Tachyon condensation in Bose-Einstein condensates

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Topological Quantum Phenomena in Condensed Matter with Broken Symmetries

Collaboration:

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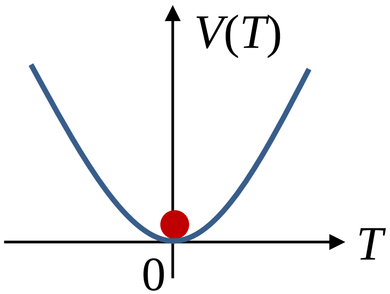
H. T., K. Kasamatsu, M. Tsubota, and M. Nitta, arXiv:1205.2330

Tachyon condensation

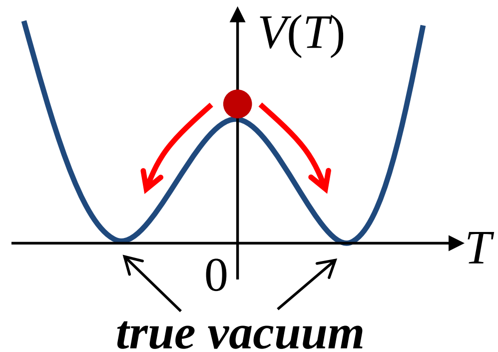
A tachyon is a hypothetical superluminal particle in special relativity. However, in quantum field theory, a tachyon *field* T can exist due to the instability of quantum vacuum.

Stable vacuum



Local minimum

Unstable vacuum



Spinodal (tachyonic) instability

The tachyon field T rolls down toward the true vacuum at a minimum of potential.

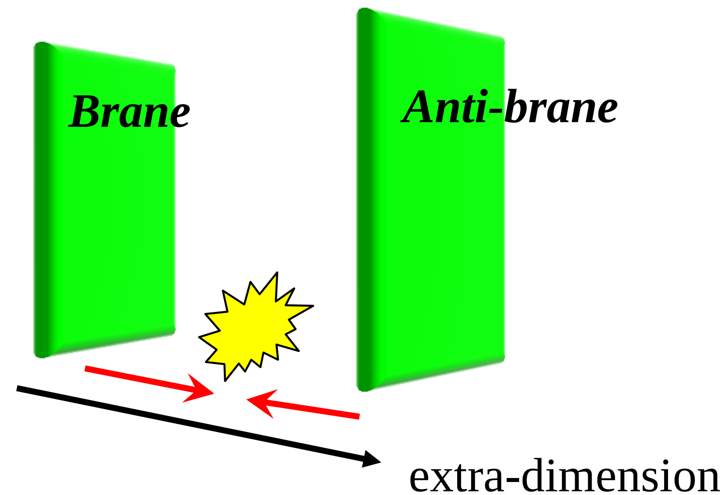
$\Rightarrow$ ***Tachyon condensation***

Brane annihilation in string theory

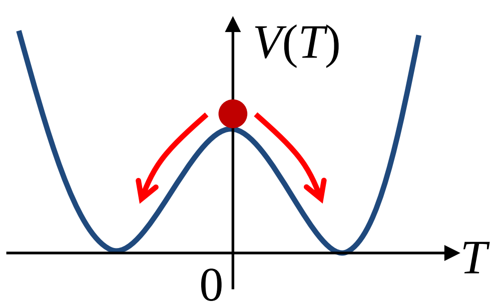
J. Polchinski, *String Theory* (Cambridge University Press, Cambridge, 1998), Vols. 1 and 2.

In string theory, a tachyon exists in a system containing a D-brane and an anti-D-brane.
(D-brane is an extended solitonic object in higher-dimensional space.)

A brane and an anti-brane annihilate in a collision like a particle and an antiparticle.



Tachyon potential



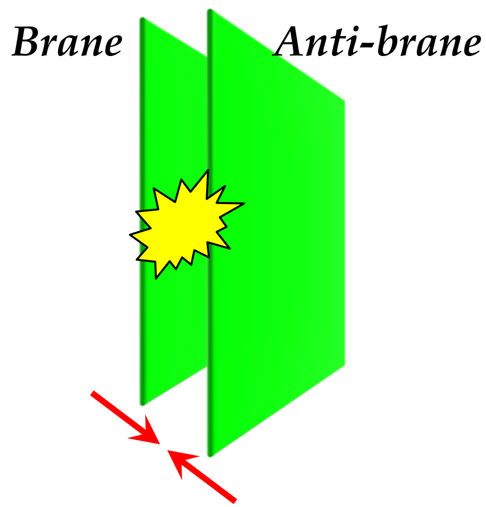
The annihilation process is interpreted as tachyon condensation, and the system falls into the true vacuum after complete annihilation.

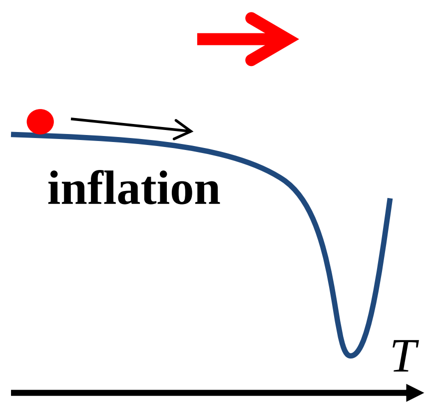
A. Sen, *Tachyon Dynamics in Open String Theory*, Int. J. Mod. Phys. A **20**, 5513-5656 (2005).

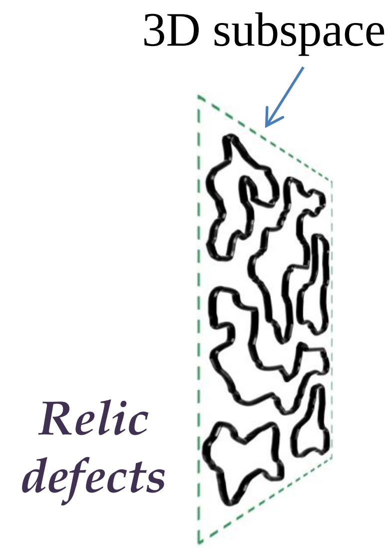
Application to cosmology

A remarkable application of tachyon condensations is in brane cosmology, in which the Big Bang is hypothesized to occur as a result of a collision of a brane and an anti-brane.

[Example]







Tachyon condensation

↓

Inflation in the early Universe?

Relic lower-dimensional branes

↓

Cosmic string?

Subspatial SSB

This situation resembles conventional phase transitions accompanied by spontaneous symmetry breaking (SSB), resulting in the formation of topological defects via the Kibble-Zurek mechanism in the early Universe.

