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Flow equations and new weak-coupling solution for the spin-polaron in a quantum antiferromagnet

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Abstract. – The t-J model for the doped two-dimensional Heisenberg quantum antiferromagnet is studied in the generalized Dyson-Maleev representation, applying a new *continuous* unitary transformation which eliminates the coupling of spin and charge degrees of freedom. The analytical solutions of the resulting flow equations are derived in the weak-coupling regime where t/J is small. This continuous transformation yields a new weak-coupling result for the dispersion of the spin-polaron, if the elimination of both the non-diagonal spin-wave contributions and the terms coupling holes and spin-waves is performed *simultaneously.* The associated one-particle ground state is *lower* in energy than the corresponding perturbative result, which is reproduced upon application of *subsequent* transformations.

The theoretical description of the motion of a single hole injected in a two-dimensional Heisenberg quantum antiferromagnet has attracted much interest [1], since it is the doping of holes in an antiferromagnetic background which prompts the breakdown of magnetic order and the appearance of superconductivity in the layered copper-oxide superconducting materials. A variety of methods has been applied to study this problem. Besides more rigorous considerations [2-5], different approximative techniques have been utilized, including perturbational and expansion methods [6-8], numerical simulations on finite clusters [9-11], and variants of effective theories [12-14] in the intermediate coupling range. The numerical results for the single-hole spectral function exhibit a clear low-energy peak which is dominant against a broad continuum at higher energies. The spectral weight of the peak remains finite in the thermodynamic limit, while its energy is described by a dispersion relation $E(k)$. This indicates the existence of a coherently propagating quasiparticle, the spin-polaron, which should be viewed as a hole dressed by virtual magnetic excitations of the antiferromagnetically ordered spin background. Analytically, the polaronic quasiparticle picture is supported by particular representations which map the holes on spinless fermions, while the magnetic excitations are described by bosonic degrees of freedom. This results in effective models which bear resemblance with the Fröhlich Hamiltonian for polarons in solids. An experimentally important prediction of all these theoretical considerations is the detailed form of the quasiparticle dispersion $E(k)$, which has received considerable physical interest, since it may help explain some peculiar features of the cuprate superconductors.

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In the present work we employ the generalized Dyson-Maleev representation [15] for the t-J model of the doped quantum antiferromagnet and diagonalize the resulting linearized Hamiltonian in the weak-coupling limit. This is achieved by a *continuous* unitary transformation which leads to flow equations [16, 17] for the various terms of the Hamiltonian. The flow equation method has been successfully applied to a number of problems for which solutions had already been obtained previously by conventional single-step unitary transformations. The continuous variants of such well-known transformations have often led to novel results and have shed new light on unsolved technical difficulties. These revisited solutions include the Fröhlich transformation for electron-phonon interactions [18], the Schrieffer-Wolff transformation for the Anderson Hamiltonian [19], the various strong-coupling expansion schemes for the Hubbard model [20], and the Bogoliubov transformation for the pure Heisenberg antiferromagnet [21]. Therefore, in view of the spin-polaron description of the doped quantum antiferromagnet, the question arises whether such a continuous transformation yields a quasiparticle dispersion for the hole excitation which is at variance with the result obtained by perturbation theory and other weak-coupling approaches utilizing unitary transformations. As is shown in this letter, this is indeed the case. The one-particle ground state corresponding to this new solution is found to have an energy which is a few percent lower than the perturbative value, due to a modified flow trajectory of the non-diagonal couplings.

We consider a doped spin-(1/2) quantum Heisenberg antiferromagnet on the square lattice,

$$H = t \sum_{i,j,\sigma} C_{i\sigma}^{\dagger} C_{j\sigma} + J \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j, \tag{1}$$

