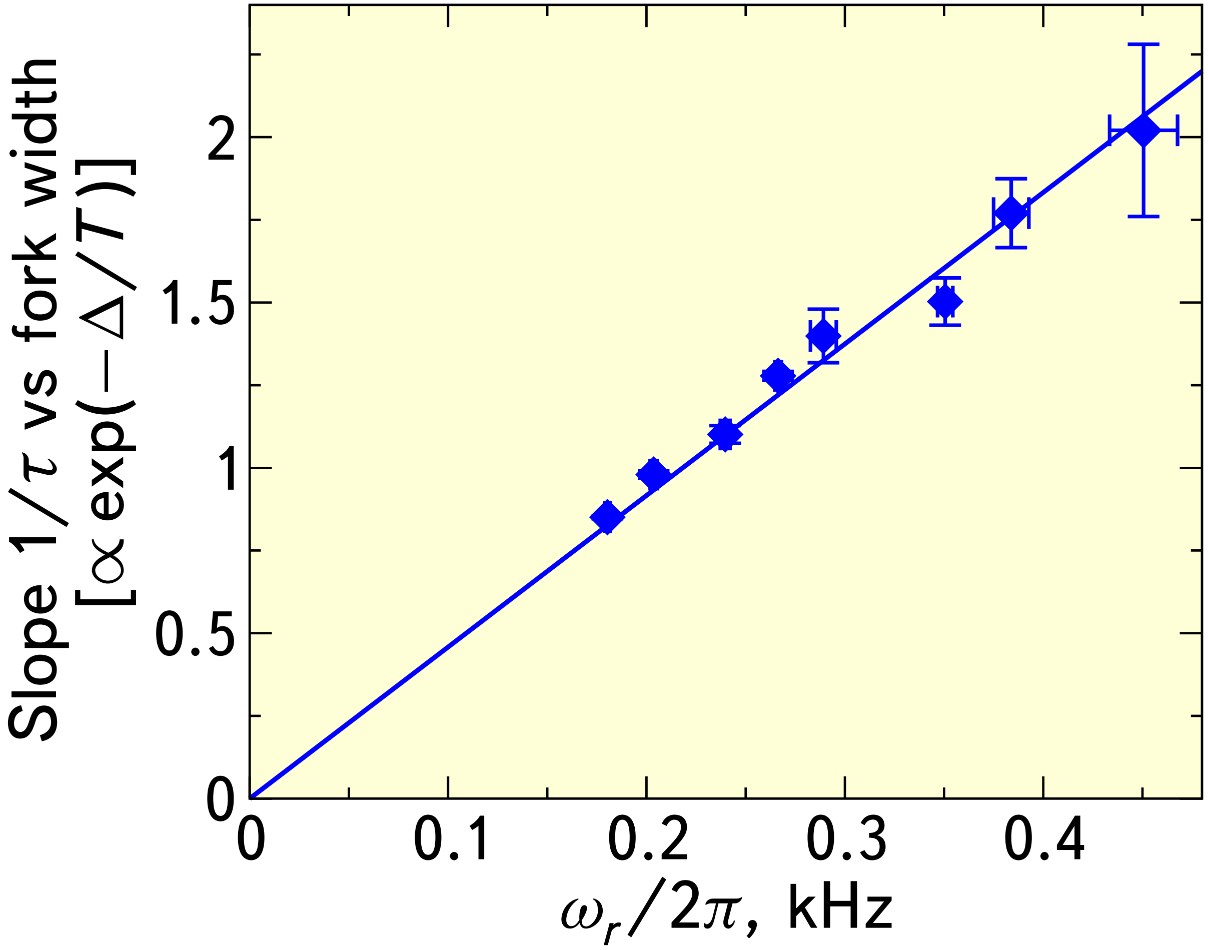


Relaxation rate: $1/\tau = 1/\tau_0(\Omega) + C(\Omega)\ \exp(-\Delta/T)$

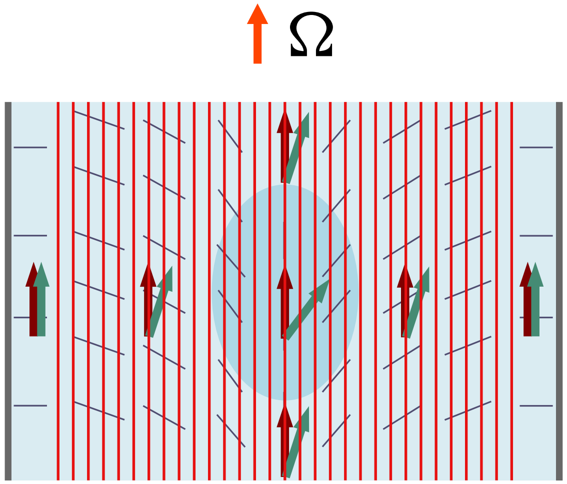