Kibble-Zurek mechanism ⟺ Analog simulations in condensed matter experiments

Conventional SSB

e.g., superfluid systems, liquid crystal……

In contrast, tachyon condensation as an SSB phenomenon have not been well understood.

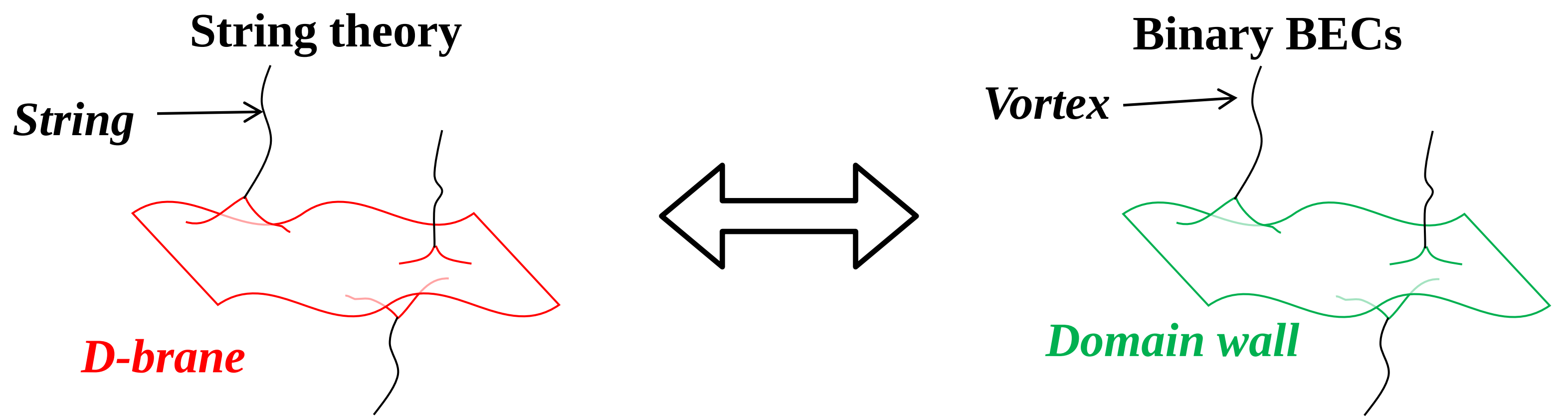
Tachyon condensation (brane annihilation) ⟺ ? Such phenomena have never been realized in actual systems.

SSB in subspace

Since brane annihilations cause defect nucleation in a lower-dimensional subspace, the dynamics may be affected by the extra dimension. However, the influence of the extra dimension has never been revealed.

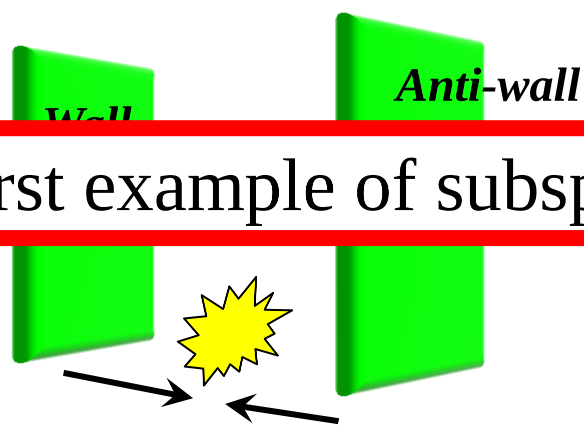
Motivation

In this work, we provide a groundbreaking system to tackle this problem in atomic Bose-Einstein condensates (BECs). Recently, we proposed that domain walls in phase-separated binary BECs can correspond to D-branes.

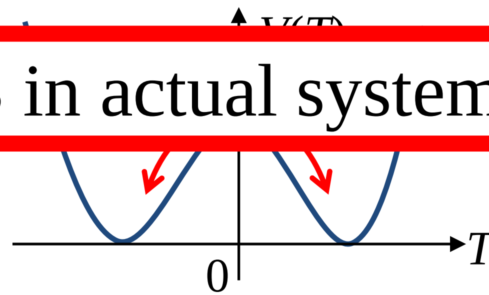


K. Kasamatsu, H. Takeuchi, M. Nitta, and M. Tsubota, JHEP **11**, 068 (2010).

Question



Subspatial SSB?



This is the first example of subspatial SSB in actual systems.

Yes!

Binary BECs

Order parameters (Macroscopic wave functions)

$$\Psi_1, \qquad \Psi_2$$

(Bose-Einstein condensations of two distinguishable bosons)

Action functional in the Gross-Pitaevskii (GP) model

$$\mathcal{S} = \int dt \int d^3x \left[i\hbar \left(\Psi_1^* \partial_t \Psi_1 + \Psi_2^* \partial_t \Psi_2 \right) - \mathcal{K} - \mathcal{V} \right]$$

$$\mathcal{K} = \frac{\hbar^2}{2m_1} |\nabla \Psi_1|^2 + \frac{\hbar^2}{2m_2} |\nabla \Psi_2|^2$$

$$\mathcal{V} = \frac{1}{2} g_{11} |\Psi_1|^4 + \frac{1}{2} g_{22} |\Psi_2|^4 + g_{12} |\Psi_1|^2 |\Psi_2|^2 - \mu_1 |\Psi_1|^2 - \mu_2 |\Psi_2|^2$$

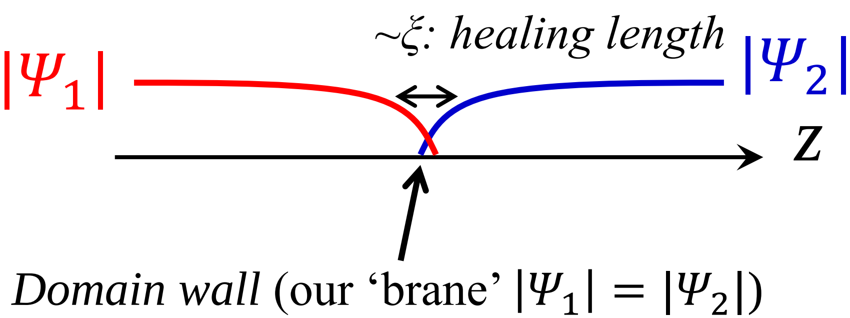
the coupled GP equations

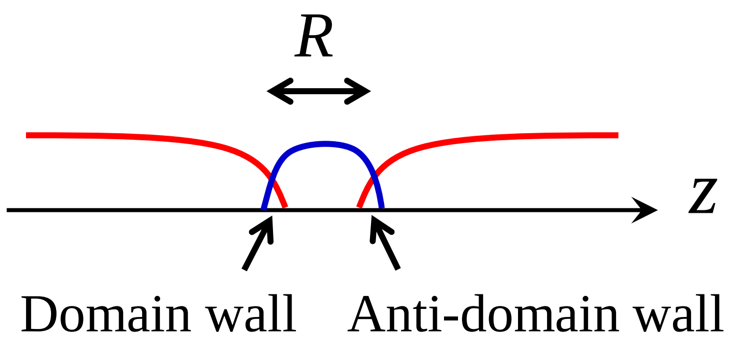
$$i\hbar \partial_t \Psi_j = \left(-\frac{\hbar^2}{2m_j} \nabla^2 - \mu_j + \sum_k g_{jk} |\Psi_k|^2 \right) \Psi_j \qquad (j, k = 1, 2)$$