where $\langle . \rangle$ denotes summation over distinct lattice bonds. The $C_{i\sigma}^{\dagger} = c_{i\sigma}^{\dagger}(1-n_{i-\sigma})$ are projected fermionic operators which enforce the constraint of no double occupancy, since $C_{i\sigma}^{\dagger} C_{i-\sigma}^{\dagger} = 0$. Note that $J = 4t^2/U$, if the t-J model is viewed as the strong-coupling limit of the Hubbard model near half band filling. To separate spin and charge of the excitations related to a hole injected in the classical Néel state, the generalized Dyson-Maleev representation [15] is employed, which properly accounts for all dynamical constraints on the hole motion. In the harmonic approximation, which is usually applied for actual calculations, it is sufficient to retain a linearized version of this representation. If one considers two sublattices A and B, one therefore has $S_i^z = \frac{1}{2} - b_i^{\dagger} b_i$, $S_i^+ = b_i$, and $S_i^- = b_i^{\dagger}$ for $i \in A$, while $S_j^z = -\frac{1}{2} + b_j^{\dagger} b_j$, $S_j^+ = b_j^{\dagger}$, and $S_j^- = b_j$ for $j \in B$. The b are local bosonic operators related to the spin fluctuations around the classical vacuum. Since we are dealing with a spin-(1/2) system, in the physical subspace of the bosons the constraint $b^{\dagger} b \leq 1$ applies. The hole operators are then given by $C_{i\uparrow} = h_i^{\dagger}$ and $C_{i\downarrow} = h_i^{\dagger} b_i$, whereas $C_{j\uparrow} = h_j^{\dagger} b_j$ and $C_{j\downarrow} = h_j^{\dagger}$. Here the h are local fermionic operators associated with the charge degree of freedom of the hole. The Néel ground state $|0\rangle$ has the property $b_q|0\rangle = 0 = h_k|0\rangle$. Inserting this representation for spin and projected fermionic operators into the Hamiltonian, one obtains [6, 15]

$$H = J \sum_q \Big(2\, b_q^{\dagger} b_q + \gamma_q \left(b_q^{\dagger} b_{-q}^{\dagger} + b_q b_{-q} \right) \Big) + 4t \sum_{k,q} h_{k+q}^{\dagger} h_k \left(\gamma_k\, b_q + \gamma_{k+q}\, b_{-q}^{\dagger} \right), \tag{2}$$

where $\gamma_q = \frac{1}{2}(\cos q_x + \cos q_y)$ is the structure factor of the square lattice. This effective Hamiltonian (2) combines linear spin-wave theory for the magnetic excitations with the description of the coupling of spin and charge degrees of freedom in leading order. It is reminiscent of the Fröhlich Hamiltonian for the electron-phonon interaction in solids [22]. However, the approach followed in the present paper is significantly different from the treatment of the electron-phonon problem by flow equations [18], since there are several fundamental aspects which do not coincide in the two models: i) Our initial effective Hamiltonian does not contain a contribution representing free fermionic excitations. Such a term is only generated in the course of the continuous transformation. Since the system is close to half-filling, corresponding to one electron per site, the Fermi energy of the holes vanishes, $\varepsilon_{\mathrm{F}} = 0$. ii) Due to the non-diagonal bilinear bosonic terms in our Hamiltonian, the diagonalizing transformation must combine a Bogoliubov-like particle-hole rotation in the bosonic degrees of freedom with a Fröhlich-like elimination of the spin-charge coupling. iii) As a consequence of the extreme quantum limit of the involved spins, the number of bosons is confined to $b^\dagger b \leq 1$. This introduces a constraint of the physical Hilbert space which is not present in the electron-phonon problem.

To remove those terms from the Hamiltonian (2) which do not conserve the number of bosons, we apply a continuous unitary transformation [16] depending on a flow parameter $\ell \geq 0$. The transformation flow of the new effective Hamiltonian, $H(\ell) = U(\ell) H U^\dagger(\ell)$, is then described by the differential equation

$$\frac{\mathrm{d}}{\mathrm{d}\ell} H(\ell) = [\eta(\ell), H(\ell)] \qquad \text{with} \qquad \eta(\ell) = \frac{\mathrm{d}U(\ell)}{\mathrm{d}\ell} U^\dagger(\ell), \tag{3}$$

where the anti-Hermitian generator $\eta(\ell)$ has to be chosen appropriately. Due to this transformation, new terms are generated in the Hamiltonian. Therefore, one finds

