Deconfinement phase transition and the quark condensate

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13. Mai 2009

C.F., arXiv:0904.2700 [hep-ph]

C.F. and R. Williams, Phys. Rev. D **78** (2008) 074006 [arXiv:0808.3372 [hep-ph]]

C.F. and R. Williams, in preparation

Content

1. Introduction
2. Infrared properties of SU(N) Yang-Mills theory
3. QCD phase transition
4. Dynamical chiral symmetry breaking: Quarks and gluons

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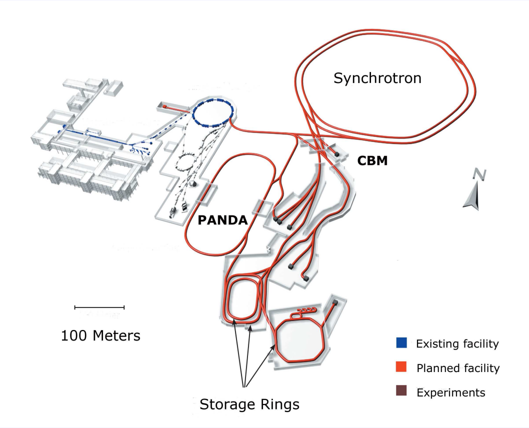
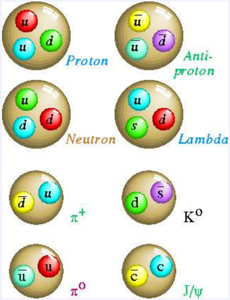
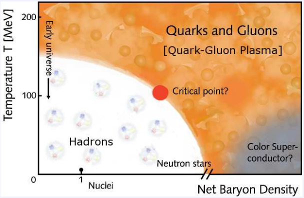
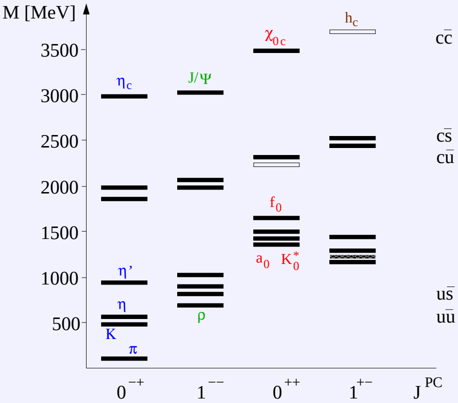
FAIR: CBM and PANDA





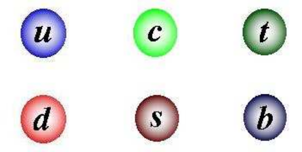


Experiment:



Goldstone-Bosons ↔ **Bound pseudoscalar Quark-Antiquark-states:** $\pi^{\pm,0}, K^{\pm,0,\bar{0}}, \eta$

Dynamical mass generation

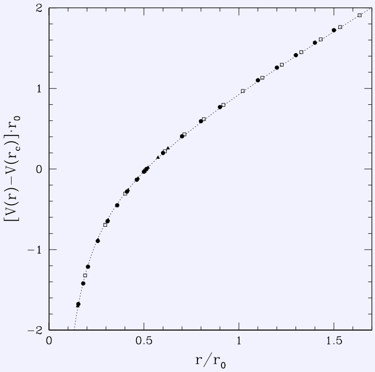


Quark mass generated by weak and strong interaction (QCD: explicit vs. dynamical breaking of chiral symmetry)

	u	d	s	c	b	t
M_{weak} [MeV]	3	5	100	1300	4000	175000
M_{strong} [MeV]	400	400	400	400	400	400
M_{tot} [MeV]	400	400	500	1700	4400	175000

M_{strong}: **Nonperturbative effect!**

Confinement

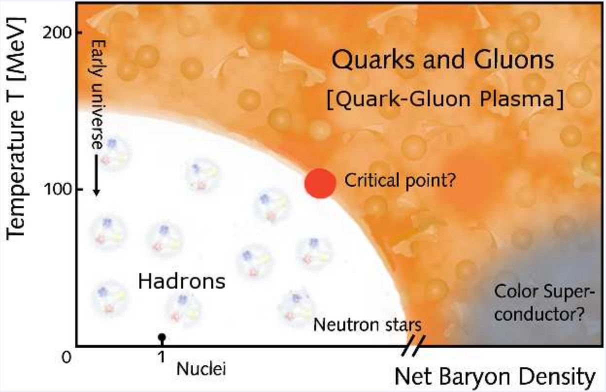


S. Necco and R. Sommer, Nucl. Phys. B **622** (2002) 328

- Linear rising potential: $V(r) \sim r$
- Quark-Antiquark system cannot be split! 'Quark-Confinement'
- in addition: Gluon-Confinement

What are the driving mechanisms?