$m_1 = m_2 \equiv m, g_{11} = g_{22} \equiv g > 0, g_{12} = 2g$ (strong segregation)

S. B. Papp, J. M. Pino, and C. E. Wieman, Phys. Rev. Lett. **101**, 040402 (2008).

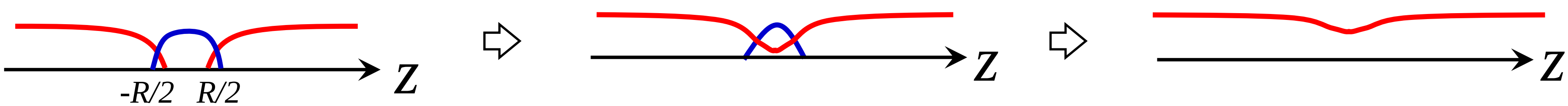
Domain wall





Since the interaction between branes is characterized by the penetration depth $\sim\xi$ of the order parameters, the annihilation process starts substantially for $R\sim\xi$.
(The inter-brane distance R is a increasing function of $\nu = \mu_2/\mu_1$.)

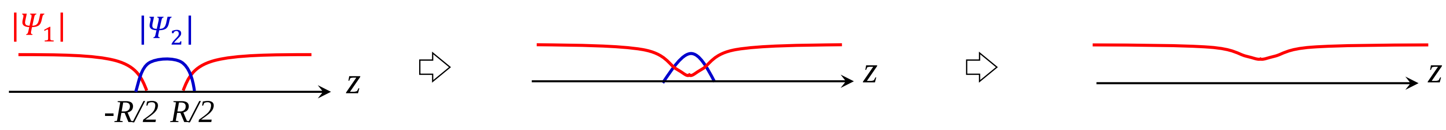
Trivial annihilation process



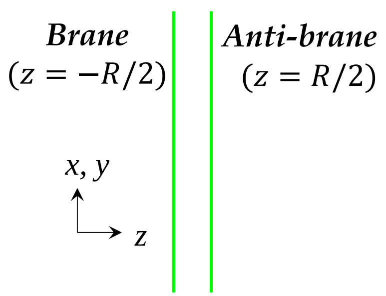
In general, the annihilation process becomes nontrivial depending on the phase difference between the two domains (z>R/2 and z< -R/2).