$$\begin{aligned} H(\ell) \;=\; & 2J \sum_q \omega_q(\ell)\, b_q^\dagger b_q + J \sum_q \alpha_q(\ell)\, \big(b_q^\dagger b_{-q}^\dagger + b_q b_{-q}\big) + 4t \sum_k \varepsilon_k(\ell)\, h_k^\dagger h_k + \\ & + 4t \sum_{q,k} h_{k+q}^\dagger h_k \,\big(g_{k,q}(\ell)\, b_q + g_{k+q,-q}(\ell)\, b_{-q}^\dagger\big) + 2t \sum_{q,k,k'} V_{q,k,k'}(\ell)\, h_{k+q}^\dagger h_{k'-q}^\dagger h_{k'} h_k + \\ & + 4t \sum_{q,k} f_{k,q}(\ell)\, h_k^\dagger h_k\, b_q^\dagger b_q + 4t \sum_{q,k} \zeta_{k,q}(\ell)\, h_k^\dagger h_k \,\big(b_q^\dagger b_{-q}^\dagger + b_q b_{-q}\big). \end{aligned} \tag{4}$$

The ratio of the couplings, $\lambda = 2t/J$, describes the crossover from the weak-coupling ($\lambda \ll 1$) to the strong-coupling ($\lambda \gg 1$) regime. For the Hubbard antiferromagnet, $\lambda = U/2t$. The effective Hamiltonian (4) contains all terms which contribute to the quasiparticle energy $\varepsilon_k(\ell)$ at order $\mathcal{O}(\lambda)$. Corresponding to the original Hamiltonian, one has the initial conditions $\omega_q(0) = 1$, $\alpha_q(0) = \gamma_q$ for the pure spin-wave part, $\varepsilon_k(0) = 0$, $V_{q,k,k'}(0) = 0$ for the hole terms, and $g_{k,q}(0) = \gamma_k$, $f_{k,q}(0) = \zeta_{k,q}(0) = 0$ for the contributions which couple spin and hole excitations. The appearance of the terms related to $\zeta_{k,q}(\ell)$ describing scattering of spin and charge excitations is peculiar for a continuous transformation which involves *both* the spin-wave part of the Hamiltonian *and* the coupling of both degrees of freedom. If the two transformations are performed subsequently, one finds that these terms vanish. In the limit of weak coupling, the non-diagonal contributions to the effective Hamiltonian (4) may be eliminated by a transformation specified on an infinitesimal scale by $\eta(\ell) = \eta_0(\ell) + \lambda\, \eta_1(\ell)$, with

$$\begin{aligned} \eta_0(\ell) \;&=\; \frac{1}{4} \sum_q \alpha_q(\ell)\, \big(b_q^\dagger b_{-q}^\dagger - b_q b_{-q}\big), \\ \eta_1(\ell) \;&=\; \frac{1}{2} \sum_{q,k} \Big(h_{k+q}^\dagger h_k \,\big(g_{k+q,-q}(\ell)\, b_{-q}^\dagger - g_{k,q}(\ell)\, b_q\big) + \zeta_{k,q}(\ell)\, h_k^\dagger h_k \,\big(b_q^\dagger b_{-q}^\dagger - b_q b_{-q}\big)\Big). \end{aligned} \tag{5}$$

This choice for the generator which contains only the non-diagonal entries of the Hamiltonian itself, albeit with modified sign, is ideally suited to preserve the banded structure of the original

Hamiltonian, since it does not result in new types of interactions which lead to transitions between initially unconnected parts of the Hilbert space [21,23]. The pure spin-wave part of the Hamiltonian is then described by the differential equations

$$\frac{\mathrm{d}}{\mathrm{d}\ell}\,\alpha_q(\ell) = -\alpha_q(\ell)\,\omega_q(\ell) \qquad \text{and} \qquad \frac{\mathrm{d}}{\mathrm{d}\ell}\,\omega_q(\ell) = -\alpha_q^2(\ell). \tag{6}$$

In the leading order of λ, the flow of the non-diagonal terms which couple spin and charge of a single hole is given by

$$\frac{\mathrm{d}}{\mathrm{d}\ell}\,g_{k,q}(\ell) = -\frac{1}{2}\omega_q(\ell)\,g_{k,q}(\ell) - \alpha_q(\ell)\,g_{k+q,-q}(\ell) + \mathcal{O}(\lambda^2), \tag{7}$$

whereas the quasiparticle dispersion of the spin-polaron due to the transformation is encapsulated in the flow equation

$$\frac{\mathrm{d}}{\mathrm{d}\ell}\,\varepsilon_k(\ell) = -4\sum_q \alpha_q(\ell)\,\zeta_{k,q}(\ell) - \lambda\sum_q \big(g_{k+q,-q}^2(\ell) + 4\,\zeta_{k,q}^2(\ell)\big). \tag{8}$$