QCD phase transitions



Chiral limit ($M_{weak} = 0$):

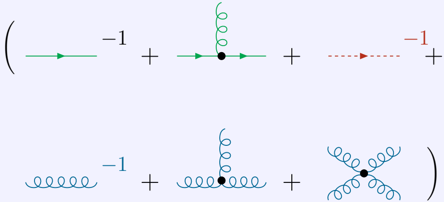
- Chiral limit ($M_{weak} = 0$): order parameter chiral condensate
- Heavy quarks ($M_{weak} = \infty$): order parameter Polyakov-loop

QCD in covariant gauge

quarks, gluons and ghosts:

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A, c] \; \exp\left\{ - \int d^4x \left(\overline{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \frac{(\partial A)^2}{2\xi} + \bar{c}(-\partial D)c \right) \right\}$$

$S_{QCD} = \int d^4x$



QCD in covariant gauge

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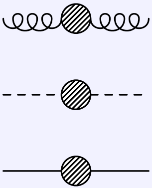
$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A, c] \; \exp\left\{ -\int d^4x \left(\bar{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \frac{(\partial A)^2}{2\xi} + \bar{c}(-\partial D)c \right) \right\}$$

Propagators in momentum space:



$$D^{\text{Gluon}}_{\mu\nu}(p) = \frac{\mathbf{Z}(p^2)}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

$$D^{\text{Geist}}(p) = -\frac{\mathbf{G}(p^2)}{p^2}$$

$$S^{\text{Quark}}(p) = \frac{Z_f(p^2)}{-i\not{p} + M(p^2)}$$

Green's functions

QCD Green's functions

- are connected to confinement:
  - Gribov-Zwanziger and Kugo-Ojima scenarios
  - Positivity
  - Running Coupling
  - Polyakov Loop
- encode DχSB
- are ingredients for hadron phenomenology
  - Bound state equations:
    Bethe–Salpeter equation / Faddeev equation

The Goal:

Gauge invariant information from gauge fixed functional approach

The Tool:

Dyson-Schwinger and Bethe-Salpeter-equations (DSE/BSE)

Lattice vs. DSE/FRG/BSE: Complementary!

- Lattice simulations
  - Ab initio
  - Gauge invariant

- Functional approaches:
  Dyson-Schwinger equations (DSE)
  Functional renormalisation group (FRG)
  Bethe-Salpeter-equations (BSE)
  - Analytic solutions at small momenta
  - Chiral symmetry: light quarks and mesons
  - Space-Time-Continuum
  - Chemical potential: no sign problem

