EUROPHYSICS LETTERS 20 June 1996

Europhys. Lett., **34** (9), pp. 717-721 (1996)

Critical exponents of the $U(n)$ vector spin glasses

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(received 20 March 1996; accepted 3 May 1996)

PACS. 75.10Nr – Spin-glass and other random models.
PACS. 75.40Cx – Static properties (order parameter, static susceptibility, heat capacities, critical exponents, etc.).

Abstract. – The critical exponents of the classical spin glasses with $O(m-2n) \otimes U(n)$ symmetry are obtained to order ϵ^2 of the ϵ-expansion around six dimensions, where $\epsilon = 6 - d$. This new unitary symmetry is realized in the m-vector ($m \geq 2$) spin glasses with anisotropic quenched random Dzyaloshinskii-Moriya (DM) interaction. Such a random DM interaction with n correlated couplings leads to critical exponents which are different from those of any isotropic m-vector spin glass and thus belong to new universality classes.

The antisymmetric Dzyaloshinskii-Moriya (DM) interaction which is mediated by spin-orbit scattering [1] breaks the global rotational invariance in magnetic systems and is well known to lead to macroscopic spin anisotropies [2]. The effect of a quenched random DM interaction on the properties of spin glasses [3] has been studied in some detail [4]-[6]. In particular, in the m-vector spin glass the random DM interaction with no preferred orientation of the couplings causes a crossover to Ising-like critical behaviour [4], [5]. The critical behaviour of the classical XY spin glass with random DM interaction was found to belong to the same universality class as the random gauge XY ("gauge glass") model [7], which is different from those of the isotropic vector spin glasses [8]. The equivalence of these XY-type models under renormalization has been attributed to the broken global reflection invariance [9] which is common to both models. On the other hand, it has been noted recently that in the general m-vector spin glass an *anisotropic* quenched random DM interaction with two *correlated* DM couplings leads to an additional new universality class [10].

In the present letter we consider the random magnetic interaction which naturally generalizes the peculiar symmetry which gave rise to new critical behaviour in the previously investigated situations and which allows to identify these models as special cases of a spin glass with a novel unitary symmetry. This symmetry describes a class of DM spin glasses which exhibit other than Ising-type critical behaviour and thus defines a new nontrivial model

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of random antisymmetric magnetic anisotropy. Additionally, it seems to be the *only* realization of a classical DM spin glass which is not Ising-like. Such an anisotropic interaction should be relevant for the description of magnetic systems with spin-orbit scattering and another correlative source for the resulting DM couplings. It also provides a generalizing and unifying treatment of other than Ising critical behaviour of existing spin glass models with DM interaction, although an immediate physical application for this model appears not to be obvious at present. Moreover, the knowledge of the critical properties of the effective field theory corresponding to this type of random magnetic Hamiltonian amends the general understanding of the critical behaviour of field theories with cubic interaction terms.

We study the m-component ($m \geq 2$) vector spin glasses with an anisotropic quenched random Dzyaloshinskii-Moriya interaction with separated couplings. The Hamiltonian of this model reads

$$\mathcal{H} = \sum_{\langle i,j\rangle} \Big(J_{ij} \sum_a S_{ia} S_{ja} + \sum_{ab} D_{ij}^{ab}\, S_{ia}\, S_{jb} \Big) , \tag{1}$$

where the summation $\langle i,j\rangle$ extends over the bonds of the lattice, and the S_{ia} ($a = 1, \ldots, m$) are the components of classical spins. We consider a DM interaction with an anisotropy of the couplings $D_{ij}^{ab} = -D_{ij}^{ba}$ which is introduced by the assumption that the D_{ij}^{ab} are nonzero only for $1 \leq n \leq m/2$ pairs (ab) of spin components. We also assume that there is no common spin component among any two different such pairs. This requirement of *separated* couplings is essential to find other than Ising-like critical behaviour [10]. The D_{ij}^{ab} for different pairs (ab) are chosen to be *not independent* of each other; we describe the correlation between the couplings by the Hamiltonian

$$\mathcal{H} = \sum_{\langle i,j\rangle} \Big[\sum_{a=1}^{n} \Big(J_{ij}(S_{ia}S_{ja} + S_{i\bar{a}}S_{j\bar{a}}) + D_{ij}(S_{ia}S_{j\bar{a}} - S_{i\bar{a}}S_{ja}) \Big) + \sum_{b=2n+1}^{m} J_{ij}(S_{ib}S_{jb}) \Big] , \tag{2}$$

and the Gaussian probability distributions

$$\mathcal{P}(J_{ij}) = \frac{1}{J\sqrt{2\pi}} \exp\Big[-\frac{(J_{ij})^2}{2J^2} \Big] \quad \text{and} \quad \mathcal{P}(D_{ij}) = \frac{1}{D\sqrt{2\pi}} \exp\Big[-\frac{(D_{ij})^2}{2D^2} \Big] . \tag{3}$$

The summation labelled by a in (2) is over the n pairs $(a\bar{a})$ of spin components (where $a < \bar{a}$) with nonvanishing $D_{ij}^{a\bar{a}}$, and the one labelled by b over the remaining $m - 2n$ spin components not involved in the DM interaction.