TEMPERATURE DEPENDENCE OF RELAXATION

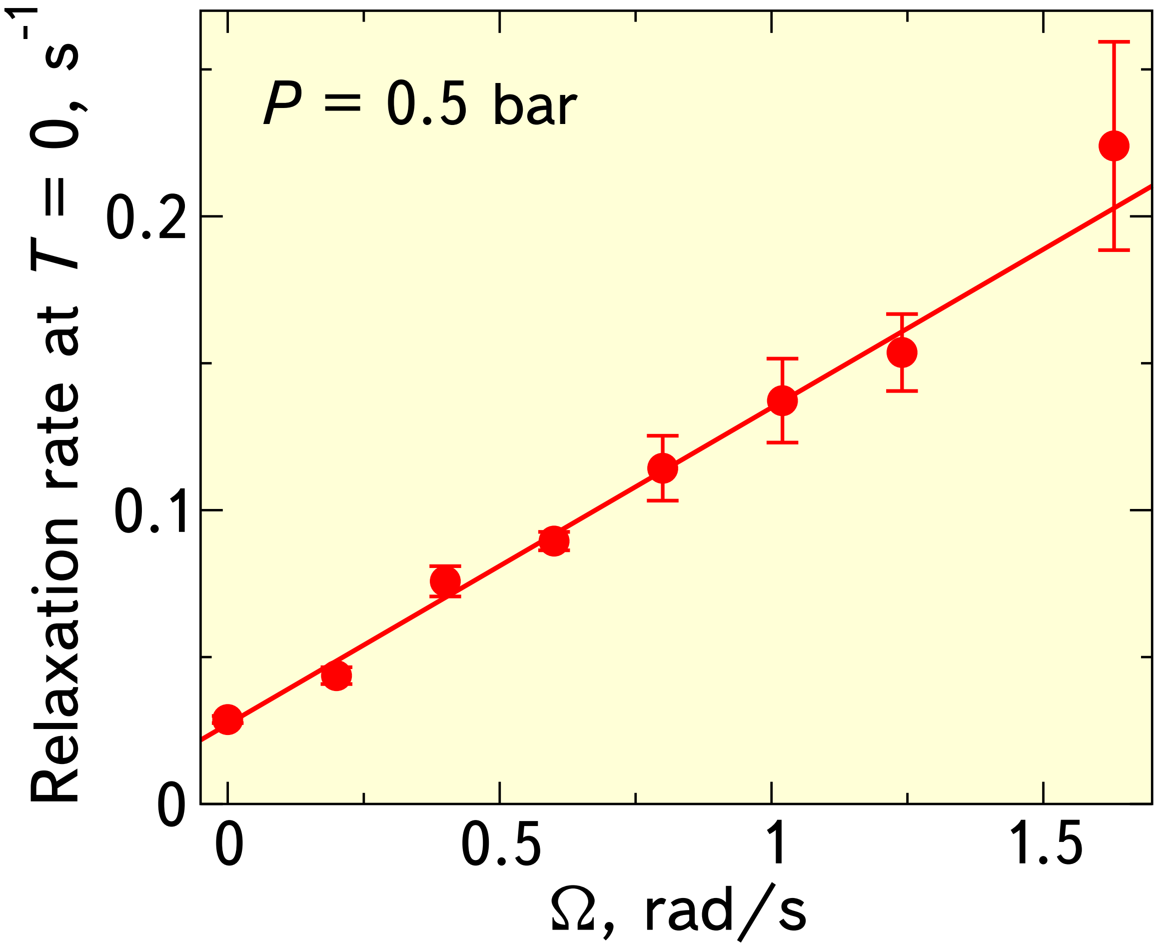
Spin diffusion via normal component (bulk thermal quasiparticles):