Additionally, the transformation generates a two-particle interaction $V_{q,k,k'}$ between the holes, which for the single-particle states can be disregarded. The interactions between spin-waves and charge carriers are found to obey

$$\frac{\mathrm{d}}{\mathrm{d}\ell}\,f_{k,q}(\ell) = -4\,\alpha_q(\ell)\,\zeta_{k,q}(\ell) - \lambda\big(g_{k+q,-q}^2(\ell) - g_{k,q}^2(\ell) + 4\,\zeta_{k,q}^2(\ell)\big), \tag{9}$$

$$\frac{\mathrm{d}}{\mathrm{d}\ell}\,\zeta_{k,q}(\ell) = -\omega_q(\ell)\,\zeta_{k,q}(\ell) - \frac{1}{2}\alpha_q(\ell)\,f_{k,q}(\ell) - \lambda\,f_{k,q}(\ell)\,\zeta_{k,q}(\ell). \tag{10}$$

Note that in the derivation of these flow equations using the prescription (3) all newly generated terms are written in normal ordered form and higher-order contributions other than those given in (4) are neglected. This leads to the closure of the flow equations at order $\mathcal{O}(\lambda)$.

From (6) one can immediately deduce the hyperbolic invariant $\omega_q^2(\ell) - \alpha_q^2(\ell) = \omega_q^2$, where $\omega_q = (1-\gamma_q^2)^{1/2}$ is the spin-wave energy, expressed in units of $2J$. (Throughout the paper we use the convention that for any ℓ-dependent function $f(\ell)$ its asymptotic value in the large-ℓ limit is denoted by $f \equiv f(\infty)$.) This suggests the solutions

$$\omega_q(\ell) = \omega_q\,\coth\big(\ell\omega_q + \varphi_q\big) \qquad \text{and} \qquad \alpha_q(\ell) = \frac{\omega_q\,\mathrm{sgn}\gamma_q}{\sinh\big(\ell\omega_q + \varphi_q\big)}, \tag{11}$$

where the relation $\tanh\varphi_q = \omega_q$ reflects the initial conditions. Obviously, the functions $\omega_q(\ell)$ and $\alpha_q(\ell)$ converge exponentially fast to their asymptotic values in the $\ell \to \infty$ limit, $\omega_q(\infty) = \omega_q$ and $\alpha_q(\infty) = 0$. In the degenerate case, the solutions (11) yield a weaker algebraic decay of the couplings, $\omega_q(\ell) = |\alpha_q(\ell)| = (\ell+1)^{-1}$ for $\omega_q = 0$. In contrast to the conventional single-step Bogoliubov transformation for the spin-wave part of the Hamiltonian, also these degenerate terms are eventually removed by the continuous transformation, even if this happens only algebraically fast. Such a slow but feasible elimination of degenerate contributions is typical for the continuous extension of the Bogoliubov transformation of a spin-wave Hamiltonian [21].

The differential equation (7) for the non-diagonal couplings can be solved in a closed analytical form, if we follow a perturbative treatment around the weak-coupling limit. For

$\lambda \ll 1$, in (7) we may neglect the terms of order $\mathcal{O}(\lambda^2)$, which involve the $\varepsilon_k(\ell)$. Up to corrections of this order one thus obtains

$$g_{k,q}(\ell) = \frac{1}{\omega_q^2}\sqrt{\frac{\alpha_q(\ell)}{\gamma_q}}\Big(\gamma_k\big(\omega_q(\ell) - \gamma_q\,\alpha_q(\ell)\big) - \gamma_{k+q}\big(\gamma_q\,\omega_q(\ell) - \alpha_q(\ell)\big)\Big). \tag{12}$$

Asymptotically, for sufficiently large ℓ one again recovers an exponential decay from this result, $g_{k,q}(\ell) \sim \exp[-\frac{1}{2}\ell\omega_q]$ for $\ell\omega_q \gg 1$, whereas degeneracy of the involved spin-wave energy with the vacuum again leads to purely algebraic behaviour, $g_{k,q}(\ell) = \gamma_k\,(\ell+1)^{-3/2}$ for $\omega_q = 0$. Generally, the generator (5) does not ensure the elimination of all the $g_{k,q}(\ell)$ in the large-ℓ limit, if the coupling parameter λ is not sufficiently small. For $\lambda \gtrsim 1$, the flow equation (7) including the $\mathcal{O}(\lambda^2)$ contributions leads to an exponentially diverging $g_{k,q}(\ell)$ in this limit instead, if ω_q becomes small. To model the crossover to strong coupling, one therefore has to employ the canonical choice [16] for the generator, which finally removes all the non-diagonal terms of the Hamiltonian for *general* coupling strength λ. In contrast to the transformation (5), however, this generator no longer preserves the banded structure of the Hamiltonian.