Content

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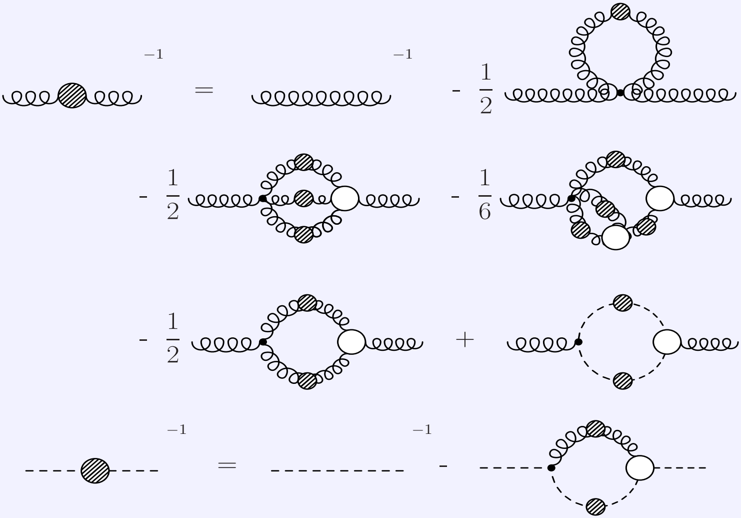
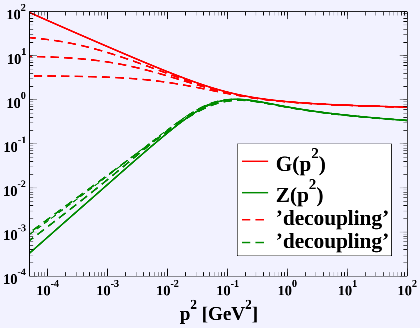
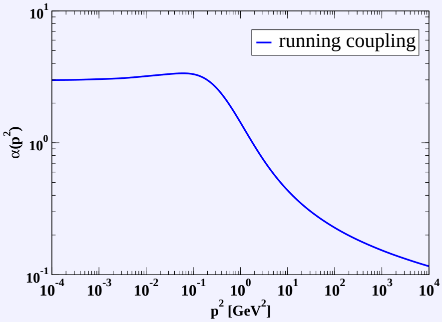
Dyson-Schwinger equations (DSEs)







- dynamically generated scale
- fixed point of coupling $\alpha(p^2) = g^2/(4\pi) Z(p^2) G(p^2) \approx 9/N_c$
- deep infrared ($p < 50$ MeV): scaling vs. decoupling

CF and Alkofer, PLB 536 (2002) 177.

C. Lerche and L. von Smekal, PRD **65**, 125006 (2002)

C.F., A. Maas and J. M. Pawlowski, arXive 0810.1987 [hep-ph]

Infrared Structure of YM-theory: $p^2 << \Lambda_{QCD}$

Two type of analytic solutions for complete tower of DSEs:

Scaling:

n ghost, *m* gluon legs

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n/2-m)\kappa}$$

- $G(0) = \infty$
- $\kappa > 0$
- Kugo Ojima confinement szenario supported!

Decoupling:

n ghost, *m* gluon legs

$$\Gamma^{0,2}(p^2) \sim (p^2)$$

all others finite

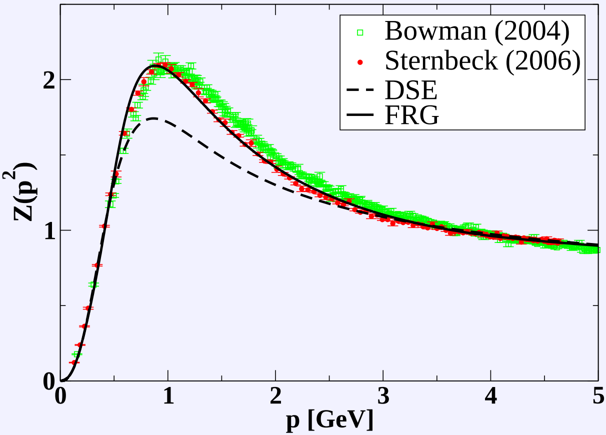
- $G(0) = \textit{finite}$
- gluon ’mass’ not determined
- BRST symmetry violated

R. Alkofer, C. F., F. Llanes-Estrada, Phys. Lett. B **611** (2005)

C.F. and J. M. Pawlowski, Phys. Rev. D **75** (2007) 025012.

C.F. and J. M. Pawlowski, arXiv:0903.2193 [hep-th]

DSEs vs Lattice



- $p^2 \approx 1GeV$: Systematic improvement possible for **DSEs**
- Deep infrared: Interesting and subtle questions

C.F., A. Maas and J. M. Pawlowski, arXive:0810.1987 [hep-ph]

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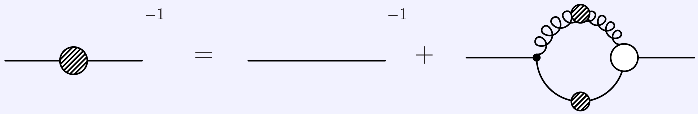
Finite Temperature: framework