$$1/\tau \propto \rho_n |\nabla\Psi|^2 \propto \rho_n R_b^{-2} \propto \exp(-\Delta/T)\,\omega_r$$



DEPENDENCE OF RELAXATION ON VORTEX DENSITY

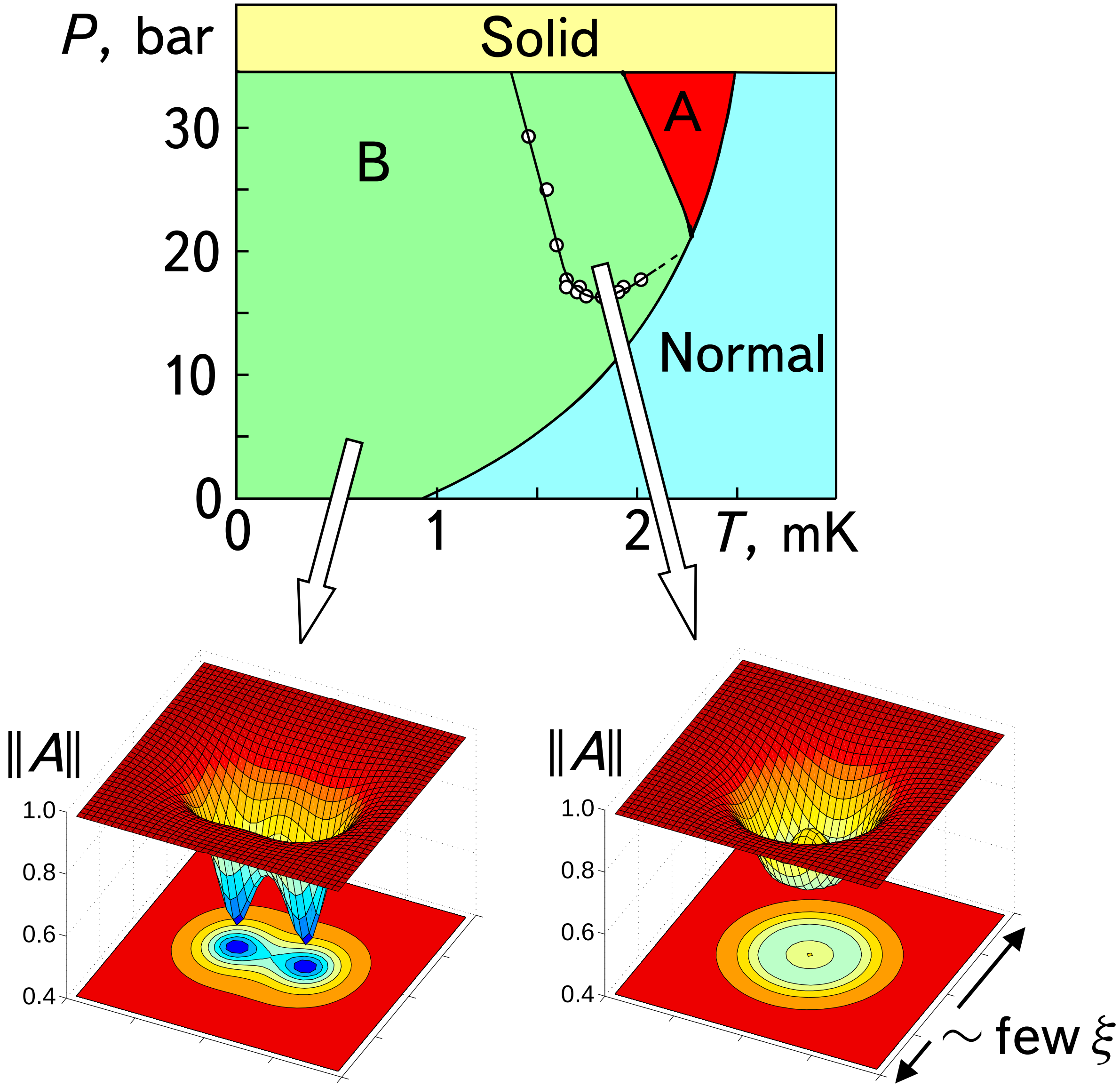




Vortices definitely contribute to the relaxation of magnon condensates.

Is the effect related to the fermions bound to vortex cores?