With (8) and (9), one may now integrate the flow equation for the quasiparticle dispersion in the weak-coupling limit,

$$\varepsilon_k(\infty) = \sum_q \Big(f_{k,q}(\infty) - \lambda\int_0^\infty \mathrm{d}\ell\, g_{k,q}^2(\ell)\Big). \tag{13}$$

Inserting here the result

$$f_{k,q}(\infty) = \frac{\lambda}{\omega_q}\int_0^\infty \mathrm{d}\ell\,\omega_q(\ell)\big(g_{k,q}^2(\ell) - g_{k+q,-q}^2(\ell)\big) = \frac{\lambda}{\omega_q}(\gamma_k^2 - \gamma_{k+q}^2) \tag{14}$$

and the explicit solution (12) for the $g_{k,q}(\ell)$, one obtains

$$\varepsilon_k(\infty) = \lambda\sum_q\Big(\frac{1}{4\gamma_q}(\gamma_{k+q}^2 - \gamma_k^2)\,\ln\frac{1+\gamma_q}{1-\gamma_q} - \frac{(\gamma_{k+q}^2 - \gamma_k^2)}{\omega_q} - \frac{(\gamma_{k+q}+\gamma_k)^2}{2(1+\gamma_q)}\Big). \tag{15}$$

From this expression, the anisotropic one-particle energy $E(k) = E(k_x, k_y)$ of the spin-polaron for $t \ll J$ is finally derived,

$$E(k_x,k_y) = 4t\,\varepsilon_k(\infty) = \frac{2t^2}{J}\Big(A\left(\cos^2 k_x + \cos^2 k_y\right) + B\,\cos k_x\,\cos k_y\, - C\Big). \tag{16}$$

For the square lattice, the numerical evaluation of the Brillouin zone integrals in (15) yields the constants $A \simeq 0.3439$, $B \simeq 0.4186$, and $C \simeq 1.344$. Since $2A > B$, four degenerate minima of $E(k_x, k_y)$ are located at $(\pm\pi/2, \pm\pi/2)$, which corresponds to the hopping of the hole on a single magnetic sublattice. Such a hole motion without remanent spin distortions is related to the one-particle ground state energy $E_0 \simeq -0.6579\,NJ - 2.688\,t^2/J$, where N is the number of lattice sites and the first contribution equals the result for the continuous version of linear spin-wave theory [21]. The quasiparticle band has a width $\Delta E \simeq 2.213\,t^2/J$. The band minima are at the bottom of quite shallow energy pockets of depth $\Delta E \simeq 0.538\,t^2/J$, which are immersed in an extended flat region around $(0, \pm\pi)$ and $(\pm\pi, 0)$. The quasiparticle weight Z_k, defined as the overlap of a bare hole excitation with the true one-particle ground state, $Z_k(\ell) = \left|\langle 0|h_k\,U^\dagger(\ell)\,h_k^\dagger\,|0\rangle\right|^2$, deviates from unity only by a small correction which is quadratic in the coupling strength, $Z_k(\infty) = 1 - \mathcal{O}(\lambda^2)$, and thus lends support to the quasiparticle picture of the spin-polaron.

Expression (15) for the one-particle dispersion of the spin-polaron is our central result. It is to be compared with the corresponding expression resulting from perturbation theory and the self-consistent Born approximation [6] and also from the weak-coupling limit of the various effective theories [12-14] devised for the effective Hamiltonian (2) of the t-J model. In the leading linear order of λ, these approaches yield

$$\varepsilon_k(\infty) = -\lambda \sum_q \frac{(u_q\,\gamma_{k+q} + v_q\,\gamma_k)^2}{\omega_q}, \tag{17}$$

where u_q and v_q are the conventional coefficients of the Bogoliubov transformation for the bosons,

$$u_q = \sqrt{\frac{1+\omega_q}{2\omega_q}} \qquad \text{and} \quad v_q = -\mathrm{sgn}\gamma_q \sqrt{\frac{1-\omega_q}{2\omega_q}}. \tag{18}$$