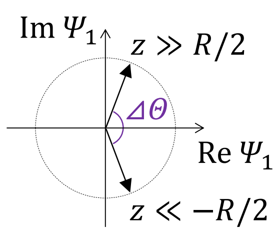
Non-trivial annihilation

1D diagram



2D diagram

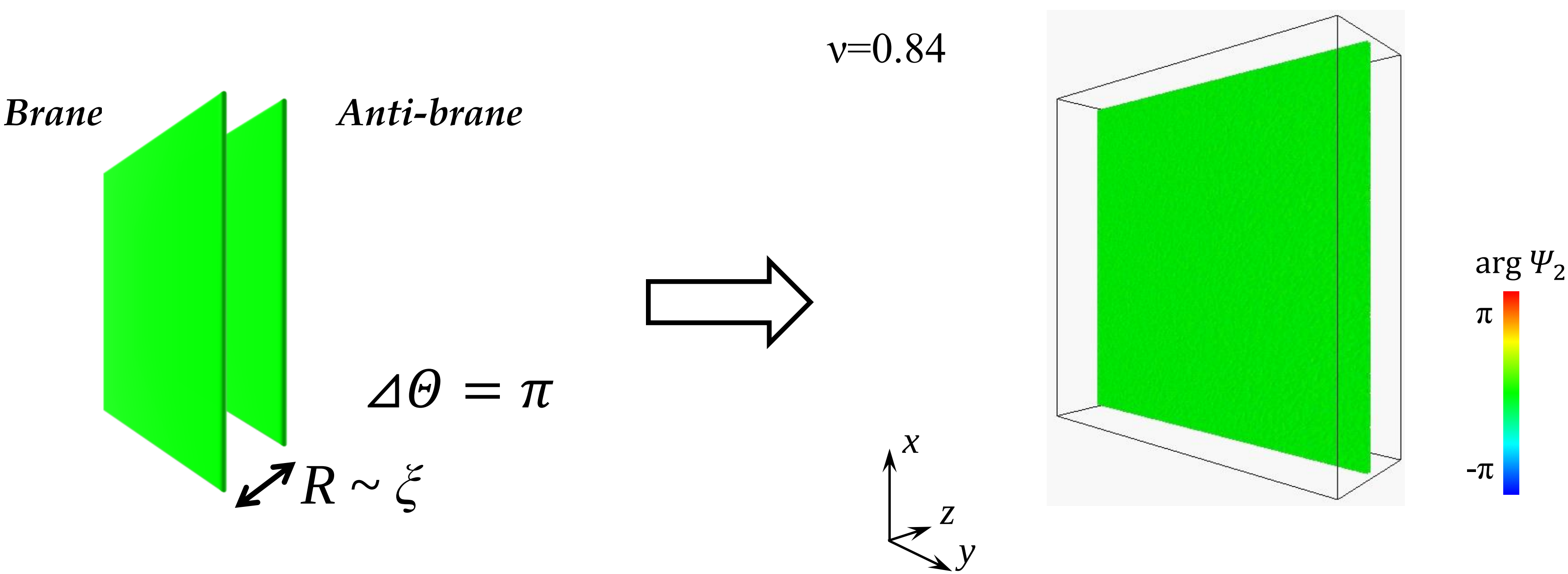




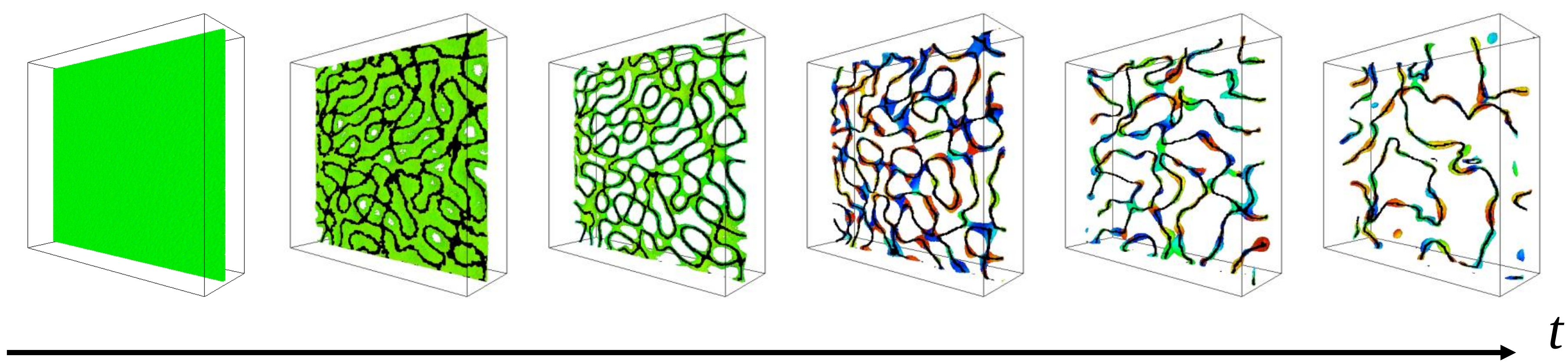
3D diagram

Numerical simulation

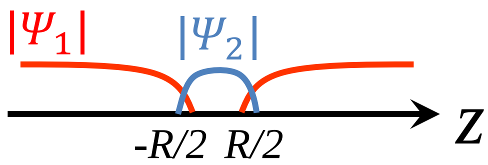
isosurface of $|\Psi_1| = |\Psi_2|$



box size $80\xi \times 80\xi \times 25.6\xi$



Effective theory of tachyon condensation

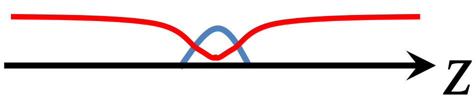


Effective tachyon field $T(x, y, t)$

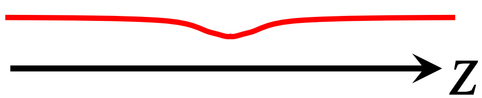
Variational ansatz

$$\Psi_1(\boldsymbol{r}, t) = \Psi_1^0(z) + T(x, y, t)\sqrt{\frac{\mu_1}{g}}\mathrm{sech}^2\left(\frac{z}{\xi_\perp}\right),$$

$$\Psi_2(\boldsymbol{r}, t) = \sqrt{n_2(T)}\mathrm{sech}\left(\frac{z}{\xi_\perp}\right).$$



Initial state T=0 at $t=0$



$$\Psi_1(\boldsymbol{r}, 0) = \Psi_1^0 = \sqrt{\frac{\mu_1}{g}\tanh^2\left(\frac{z}{\xi_\perp}\right)e^{\pm i\Delta\Theta}},$$

Effective potential $V(T)$ for the field T

$$V(T) = \int_{-\infty}^{+\infty} dz(\mathcal{K} + \mathcal{V}) \quad \text{with } \partial_x\Psi_j = \partial_y\Psi_j = 0$$

$n_2(T)$ is determined so as to minimize $V(T)$

Effective theory of tachyon condensation

Effective energy functional $$E_{\mathrm{2D}} = \int dxdy\,\left[G(T)(\xi \boldsymbol{\nabla}_{\parallel} T)^2 + V(T)\right]$$

Effective potential $$V = \frac{\mu_1^2 \xi_\perp}{g} \sum_{n=0}^{4} F_n(\Delta\Theta, \nu, \gamma) T^n \qquad (\mathrm{T}{\sim}0)$$

$$\left(\gamma = \frac{g_{12}}{g} = 2\right)$$

$$F_{1,3} \propto \cos\frac{\Delta\Theta}{2} = 0 \quad \text{for} \quad \Delta\Theta = \pi$$

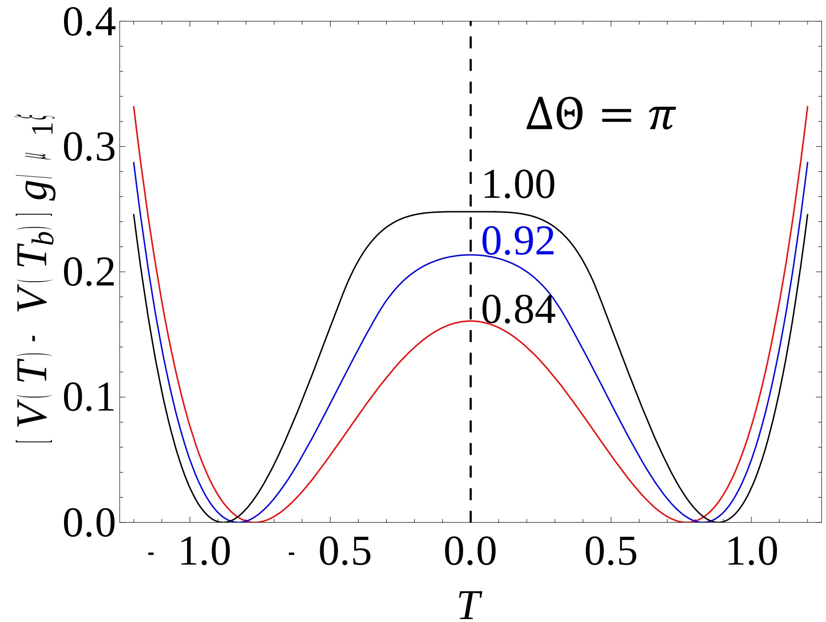
Stability of the brane-anti-brane state for $\Delta\Theta = \pi$

$$V''(T = 0) \propto F_2$$

$$\begin{cases} F_2 > 0 \Rightarrow \text{Stable} \\ F_2 < 0 \Rightarrow \text{Unstable (tachyonic!)} \end{cases}$$