BROKEN SYMMETRY OF VORTEX CORES IN ^{3}He-B



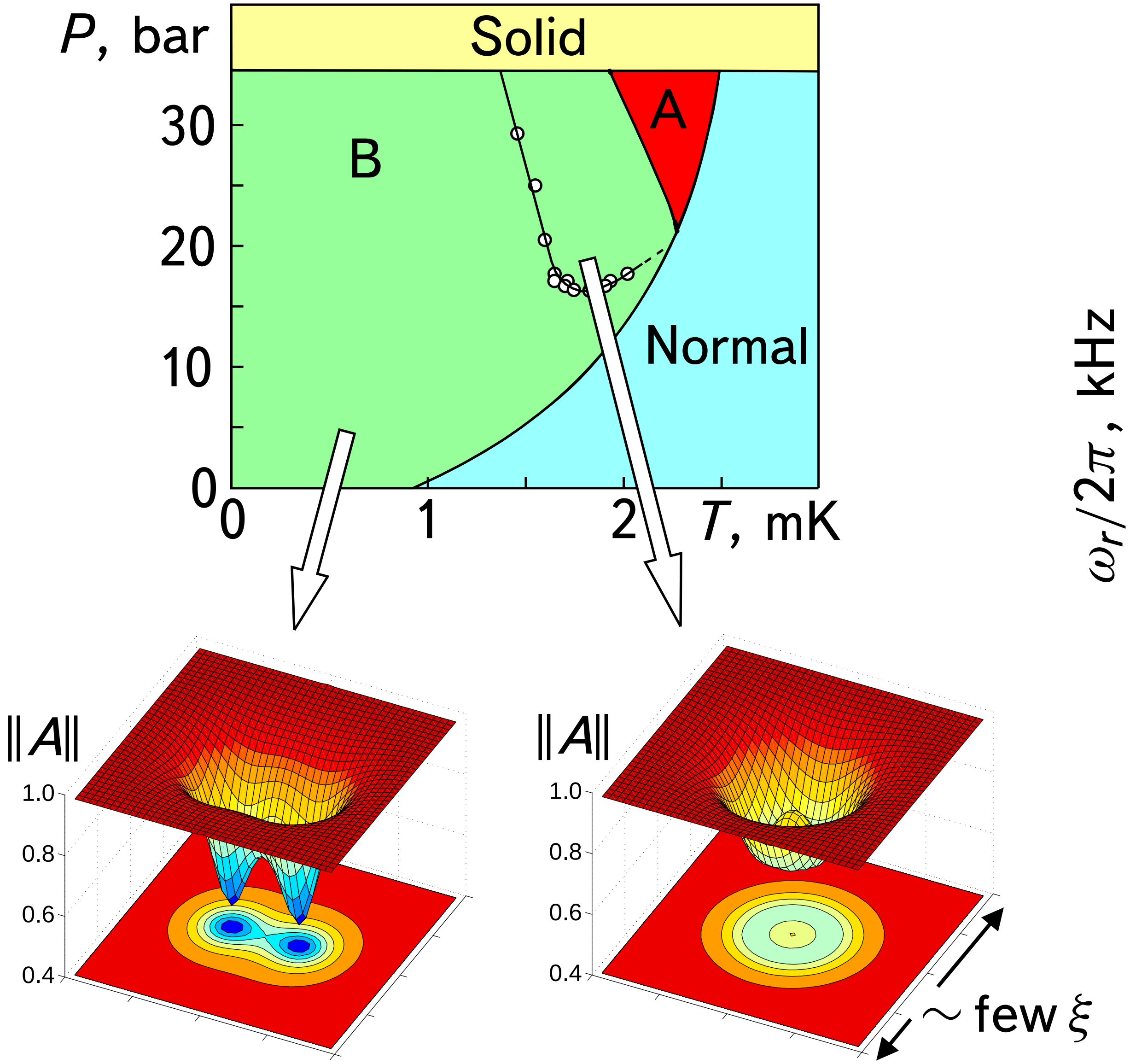
Broken symmetry core

Axisymmetric core