A possible physical motivation for this form of the spin Hamiltonian is given by magnetic systems in which the existence of the DM interaction requires both the spin-orbit coupling of the magnetic ions *and* an additional source which may correlate different DM components D_{ij}^{ab}, *e.g.* structural distortions of the crystal lattice [11]. It is apparent from the Hamiltonian (2) that the DM interaction with n separated but correlated couplings generates a real-symplectic invariance under Sp_{2n} and reduces the symmetry of the m-vector spin glass from $O(m)$ to $O(m-2n) \otimes U(n)$, due to the isomorphism $U(n) \simeq O(2n) \cap Sp_{2n}$ [12]. Note that $U(1)$ as the simplest case of this general unitary symmetry comprises both the XY gauge glass, for which J_{ij} and D_{ij} are not independent, and the conventional XY spin glass with independent random isotropic and DM couplings, in accordance with the identical critical behaviour of these two models.

To perform the quenched disorder average, we employ the replica method and label the spins S_{ia}^{α} with the additional replica index $\alpha = 1, \ldots, N$. After integration of the Gaussian couplings (3), we introduce the spin glass fields $Q_{iab}^{\alpha\beta} = S_{ia}^{\alpha} S_{ib}^{\beta}$ and disregard the quadrupole fields $Q_{iab}^{\alpha\alpha}$ which do not control the critical behaviour. With the probability distribution of the spin glass fields [13],

$$\mathcal{P}(\{Q\}) \simeq \operatorname{Tr}\delta(Q^{\alpha\beta}_{iab} - S^{\alpha}_{ia}S^{\beta}_{ib}) \simeq \exp\Big[-\frac{1}{2}\sum_{\substack{ab\\ \alpha<\beta}}(Q^{\alpha\beta}_{iab})^2 + \sum_{\substack{abc\\ \alpha<\beta<\gamma}} Q^{\alpha\beta}_{iab}Q^{\beta\gamma}_{ibc}Q^{\gamma\alpha}_{ica} + \mathcal{O}(Q^4_i)\Big], \quad (4)$$

we obtain the replicated partition function

$$\langle Z^N\rangle = \int \mathcal{D}Q\ \exp\Big[\sum_{\substack{ab\\ \alpha<\beta}}\Big[-\frac{1}{2}\sum_i (Q^{\alpha\beta}_{iab})^2 + (\beta J)^2\sum_{\langle i,j\rangle} Q^{\alpha\beta}_{iab}Q^{\alpha\beta}_{jab}\Big] + \sum_{\substack{abc\\ \alpha<\beta<\gamma}}\sum_i Q^{\alpha\beta}_{iab}Q^{\beta\gamma}_{ibc}Q^{\gamma\alpha}_{ica} +$$

$$+(\beta D)^2 \sum_{\substack{\langle i,j\rangle\\ \alpha<\beta}}\Big[\sum_{a,b=1}^{n}(Q^{\alpha\beta}_{iab}Q^{\alpha\beta}_{j\bar a\bar b} - Q^{\alpha\beta}_{i\bar a b}Q^{\alpha\beta}_{ja\bar b}) + (i\leftrightarrow j)\Big]\Big], \quad (5)$$

where we have omitted (irrelevant) powers of Q^4_i or higher. To extract the critical behaviour of this effective field theory, it is sufficient to Fourier transform to momentum space and consider only the $\mathbf{k}=0$ components of the Q-fields. The propagators of the critical modes obtained in this way are then derived from the fully $\mathbf{k}$-dependent bilinear terms in (5). Neglecting higher powers of Q for the moment one thus obtains

$$\langle Z^N\rangle = \int \mathcal{D}Q\ \exp\Big[-\frac{1}{2}\sum_{\substack{abcd\\ \alpha<\beta}} Q^{\alpha\beta}_{ab}\mathbf{M}^{\alpha\beta}_{ab,cd}Q^{\alpha\beta}_{cd} + \mathcal{O}(Q^3)\Big], \quad (6)$$

where the inverse zero-momentum propagator $\mathbf{M}$ of the Q-fields is a $(m^2\times m^2)$-matrix. Its eigenvalues λ and the corresponding degeneracies are

$$\begin{aligned} \lambda_1 &= 1 - z\beta^2(J^2+D^2) \quad && 2n^2\text{-fold degenerate},\\ \lambda_2 &= 1 - z\beta^2(J^2-D^2) \quad && 2n^2\text{-fold degenerate},\\ \lambda_3 &= 1 - z(\beta J)^2 && (m^2-4n^2)\text{-fold degenerate}, \end{aligned} \quad (7)$$

where z is the number of nearest neighbours in the lattice. For nonzero D, the smallest eigenvalue is λ_1 and the critical behaviour is controlled by a $2n^2$-component field. This is in contrast to the isotropic m-vector spin glass, whose critical properties are governed by a m^2-component field, and leads to new physical behaviour, since the $2n^2$ components do not decouple. The symmetry responsible for the critical behaviour is thus $U(n)$.