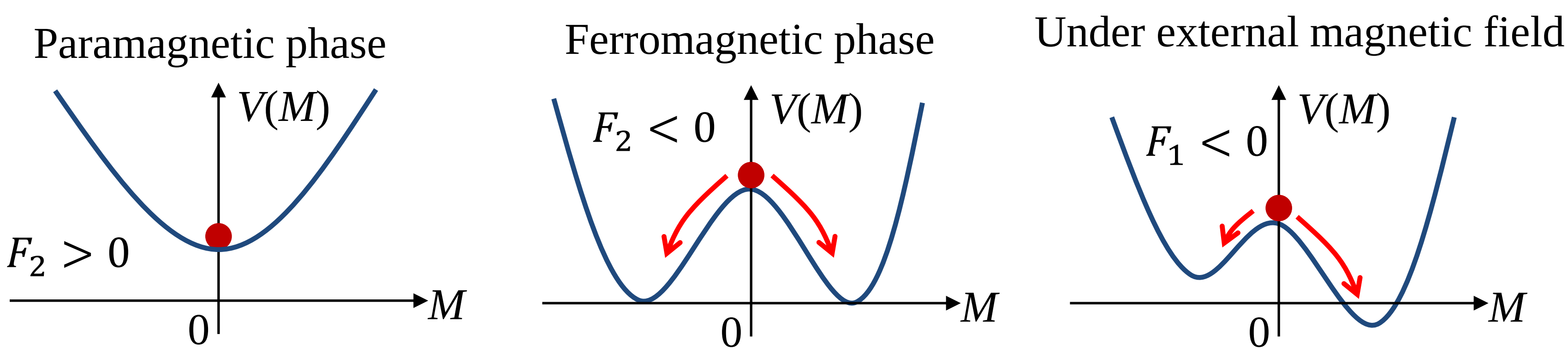
F_2 is a decreasing function of R

ν-dependence of $V(T)$



Effective theory of tachyon condensation

Analogy to the Ginzburg-Landau model for a ferromagnetic system



Tachyon field	$\Rightarrow$	'magnetization density'
Inter-brane distance	$\Rightarrow$	'temperature'
Phase difference ($\Delta\Theta \neq \pi$)	$\Rightarrow$	'external field'

Interaction between branes becomes small as R increases,
and the instability vanishes for $R \to \infty$.

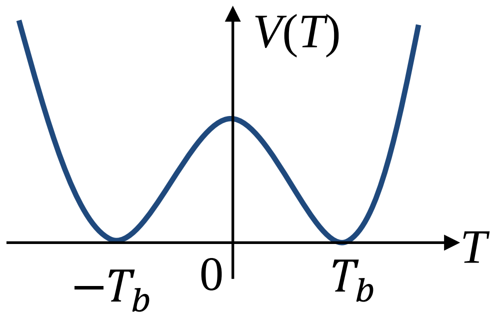
$R \to \infty$ corresponds to the transition 'temperature'

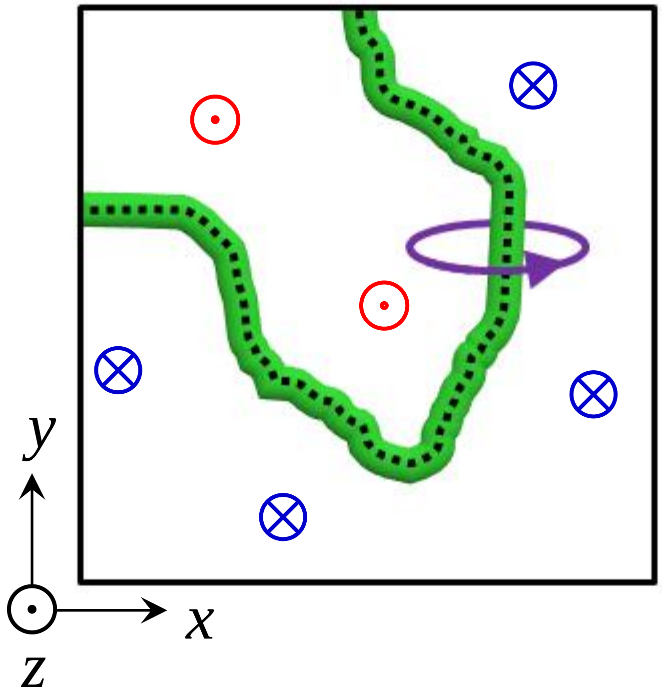
Two-dimensional SSB is formulated be 'projected' from the original order parameters.

Vortex ⇒ kink in projected-2D

$v_\perp > 0 \Longrightarrow T > 0$

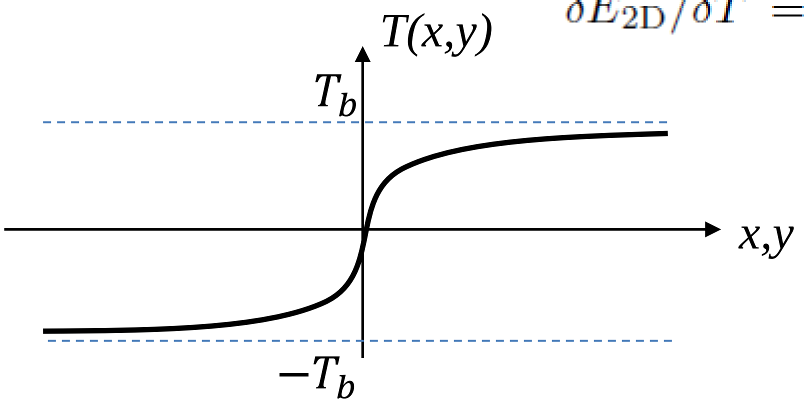
$v_\perp < 0 \Longrightarrow T < 0$





a kink solution

$\delta E_{2D}/\delta T = 0$



Projected phase ordering dynamics

To check the validity of the effective theory, we apply the scaling law of phase ordering kinetics [A. J. Bray, Adv. Phys. **43**, 357-459 (1994)] to the relaxation dynamics after the defect nucleation in the projected-2D space.
If the field *T* obeys the scaling law for 2D, its structure factor *S*(***q***, t) would be written as

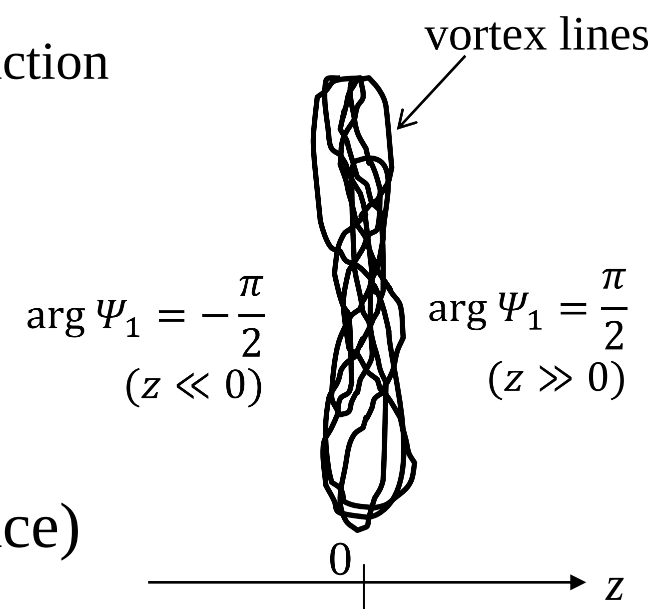
$$\mathcal{F}(q/l_{\rm 2D}) = S(\boldsymbol{q}, t) l_{\rm 2D}^2 \qquad \text{for } |\boldsymbol{q}| \ll 1/\zeta$$

l_{2D}: mean inter-kink distance
ζ: width of a kink
F: time-independent scaling function