- Quark propagator:

$$S(p_{\vec{p},\omega_p}) = [i\,\vec{\gamma}\cdot\vec{p}\;A(\vec{p},\omega_p) + i\,\gamma_4\omega_n\;C(\vec{p},\omega_p) + B(\vec{p},\omega_p)]^{-1}$$

- consider DSE on torus with $V = 1/T \times L^3$
- spatial directions: periodic boundary conditions
- temporal direction: antiperiodic boundary condition
- 'Order parameter' for chiral transition:

$$\langle\bar{\psi}\psi\rangle = Z_2 N_c \frac{T}{L^3} Tr_D \sum_{\vec{p},\omega_p} S(p_{\vec{p},\omega_p})$$

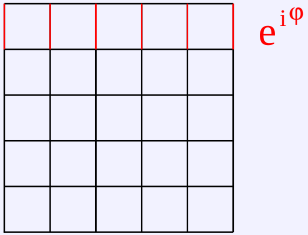
The dual condensate I

Consider general $U(1)$-valued boundary conditions in temporal direction for quark fields ψ:

$$\psi(\vec{x}, 1/T) = e^{i\varphi}\psi(\vec{x}, 0)$$

Matsubara frequencies: $\omega_p(n_t) = (2\pi T)(n_t + \varphi/2\pi)$

Lattice:

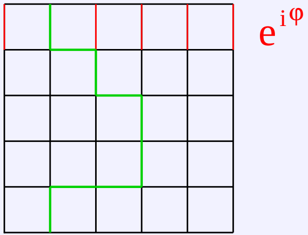


E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.

Relation of condensate to loops of link variables $U_\mu(x)$:

$$\langle\overline{\psi}\psi\rangle_\varphi = Tr\,[m + D_\varphi]^{-1} = \frac{1}{Vm}\sum_{l\in\mathcal{L}} \frac{s(l)e^{i\varphi q(l)}}{(2am)^{|l|}} Tr_c \prod_{(x,\mu)\in l} U_\mu(x)\,.$$

- geometric series of inverse staggered Dirac operator
- winding number $q(l)$ of loop l around temporal direction



E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.

The dual condensate III

Then define dual condensate Σ_n:

$$\Sigma_n \quad = \quad - \int_0^{2\pi} \frac{d\varphi}{2\pi} \, e^{-i\varphi n} \, \langle \overline{\psi}\psi \rangle_\varphi$$

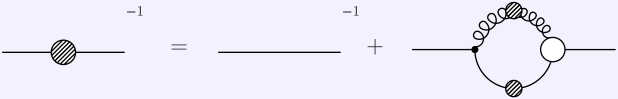
- $n = 1$ projects out loops with $q(l) = 1$: dressed Polyakov loop
- transforms under center transformation exactly like ordinary Polyakov loop
- Σ_1 is order parameter for center symmetry/deconfinement

  E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.

- Σ_1 is accessible with functional methods

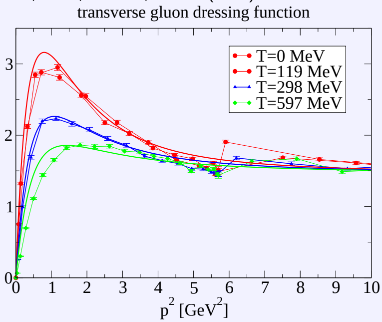
  C.F., arXiv:0904.2700 [hep-ph]

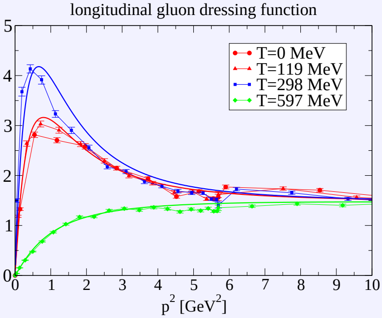
Input into quark-DSE



- *T*-dependent gluon propagator from lattice data

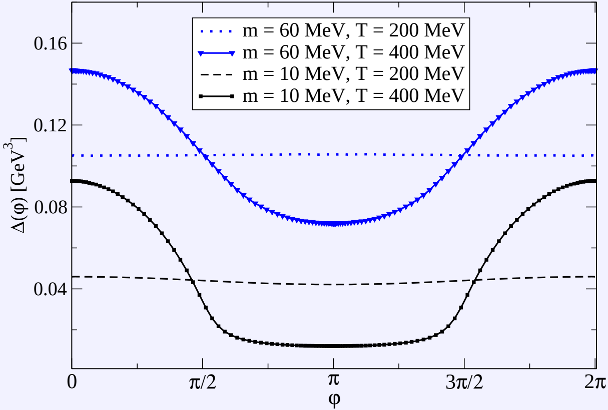
Cucchieri, Maas, Mendes, PRD75 (2007)