This result is in qualitative agreement with the explicit expression (16), with the minima of the energy also located at $(\pm\pi/2, \pm\pi/2)$. However, it implies a single-hole ground state energy $E_0 \simeq -0.6579\,NJ - 2.617\,t^2/J$. Thus, the flow equation approach yields a spin-polaron excitation energy which is about 3% *lower* than the value obtained by second-order perturbation theory. Note that the expression (17) may also be reproduced by the flow equation approach, if the Bogoliubov transformation for the spin-waves and the elimination of the spin-charge coupling terms are performed *separately.* If we first choose $\eta(\ell) = \eta_0(\ell)$ and integrate up to $\ell \to \infty$, this diagonalizes the spin-wave part of the Hamiltonian (4). Applying *then* the generator $\tilde{\eta}(\ell) = \lambda\,\eta_1(\ell)$, in the flow equation (7) for $\tilde{g}_{k,q}(\ell)$ only the first term on the r.h.s. with constant $\tilde{\omega}_q(\ell) \equiv \omega_q$ is retained. In (9) and (10), the terms coupling $f_{k,q}(\ell)$ and $\zeta_{k,q}(\ell)$ by a non-zero $\alpha_q(\ell)$ vanish, so that $\tilde{\zeta}_{k,q}(\ell) \equiv 0$ and $\tilde{f}_{k,q}(\infty)$ is again given by (14). Together with the new initial conditions $\tilde{g}_{k,q}(0) = u_q\,\gamma_k + v_q\,\gamma_{k+q}$ which are modified due to the completed Bogoliubov transformation, and the flow equation (8) for $\varepsilon_k(\ell)$, this leads to the perturbative result (17) for the one-particle dispersion. More specifically, the difference of the hole energies obtained by simultaneous and by subsequent transformations is given by

$$\Delta\varepsilon_k = \lambda \sum_q \int_0^\infty \mathrm{d}\ell\,\big(\tilde{g}^2_{k,q}(\ell) - g^2_{k,q}(\ell)\big). \tag{19}$$

It is thus the *simultaneous* application of both $\eta_0(\ell)$ and $\eta_1(\ell)$ which results in a modified flow trajectory for the $g_{k,q}(\ell)$ and finally yields a lower ground state energy.

This statement can be made more explicit. Since $[\eta_0(\ell), \eta_0(\ell')] = 0$, the pure Bogoliubov transformation of the spin-waves is simply given by $U_0(\ell) = \exp\left[\int_0^\ell \mathrm{d}\ell'\,\eta_0(\ell')\right]$, so that the transformed ground state reads

$$U_0(\infty)\,|0\rangle = \exp\left[\frac{1}{8}\sum_q \ln\frac{1+\gamma_q}{1-\gamma_q}\,\big(b^\dagger_q b^\dagger_{-q} - b_q b_{-q}\big)\right]\,|0\rangle. \tag{20}$$

However, generally the complete generator $\eta(\ell) = \eta_0(\ell) + \lambda\,\eta_1(\ell)$ does not commute with itself at differing ℓ any more,

$$[\eta(\ell), \eta(\ell')] = \frac{\lambda}{4}\sum_{q,k}\Big(h^\dagger_{k+q} h_k\, b^\dagger_{-q}\,\big(\alpha_q(\ell)\,g_{k,q}(\ell') - \alpha_q(\ell')\,g_{k,q}(\ell)\big) - \mathrm{h.c.}\Big) + \mathcal{O}(\lambda^2). \tag{21}$$

Since in the leading order $\mathcal{O}(\lambda)$ this commutator reproduces the structure of $\eta_1(\ell)$, also all higher commutators which contribute to the full transformation involve terms of this form at the same order, *e.g.*, $[\eta(\ell),[\eta(\ell'),\eta(\ell'')]] = \mathcal{O}(\lambda)$. This prevents the explicit calculation of the completed transformation $U(\infty)$. However, including the terms due to $\eta_1(\ell)$ in the leading linear order of λ but *subsequent* to the continuous Bogoliubov transformation, $U(\ell) \simeq \exp\left[\lambda V(\ell)\right] U_0(\ell)$, one finds

$$V(\ell) = \frac{1}{2} \sum_{q,k} h^\dagger_{k+q} h_k \left(G_{k+q,-q}(\ell)\, b^\dagger_{-q} - G_{k,q}(\ell)\, b_q\right). \tag{22}$$