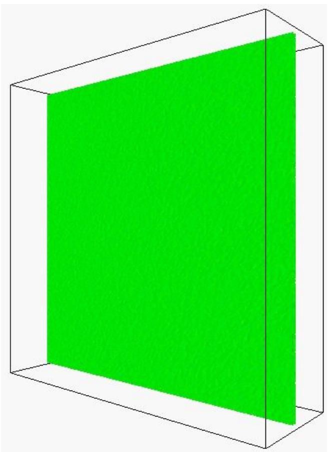
Projected field

$$\phi(x, y, t) \equiv \frac{m}{\hbar} \int_{-\infty}^{+\infty} dz v_{\perp}(\mathbf{r}, t)$$

$T \propto \phi \approx \pm\pi$ (far from kinks in 2D space)

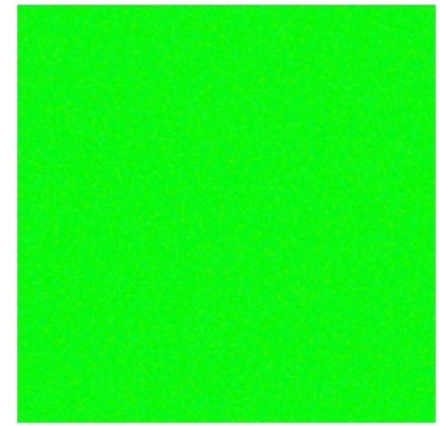


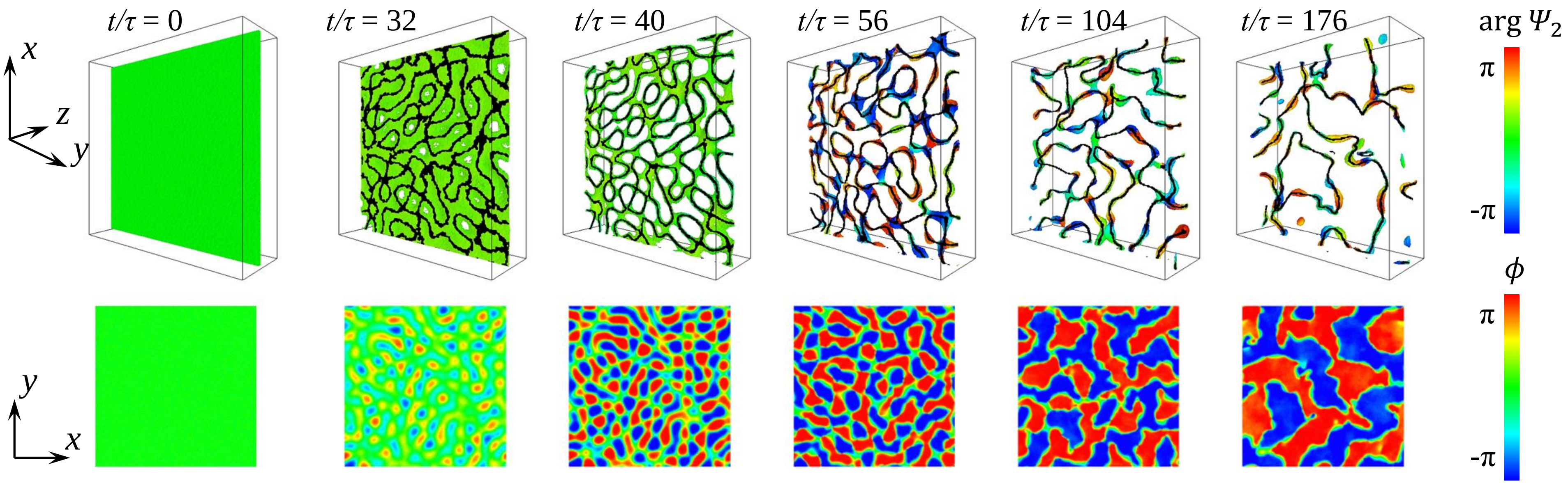
Projected phase ordering dynamics



‘projection’
from 3D to 2D

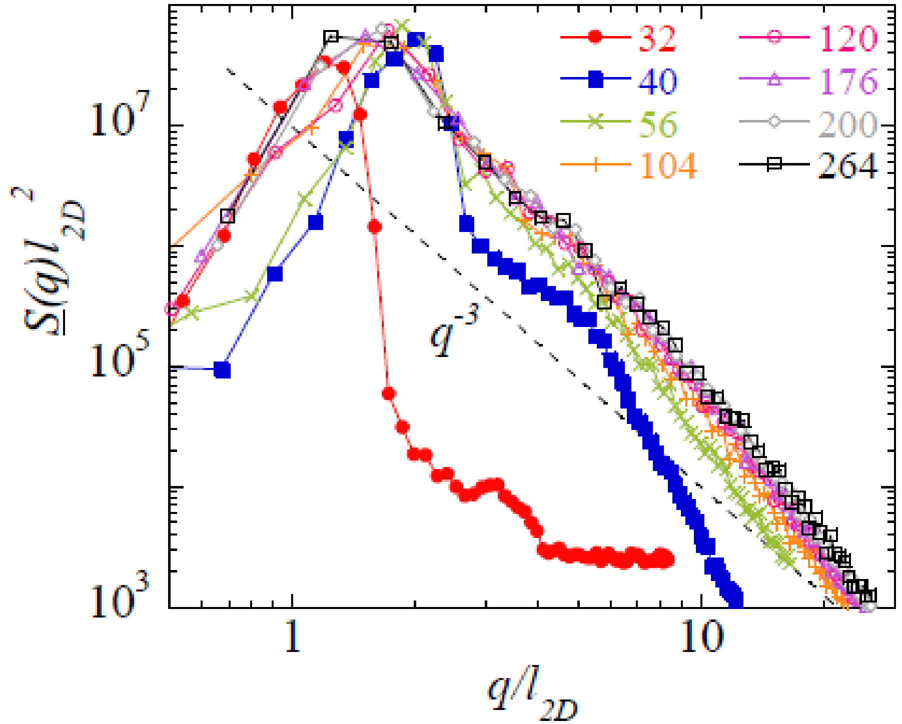
$\Psi_j(x, y, z, t) \longrightarrow \phi(x, y, t)$

