University of Heidelberg WS 2010/2011

## 2. Presence exercise for the lecture Statistical Physics

for Thursday 28.10.2010 and Friday, 29.10.2010
For active participation you get 2 points !

**Exercise P2.1:** *Correlation function*
Consider a quantum-mechanical spin $\vec{\hat{S}}$ and the three observables $\hat{A} = \hat{S}_z + \frac{\hbar}{2}$(= measuring the z-component of the spin), $\hat{B} = \hat{S}_x + \frac{\hbar}{2}$ (=measuring the x-component of the spin) and $\hat{C} = \hat{S}_z + \frac{\hbar}{2}$ (= also measuring the z-component of the spin). The possible results for the measurement of the observables $\hat{A}, \hat{B}$ and $\hat{C}$(in units of $\frac{\hbar}{2}$) are

$$\lambda_A \in \{1, 0\}, \quad \lambda_B \in \{1, 0\}, \quad \lambda_C \in \{1, 0\}.$$

Assume that we first measure $\hat{A}$ and let $P_A = P_A(1)$ be the probability that one gets $\lambda_A = 1$. We define the probabilities

(i) $P_{BA}(1, 1) :=$ probability to measure $\lambda_B = 1$ after one already measured $\lambda_A = 1$

(ii) $P_{CB}(1, 1) :=$ probability to measure $\lambda_C = 1$ after one already measured $\lambda_B = 1$

(iii) $P_{BC}(1, 1) :=$ probability to measure $\lambda_B = 1$ after one already measured $\lambda_C = 1$

(iv) $P_{CA}(1, 1) :=$ probability to measure $\lambda_C = 1$ after one already measured $\lambda_A = 1$

Give the probabilities $P_{BA}(1, 1), P_{CB}(1, 1), P_{BC}(1, 1), P_{CA}(1, 1)$ and calculate the corresponding correlation functions $\langle \hat{C} \circ \hat{B} \circ \hat{A} \rangle$ and $\langle \hat{B} \circ \hat{C} \circ \hat{A} \rangle$.
Compare the results.

**If there is still time left:**

**Exercise P2.2:** *Poisson distribution*
The probability $P(n; p, N)$ that an event characterized by a probability $p$ occurs $n$ times in $N$ trials is given by the binominal distribution (see lecture)

$$P(n; p, N) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n} \qquad (1)$$

Consider a situation where the probability $p$ is very small ($p \ll 1$) and where one is interested in the case $n \ll N$. (Note that if $N$ is large, $P(n; p, N)$ becomes very small if $n \to N$ because of the smallness of the factor $p^n$ when $p \ll 1$. Hence $P(n; p, N)$ is indeed only appreciable when $n \ll N$.) Several approximations can then be made to reduce (1) to simpler form.

a) Using the result $\ln(1-p) \approx -p$, show that $(1-p)^{N-n} \approx e^{-Np}$.

b) Show that $N!/(N-n)! \approx N^n$.

c) Hence show that (1) reduces to

$$P(n; p, N) = \frac{\lambda^n}{n!} e^{-\lambda} =: P(n; \lambda) \qquad (2)$$

where $\lambda := Np$ is the mean number of events (mean of the binominal distribution, see lecture). The distribution (2) is called the "Poisson distribution".

d) Show that die Poisson distribution is properly normalized in the sense that $\sum_{n=0}^{N} P(n; \lambda) = 1$.

e) Calculate the mean value $\langle n \rangle$ and the dispersion $\langle n \rangle^2 = \langle (n - \langle n \rangle)^2 \rangle = \langle n^2 \rangle - \langle n \rangle^2$ of the Poisson distribution (2).