Ikkala, Hakonen, Bunkov, Krusius et al 1982-

Salomaa, Volovik, Thuneberg et al

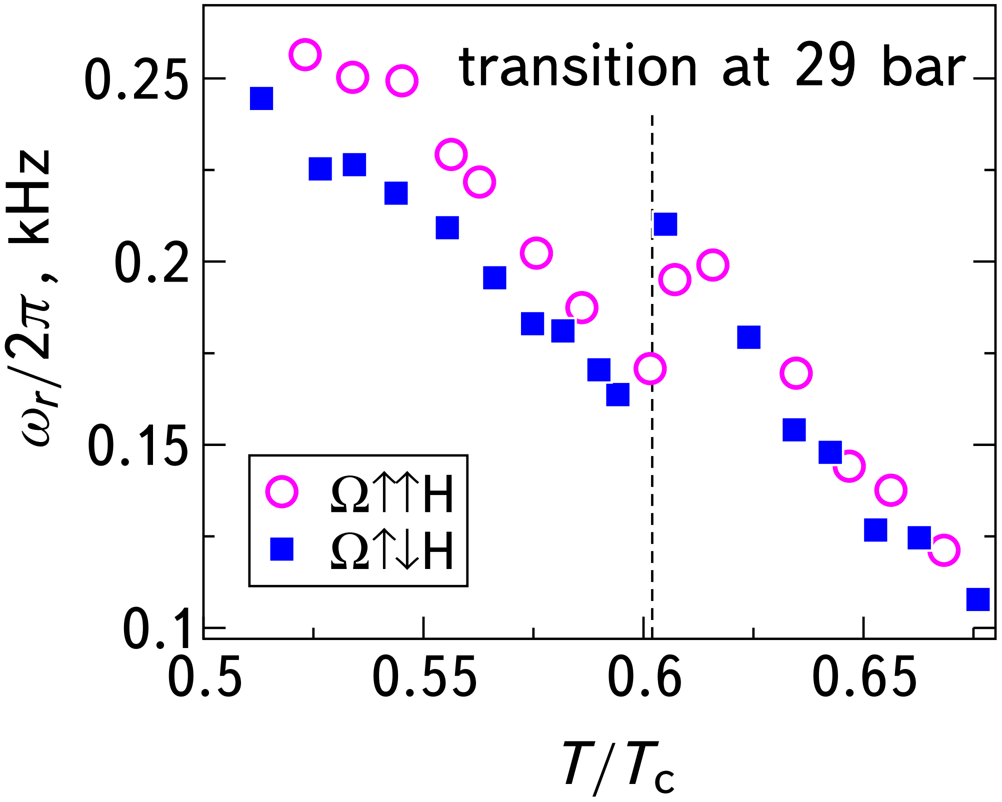
BROKEN SYMMETRY OF VORTEX CORES IN ^{3}He-B