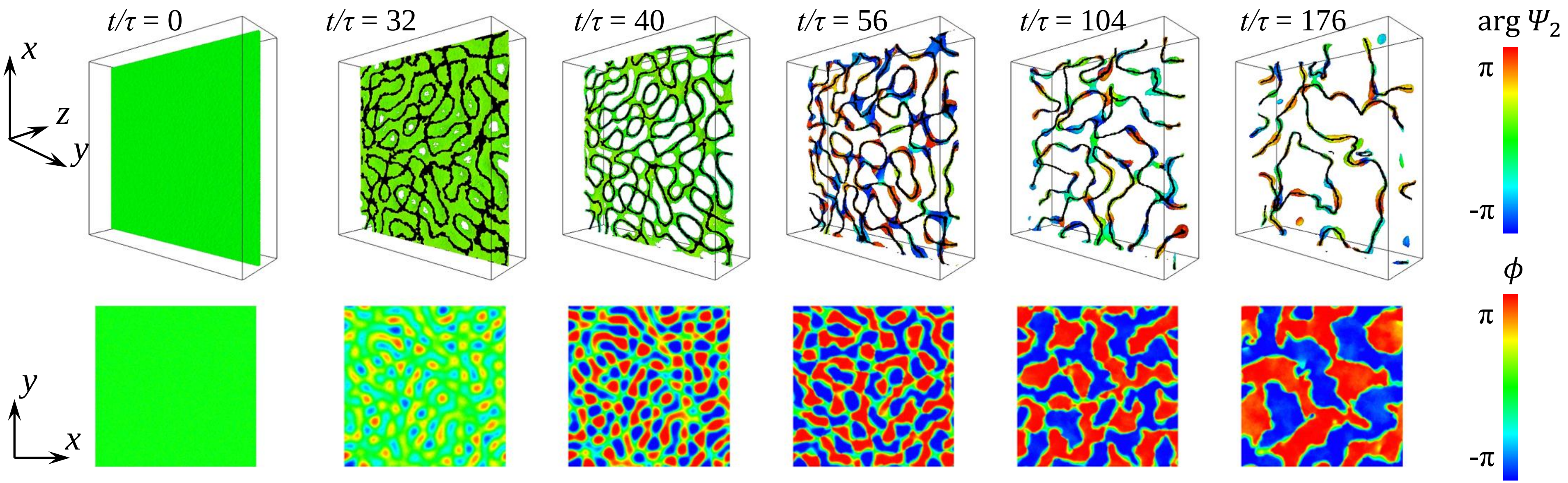




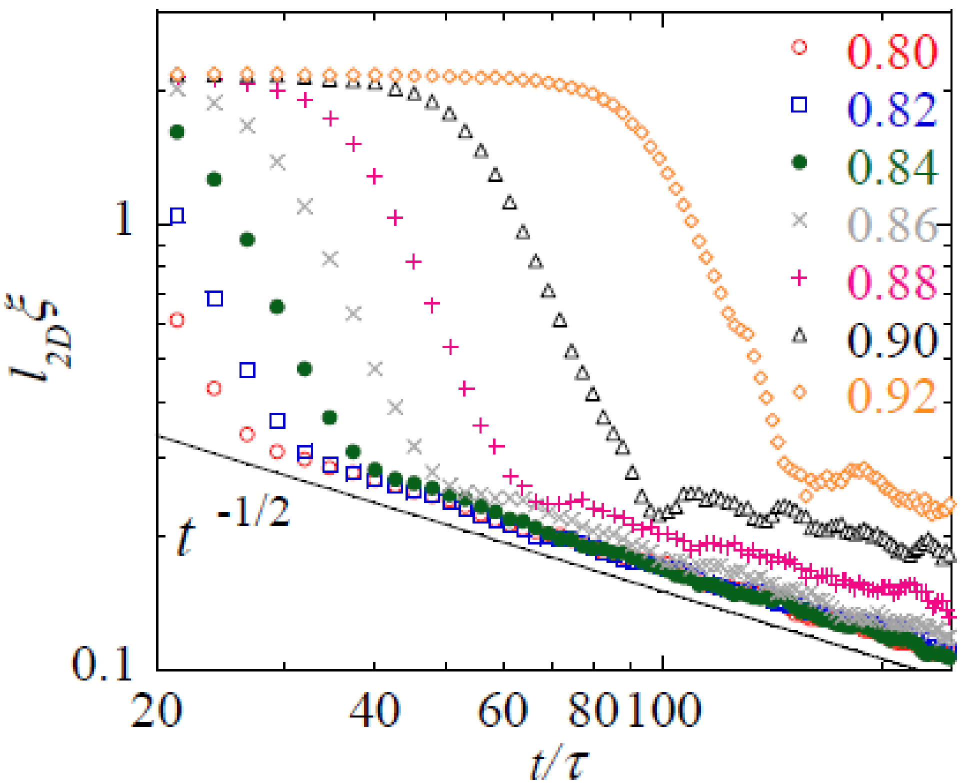
Scaling law

The scaling law is obeyed after the domain structure becomes clear.



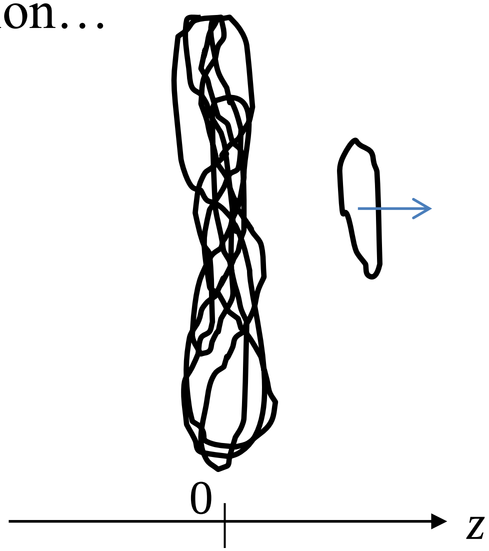


Decay law



Although the total energy is conserved in the original 3D system,
kink decay implies that the projected-2D system is dissipative.
This is because the degree of freedom associated to the extra dimension is neglected in the projected-2D system,

e.g., emission of vortex ring to the extra-dimension…



Summary

We propose the first example of subspatial SSB in condensed matter system.

