Universität Heidelberg SS 2009

1. Quantum Mechanics exercise sheet (PTP 4)

These exercises (Präsenzübung) will be discussed together in the exercise class on the 7th and 8th of April

For active participation you get **2 points** !

Exercise P1:
The scalarproduct of two two-state vectors in a two-state system were defined in the lectures:
$\langle\varphi|\psi\rangle = (\varphi_1^*, \varphi_2^*) \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \varphi_1^*\psi_1 + \varphi_2^*\psi_2.$
Show that $\langle\psi|\varphi\rangle = (\langle\varphi|\psi\rangle)^*$.

Exercise P2:

Consider the spin vector

$$\hat{\vec{S}} = \frac{\hbar}{2}\vec{\tau}$$

with the Pauli matrices defined by

$$\hat{\tau}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{\tau}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \hat{\tau}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

a) Calculate $\langle\hat{S}_y\rangle$ for the two-state vector $|\varphi\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

b) Calculate $\langle\hat{S}_z\rangle$ for $|\varphi\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$.

c) Calculate $\langle\hat{S}_z\rangle$ for $|\varphi\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

d) Calculate $\langle\hat{S}_z\rangle$ for $|\varphi\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Exercise P3:
Using matrix multiplication calculate the commutator

$$\left[\hat{S}_x \,,\, \hat{S}_y\right] \;=\; \hat{S}_x\hat{S}_y \;-\; \hat{S}_y\hat{S}_x$$

Exercise P4: Eigenvalue problem

Solve the Eigenvalue problem for the self-adjoint matrix

$$A = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

Verify that the Eigenvectors are orthogonal and normalise them.

Exercise P5: Linear operators

Consider a linear operator with the property that $AA^+ = A^+A$, which has the Eigenvector $|a\rangle$ with Eigenvalue a and it is normalised, $\langle a|a\rangle = 1$. Show that $|a\rangle$ is also an Eigenvector of A^+ and that the corresponding Eigenvalue of A^+ is a^*.