For any of the n^2 combinations of two component pairs $(a\bar a)$ and $(b\bar b)$ with nonzero $D^{a\bar a}_{ij}$ and $D^{b\bar b}_{ij}$ one finds two critical fluctuation modes,

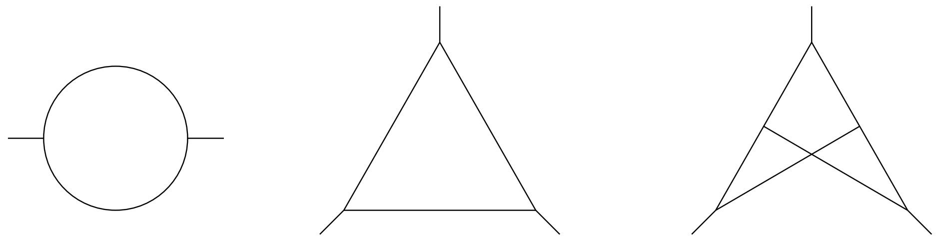
$$\phi^{\alpha\beta}_{ab} = Q^{\alpha\beta}_{ab} + Q^{\alpha\beta}_{\bar a\bar b} \quad\text{and}\quad \psi^{\alpha\beta}_{ab} = Q^{\alpha\beta}_{a\bar b} - Q^{\alpha\beta}_{\bar a b}. \quad (8)$$

As a consequence of the symmetry of the spin glass order parameter, $Q^{\alpha\beta}_{ab} = Q^{\beta\alpha}_{ba}$, the relations $\phi^{\beta\alpha}_{ba} = +\phi^{\alpha\beta}_{ab}$ and $\psi^{\beta\alpha}_{ba} = -\psi^{\alpha\beta}_{ab}$ hold. This property of the fields determines the structure of the corresponding propagators,

$$\langle\phi^{\alpha\beta}_{ab}(\mathbf{k})\phi^{\alpha\beta}_{ab}(-\mathbf{k})\rangle = \langle\phi^{\alpha\beta}_{ab}(\mathbf{k})\phi^{\beta\alpha}_{ba}(-\mathbf{k})\rangle = \langle\psi^{\alpha\beta}_{ab}(\mathbf{k})\psi^{\alpha\beta}_{ab}(-\mathbf{k})\rangle = -\langle\psi^{\alpha\beta}_{ab}(\mathbf{k})\psi^{\beta\alpha}_{ba}(-\mathbf{k})\rangle, \quad (9)$$

where $\alpha\neq\beta$. Keeping only the terms which couple to the $2n^2$ soft modes, we evaluate the contribution to the action cubic in the Q-fields,

$$\operatorname{Tr} Q^3 = \sum_{\substack{abc\\ \alpha\beta\gamma}} Q^{\alpha\beta}_{ab}Q^{\beta\gamma}_{bc}Q^{\gamma\alpha}_{ca} \simeq \sum_{\substack{abc\\ \alpha\beta\gamma}}\big(\phi^{\alpha\beta}_{ab}\phi^{\beta\gamma}_{bc}\phi^{\gamma\alpha}_{ca} - 3\phi^{\alpha\beta}_{ab}\psi^{\beta\gamma}_{bc}\psi^{\gamma\alpha}_{ca}\big). \quad (10)$$



α β γ

Fig. 1. – The contractions α, β, and γ of the interaction tensor d_{ijk} which contribute at order ϵ^2.

Recovering the full momentum dependence from (5), the effective Hamiltonian of the $U(n)$ vector spin glass can then be written in the form

$$\mathcal{H}_{\text{eff}} = \int \mathrm{d}^d r \Big[\frac{1}{2}\sum_{\alpha\neq\beta}\sum_{i=1}^{2n^2}\varphi_i^{\alpha\beta}(r-\nabla^2)\varphi_i^{\alpha\beta} + \frac{g_0}{3!}\sum_{\alpha\beta\gamma}\sum_{i,j,k=1}^{2n^2} d_{ijk}\,\varphi_i^{\alpha\beta}\varphi_j^{\beta\gamma}\varphi_k^{\gamma\alpha}\Big]\,, \tag{11}$$