- *T*-dependent ansatz for quark-gluon vertex

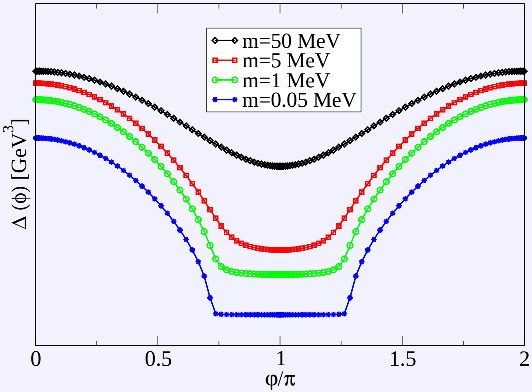
Results: angular dependence of condensate



$$\Delta(\varphi) \equiv \langle \overline{\psi}\psi \rangle_\varphi = Tr\,[m + D_\varphi]^{-1} = \frac{1}{Vm} \sum_{l \in \mathcal{L}} \frac{s(l) e^{i\varphi q(l)}}{(2am)^{|l|}} Tr_c \prod_{(x,\mu)\in l} U_\mu(x)\,.$$

- Smaller mass: more contributions from loop with larger $q(l)$!

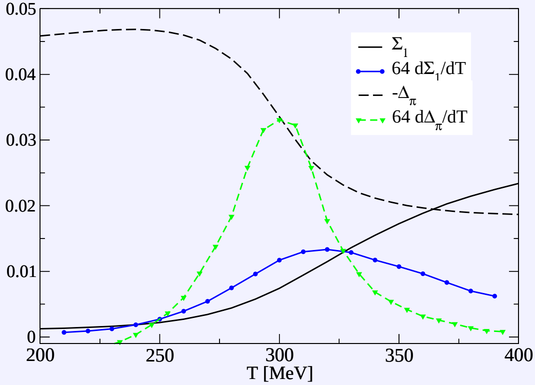
Results: angular dependence in chiral limit



- Chiral limit: need DSE in continuum
- Chiral limit: all terms of expansion contribute
- Width of plateau is T-dependent!

C.F. and Jens Müller, work in progress

Results: dressed Polyakov loop Σ_1



- Deconfinement transition from functional methods
- Insensitive to scaling vs decoupling

see also J. Braun, H. Gies and J. M. Pawlowski, arXiv:0708.2413 [hep-th].

C.F., arXiv:0904.2700 [hep-ph]

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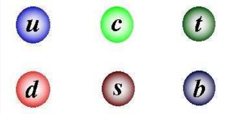
Masses of quarks in QCD

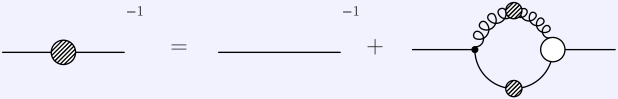


- **External** quark-masses:

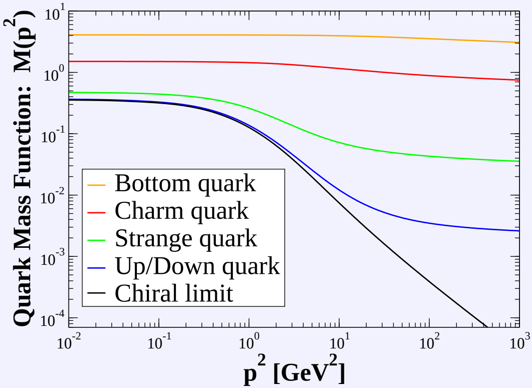
$$M_{weak} = M_{u,d,s,c,b,t} \neq 0$$

- **Dynamical** quark-masses:

$M_{strong}(p^2) \neq 0$ Non-perturbative effect!