From the basic equation (3) the flow of $V(\ell)$ is derived, $\frac{\mathrm{d}}{\mathrm{d}\ell} V(\ell) = \eta_1(\ell) + [\eta_0(\ell), V(\ell)]$. With the initial condition $G_{k,q}(0) = 0$, the solution for $G_{k,q}(\ell)$ is then determined by

$$\frac{\mathrm{d}}{\mathrm{d}\ell} G_{k,q}(\ell) = g_{k,q}(\ell) + \frac{1}{2}\alpha_q(\ell)\, G_{k+q,-q}(\ell). \tag{23}$$

The final result is $G_{k,q}(\infty) = \frac{2}{\omega_q}(u_q\,\gamma_k + v_q\,\gamma_{k+q})$, which inserted in $U(\infty)$ again reproduces the perturbative expression (17).

In summary, we have presented a new continuous unitary transformation for the weak-coupling *t-J* model. This transformation yields a result for the one-particle dispersion of the spin-polaron which is at variance with perturbation theory and which implies a lower value for the one-particle ground state energy. The modified result is due to the simultaneous elimination of the non-diagonal terms in the pure spin-wave part of the Hamiltonian and of the terms which couple the spin and charge degrees of freedom.

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REFERENCES

[1] Dagotto E., *Rev. Mod. Phys.*, **66** (1994) 763; Brenig W., *Phys. Rep.*, **251** (1995) 153.
[2] Trugman S. A., *Phys. Rev. B*, **37** (1988) 1597.
[3] Shraiman B. I. and Siggia E. D., *Phys. Rev. Lett.*, **60** (1988) 740.
[4] Zhang S. and Arovas D. P., *Phys. Rev. B*, **40** (1989) 2708.
[5] Sheng D. N., Chen Y. C. and Weng Z. Y., *Phys. Rev. Lett.*, **77** (1996) 5102.
[6] Schmitt-Rink S., Varma C. M. and Ruckenstein A. E., *Phys. Rev. Lett.*, **60** (1988) 2793.
[7] Kane C. L., Lee P. A. and Read N., *Phys. Rev. B*, **39** (1989) 6880.
[8] Becker K. W., Eder R. and Won H., *Phys. Rev. B*, **45** (1992) 4864.
[9] Poilblanc D., Schulz H. J. and Ziman T., *Phys. Rev. B*, **47** (1993) 3268.
[10] Dagotto E., Nazarenko A. and Boninsegni M., *Phys. Rev. Lett.*, **73** (1994) 728.
[11] Liu Z. P. and Manousakis E., *Phys. Rev. B*, **51** (1995) 3156.
[12] Barentzen H., *Phys. Rev. B*, **53** (1996) 5598.
[13] Yang M. F., *Phys. Rev. B*, **55** (1997) 56.
[14] Li T. and Gan Z.-Z., *J. Phys. Condens. Matter*, **10** (1998) 8007.
[15] Barentzen H. and Wróbel P., *Z. Phys. B*, **93** (1994) 375.
[16] Wegner F., *Ann. Phys. (Leipzig)*, **3** (1994) 77.
[17] Głazek S. D. and Wilson K. G., *Phys. Rev. D*, **49** (1994) 4214.
[18] Lenz P. and Wegner F., *Nucl. Phys. B*, **482** (1996) 693; Mielke A., *Ann. Phys. (Leipzig)*, **6** (1997) 215; Ragwitz M. and Wegner F., *Eur. Phys. J. B*, **8** (1999) 9.
[19] Kehrein S. K. and Mielke A., *Ann. Phys. (N.Y.)*, **252** (1996) 1.
[20] Stein J., *J. Stat. Phys.*, **88** (1997) 487.
[21] Stein J., *Eur. Phys. J. B*, **5** (1998) 193.
[22] Pines D., in *Polarons and Excitons*, edited by C. G. Kuper and G. D. Whitfield (Oliver and Boyd, Edinburgh) 1962.
[23] Mielke A., *Eur. Phys. J. B*, **5** (1998) 605.