where up to an appropriate normalization factor the critical modes (8) are denoted by φ_l, with $l = 1\ldots 2n^2$, and the third-rank interaction tensor d_{ijk} is defined by eq. (10). Of the $8n^6$ tensor elements only $4n^3$ are nonzero. The Hamiltonian (11) is a special case of the general cubic field theory studied by de Alcantara Bonfim *et al.* [14]. To find the corresponding critical exponents within an ϵ-expansion, we calculate the contractions of the tensor d_{ijk} as defined in ref. [14]. In general, a cubic field theory is unstable [15] and the ϵ-expansion is not well defined. However, for the classical isotropic m-vector spin glass it is believed [15], [16] that the replica limit $N \to 0$ ensures oscillatory behaviour of the ϵ-expansion at high orders. Such a behaviour should also persist for a classical spin glass with more general symmetry. At two-loop order we have to evaluate the tensor contractions which are represented by the diagrams depicted in fig. 1. The corresponding contraction factors ([1]) are given by

$$\alpha = 4n(N-2), \quad \beta = 2n(N-3) \quad \text{and} \quad \gamma = 8\,. \tag{12}$$

Taking now the replica limit $N \to 0$ and inserting these results for α, β and γ in the general expressions of ref. [14], we finally obtain the ϵ-expansion for the critical exponents η and ν belonging to the $U(n)$ fixed point,

$$\eta = -\frac{1}{6}\epsilon - \frac{11n^2+12}{576n^2}\epsilon^2 + \mathcal{O}(\epsilon^3) \quad \text{and} \quad \nu = \frac{1}{2} + \frac{5}{24}\epsilon - \frac{41n^2-60}{2304n^2}\epsilon^2 + \mathcal{O}(\epsilon^3)\,, \tag{13}$$

with $\epsilon = 6-d$. Other exponents may be obtained from the usual scaling relations. The special cases $n = 1$ and $n = 2$ reproduce the previously derived results for the exponents of the XY gauge glass [8] and of a spin glass with two correlated DM components [10], respectively. For the large-n limit, $n \to \infty$, the exponents (10) agree with those of the isotropic $O(m)$ vector spin glass [16], [17] in the $m \to \infty$ limit, but a finite n or m leads to different exponents for the two models. The spin glasses with $U(n)$ symmetry therefore belong to new universality classes.

([1]) Note that, due to a different choice of the basis for the critical modes, the values of α, β, γ derived in ref. [10] for $n = 2$ differ from the results (12) by a factor which does not change the critical exponents.

In summary, we have shown that in the classical m-vector spin glass the presence of n correlated and separated DM components generates a novel unitary symmetry of the random magnetic Hamiltonian which results in new nontrivial critical properties different from those of the isotropic spin glasses.

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The author wishes to thank A. Aharony for valuable conversations and acknowledges financial support by a Minerva fellowship. This work was also supported by the US-Israel Binational Science Foundation.

REFERENCES

[1] Moriya T., *Phys. Rev.*, **120** (1960) 91.
[2] Fert A. and Levy P. M., *Phys. Rev. Lett.*, **44** (1980) 1538.
[3] Binder K. and Young A. P., *Rev. Mod. Phys.*, **58** (1986) 801.
[4] Kotliar G. and Sompolinsky H., *Phys. Rev. Lett.*, **53** (1984) 1751.
[5] Bray A. J. and Moore M. A., *J. Phys. C*, **15** (1982) 3897.
[6] Goldbart P. M., *J. Phys. C*, **18** (1985) 2183.
[7] Gingras M. J. P., *Phys. Rev. B*, **43** (1991) 13747; **45** (1992) 7547.
[8] Houghton A. and Moore M. A., *Phys. Rev. B*, **38** (1988) 5045; Moore M. A. and Murphy S., *Phys. Rev. B*, **42** (1990) 2587.
[9] Gingras M. J. P., *Phys. Rev. B*, **44** (1991) 7139.
[10] Stein J., *J. Phys. A*, **29** (1996) 963.
[11] Yildirim T., Harris A. B., Entin-Wohlman O. and Aharony A., *Phys. Rev. Lett.*, **73** (1994) 2919.
[12] Fulton W. and Harris J., *Representation Theory* (Springer-Verlag, New York, N.Y.) 1991.
[13] Harris A. B., Lubensky T. C. and Chen J.-H., *Phys. Rev. Lett.*, **36** (1976) 415.
[14] de Alcantara Bonfim O. F., Kirkham J. E. and McKane A. J., *J. Phys. A*, **13** (1980) L247; **14** (1981) 2391.
[15] McKane A. J., *Nucl. Phys. B*, **152** (1979) 166.
[16] Green J. E., *J. Phys. A*, **18** (1985) L43.
[17] Elderfield D. J. and McKane A. J., *Phys. Rev. B*, **18** (1978) 3730.