Masses of quarks: numerical solutions of quark-DSE

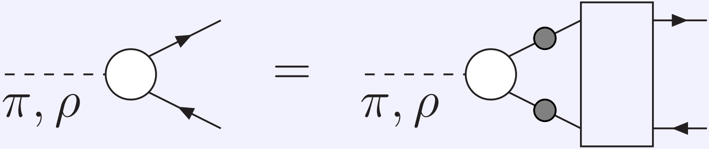


- $M(p^2)$: momentum dependent!
- Dynamical masses $M_{strong}(0) \approx 350 MeV$
- Flavour dependence because of M_{weak}

C. F., J. Phys. G **32** (2006) R253

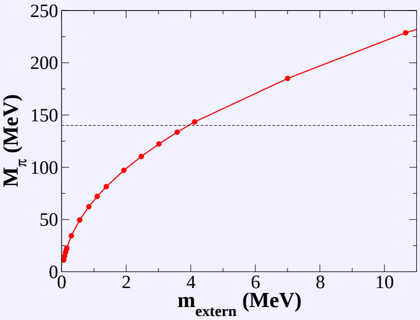
P. Maris and C. D. Roberts, Int. J. Mod. Phys. E **12** (2003) 297

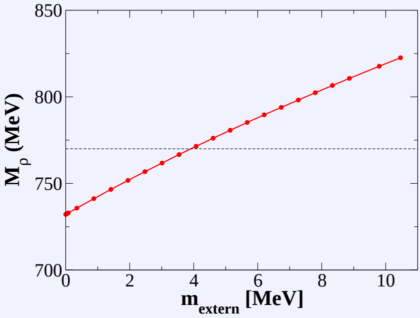
Dynamical quark masses and mesons



- Eigenvalue equation for M_π, $M_\rho, \ldots$
- π: pseudoskalar meson $\longleftrightarrow$ ρ: vector meson
  (built from up- and down-quarks)

Pions and Rho-Mesons

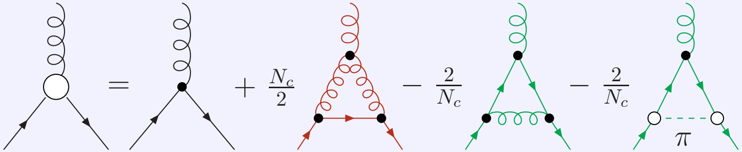




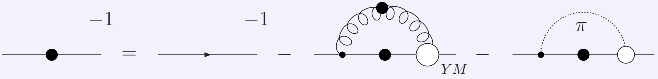
- $m_{extern} \to 0$: Pion is masseless Goldstone-boson with two massive constituents
- Simple model: 'rainbow-ladder approximation'
  only vector coupling between quark and gluon included

P. Maris and C. D. Roberts, Phys. Rev. C **56** (1997) 3369

Quark-gluon vertex



- Pion cloud effects

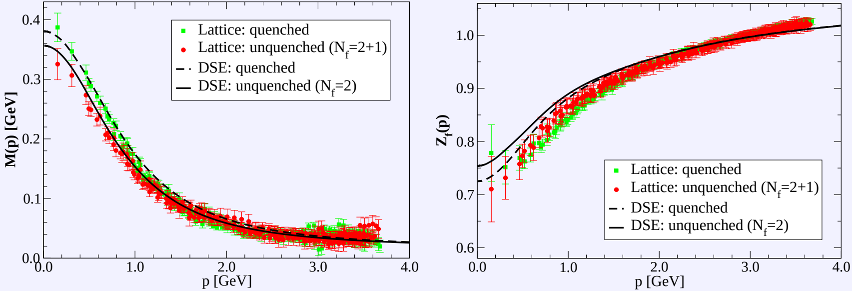


  - Large pion cloud effects in quark propagator
  - Large pion cloud effects in light meson spectrum

C.F., D. Nickel and R. Williams, EPJC **60**, 1434 (2008)

C.F. and R. Williams, PRD **78**, 074006 (2008).