Broken symmetry core

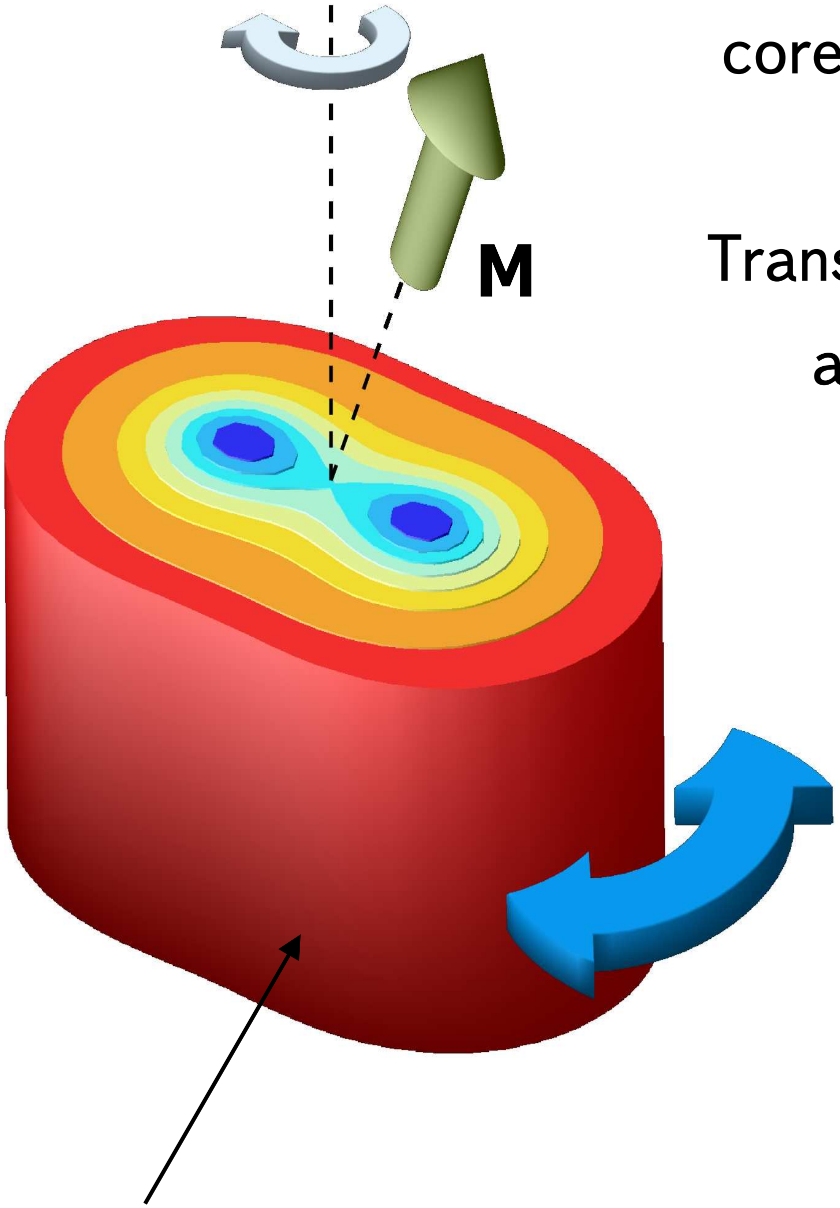
Axisymmetric core

Effect on the textural potential well for magnons:



Ikkala, Hakonen, Bunkov, Krusius et al 1982-
Salomaa, Volovik, Thuneberg et al

DAMPING OF SPIN PRECESSION VIA VORTEX CORES



Torque from precessing magnetic moment puts vortex core in twisting motion (oscillations / precession)

⇓

Transitions between the core-bound fermion states are triggered and the core gets overheated

⇓

Dissipation

(Kopnin and Volovik, 1998)

Core of the non-axisymmetric vortex

CONCLUSIONS

- For the coherently precessing magnon condensate in a magneto-textural trap in ^{3}He-B the trap transforms with increasing magnon number from a harmonic well to a cylindrical box: bosonic analogue of the electron bubble in helium and of the MIT bag model of hadrons.

- Unlike cold-atom case, in the magnon trap different excited levels can be selectively populated with condensates.

- Relaxation rate of magnon condensates depends on temperature and the trap size as expected for the spin diffusion relaxation mechanism.

- In the vortex state relaxation rate has an additional contribution, which grows linearly with the density of vortices. Whether this contribution can be attributed to the Majorana fermions bound to vortex cores remains to be established.

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