Correspondence

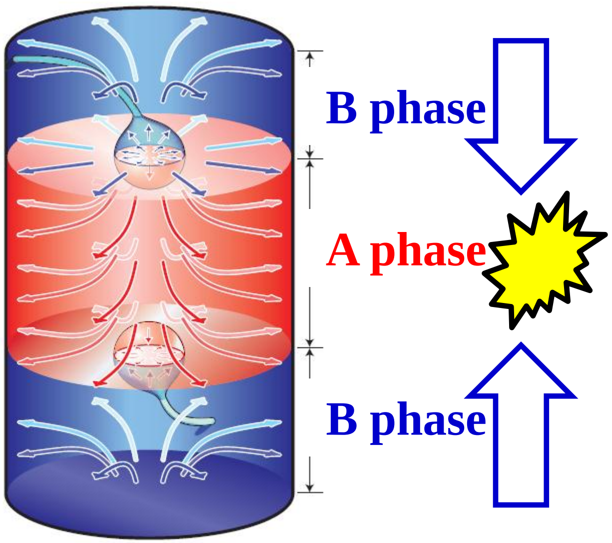
3D		Projected-2D
Inter-brane distance R	$\Rightarrow$	'*Temperature*'
Phase difference $\Delta\Theta \neq \pi$	$\Rightarrow$	'*External field*'
Vortices in Ψ_1	$\Rightarrow$	Kinks in tachyon field T
Energy conserving	$\Rightarrow$	Energy E_{2D} dissipative

Future interests

What happen for more complicated order parameter and/or in higher-dimensional space?
Applicability to other systems, spinor condensates? superfluid ^{3}He, …

Brane annihilation in superfluid ^{3}He

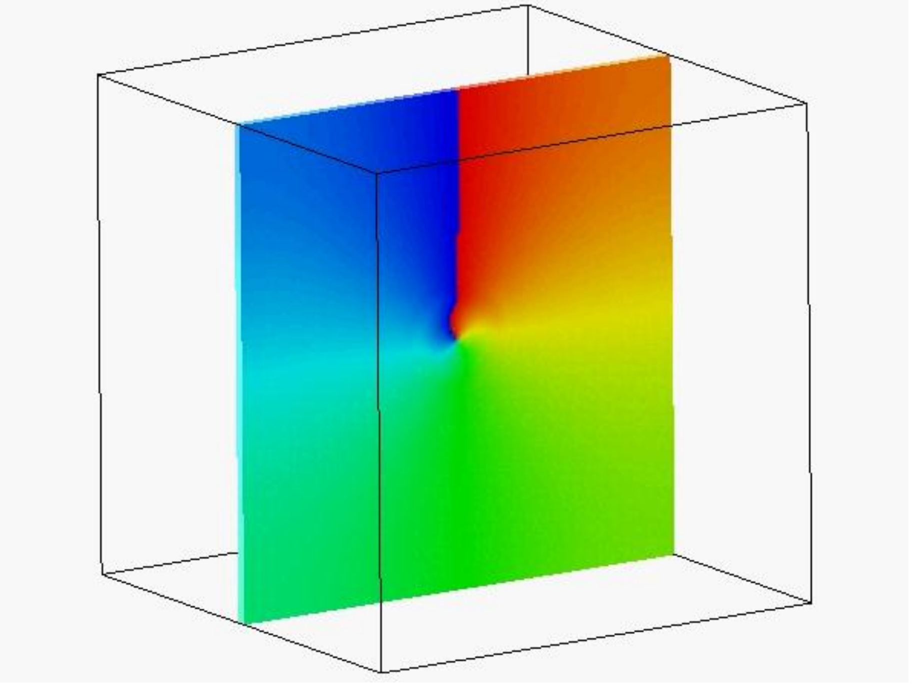
The analogue of the brane annihilation was simulated experimentally on the AB phase boundary of superfluid ^{3}He.



D. I. Bradley, S. N. Fisher, A. M. Guenault, R. P. Haley, J. Kopu, H. Martin, G. R. Pickett, J. E. Roberts and V. Tsepelin, Nat. Phys. **4**, 46-49 (2008).

But, its physical mechanism and the aspect of the unconventional SSB have never been revealed,
because of the complex order parameters in the ^{3}He system.

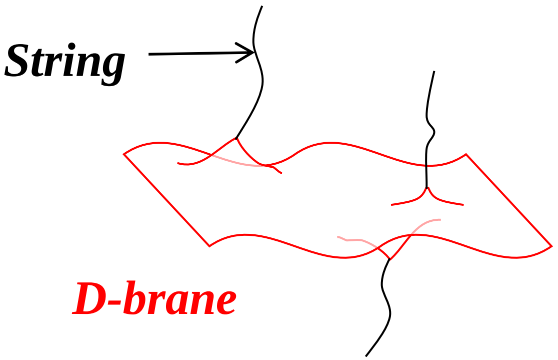
Vorton nucleation with a bridged vortex between branes



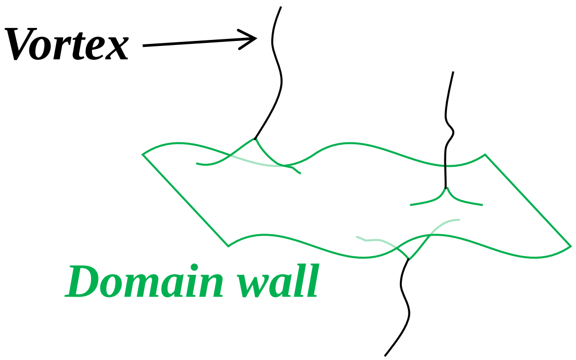
Creating vortons and three-dimensional skyrmions from domain-wall annihilation with stretched vortices in Bose-Einstein condensates
Muneto Nitta, Kenichi Kasamatsu, Makoto Tsubota, and Hiromitsu Takeuchi
Physical Review A **85**, 053639 (2012)

Analogy between two systems

String theory



Binary BECs



Analog topological structure (NLσM)

J.P. Gauntlett, R. Portugues, D. Tong, and P. K. Townsend, *Phys. Rev.* D 63 (2001) 085002.

K. Kasamatsu, H. Takeuchi, M. Nitta, and M. Tsubota, JHEP **11**, 068 (2010).

Vortex nucleation rate

Estimation by using tension formula

$$l_{\text{kink}} = \frac{L_v}{A} \sim \frac{V(0) - V(T_b)}{\sigma}$$

$$\sigma = 4\xi \int_0^{T_b} dT \sqrt{G(T)\left[V(T) - V(T_b)\right]}$$

Estimation from linear stability analysis

$$l_{\text{kink}} \sim \frac{q_{\text{max}}}{\pi}$$

$$q_{\max}\xi \sim \sqrt{-\frac{m_T^2}{2G(0)}},$$