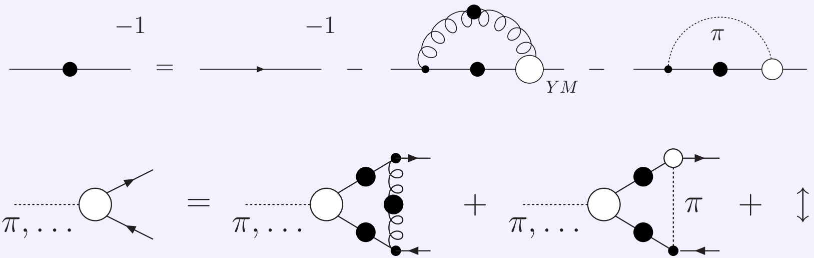
Pion cloud effects in the quark propagator



CF, D. Nickel and R. Williams, EPJC **60**, 1434 (2008)

- Unquenching effects of similar size as lattice

  P. O. Bowman, et al. Phys. Rev. D **71** (2005) 054507



- Axial Ward-Takahashi identity satisfied

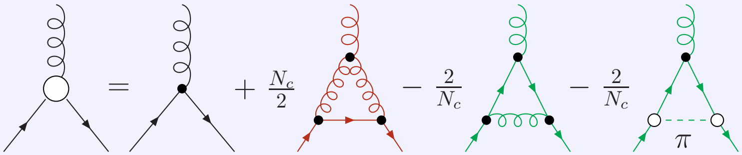
Pion cloud effects in light mesons

	wo π	with π	Experiment
M_π	125	138	138
f_π	102	94	93
M_ρ	795	703	776
f_ρ	138	159	162
M_σ	638	485	400 – 1200
M_{a_1}	941	873	1230
M_{b_1}	879	806	1230

- Attractive Effects of order of 100 MeV, similar to (extrapolated) lattice results
- Yang-Mills part of quark-gluon vertex too simple!

CF and R. Williams, PRD **78**, 074006 (2008).

Quark-gluon vertex

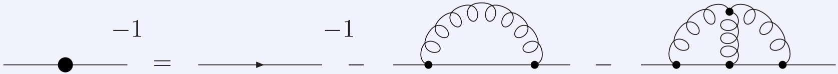


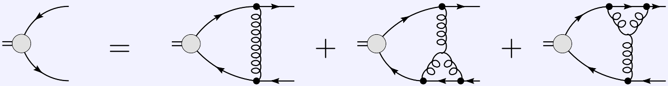
- Gluon self-interaction: Leading contribution!

R. Alkofer, C.F., F. Llanes-Estrada, Kai Schwenzer, Annals Phys. **324**, 106 (2009).

C.F, R. Williams, in preparation

Bethe-Salpeter equation





- Axial Ward-Takahashi identity satisfied
- about 100 CPU hours needed

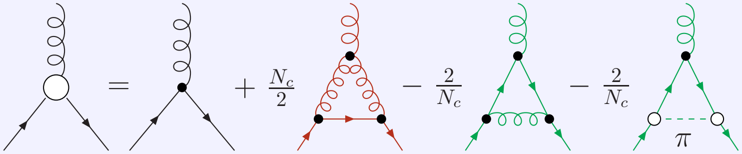
Gluon self-interaction effects in light mesons

	RL	BTR	Experiment
M_π	138	138	138
f_π	94	111	93
M_ρ	758	881	776
f_ρ	154	176	162
M_σ	645	884	400 – 1200
M_{a_1}	926	1055	1230
M_{b_1}	912	972	1230

- Repulsive effects of order of 100 MeV
- Beyond rainbow-ladder: attractive and repulsive effects cancel nonperturbatively for vector mesons

CF and R. Williams, in preparation

Quark-gluon vertex



- Gluon self-interaction: Infrared leading
  - infrared slavery (quark-gluon coupling)
  - generates linear rising quark-antiquark potential
  - generation of topological mass of η' ($U_A(1)$-problem)

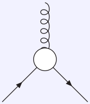
R. Alkofer, C.F., F. Llanes-Estrada, Kai Schwenzer, Annals Phys. **324**, 106 (2009).

R. Alkofer, C.F, R. Williams, Eur. Phys. J. A **38**, 53 (2008).

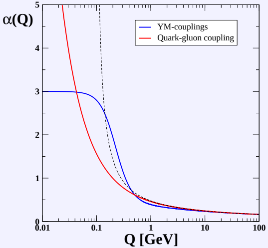
Infrared slavery (quenched QCD)

Quark-gluon vertex: $\Gamma^{quark-gluon}(p^2) \sim \begin{cases} (p^2)^{-1/2-\kappa} : \chi SB \\ (p^2)^{-\kappa} : \chi S \end{cases}$

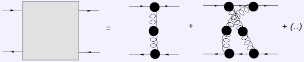
- Quark-gluon coupling:



$$\alpha^{qg}(p^2) \sim \begin{cases} \frac{1}{p^2} : \chi SB \\ const : \chi S \end{cases}$$

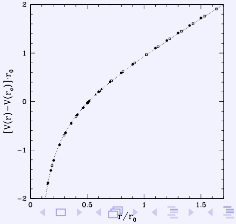


- 'Quark-Antiquark potential'



$\sim \frac{1}{p^4}$

$$\rightarrow V(\mathbf{r}) \sim \begin{cases} |r| : D\chi SB \\ \frac{1}{|r|} : \chi S \end{cases}$$



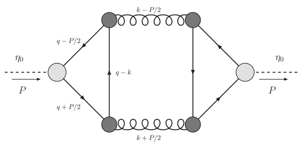
The $U_A(1)$-problem

η' too heavy for a Goldstone boson $\longrightarrow m^{\chi}_{\eta'} \neq 0$ in chiral limit

- Conjecture: Coloured infrared singularities generate $m^{\chi}_{\eta'} \neq 0$

  J. B. Kogut and L. Susskind, Phys. Rev. D **10** (1974) 3468.

- $\Gamma^{quark-gluon}(p^2) \sim (p^2)^{-1/2-\kappa}$ provides correct IR-strength



- our results:

$m^{\chi}_{\eta'}$ [MeV]	$Top.susz.$ [MeV^4]	θ	m_η [MeV]	$m_{\eta'}$ [MeV]
748	169	-23.2	479	906

R. Alkofer, C.F. and R. Williams, Eur. Phys. J. A **38**, 53 (2008)

Summary

Infrared QCD:

- Scaling solution for all correlation functions
- Fixed point and infrared slavery in couplings

Finite temperature:

- Dual condensate $\rightarrow$ order parameter for center symmetry
- Calculable with functional methods!

Light mesons:

- Bethe-Salpeter kernel beyond rainbow-ladder
- Large unquenching ('pion cloud') effects
- Large effects from gluon-selfinteraction

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TECHNISCHE UNIVERSITÄT DARMSTADT

HELMHOLTZ GEMEINSCHAFT

GSI

HIC for FAIR

LOEWE – Landes-Offensive zur Entwicklung Wissenschaftlich-ökonomischer Exzellenz

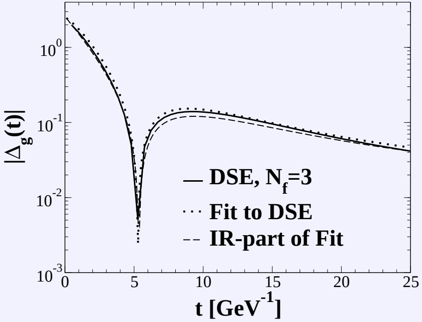
EMMI

Helmholtz-Alliance: Extremes of density and temperature; cosmic matter in the laboratory

Gluon confinement

- $$\int \frac{d^4x}{(2\pi)^4} \Delta_g(x^2) = \left(\frac{Z(p^2)}{p^2}\right)\Bigg|_{p^2=0} = \left(\frac{(p^2)^{2\kappa}}{p^2}\right)\Bigg|_{p^2=0} = 0 \quad \text{for } \kappa > 1/2$$

$\Delta_g(t) :=$
$$\int d^3x \int \frac{d^4p}{(2\pi)^4} e^{i(tp_4+\vec{p}\vec{x})} \; \frac{Z(p^2)}{p^2}$$



- Violation of positivity $\Rightarrow$ Signal for **confined gluons**

R. Alkofer, W. Detmold, C. F., P. Maris, Phys. Rev. D **70** (2004) 014014