3. Presence exercise for the lecture Statistical Physics

for Thursday/Friday October 18/19, 2010
For active participation you get 2 points !

Together with your tutor: Recap the termodynamic relations in the table 2(see next page) and the relations between the statistical ensembles and the thermodynamic potentials (table 1).

a) Derive the Maxwell relations in table 2. Find other Maxwell relations.

b) Use (see table 2) $dF = -SdT - pdV + \mu dN$ and $dJ = -SdT - pdV - Nd\mu$ and the expressions for F and J in table 1 to show that $S = k_B\left(1 - \beta\frac{\partial}{\partial\beta}\right)\ln Z_{can}$ and $S = k_B\left(1 - \beta\frac{\partial}{\partial\beta}\right)\ln Z_{gc}$.

c) Use $dS = (1/T)dE + (p/T)dV - (\mu/T)dN$ and $dJ = -SdT - pdV - Nd\mu$ to deduce the expressions forμ in table 1.

Table 1:

microcanonical	canonical	grand canonical
$Z_{micro}(V, N, E)$	$Z_{can}(V, N, \beta)$	$Z_{gr}(V, \mu, \beta)$
$\beta = \frac{\partial}{\partial E}\ln Z_{mic}$	$E = -\frac{\partial}{\partial\beta}\ln Z_{can}$	$E = \left(-\frac{\partial}{\partial\beta} + \frac{\mu}{\beta}\frac{\partial}{\partial\mu}\right)\ln Z_{gc}$
$S = k_B\ln Z_{mic}$	$F = -k_BT\ln Z_{can}$	$J = -k_BT\ln Z_{gc}$
	$S = k_B\left(1 - \beta\frac{\partial}{\partial\beta}\right)\ln Z_{can}$	$S = k_B\left(1 - \beta\frac{\partial}{\partial\beta}\right)\ln Z_{gc}$
$\mu = -\frac{1}{\beta}\frac{\partial}{\partial N}\ln Z_{mic}$	$\mu = -\frac{1}{\beta}\frac{\partial}{\partial N}\ln Z_{can}$	$N = \frac{1}{\beta}\frac{\partial}{\partial\mu}\ln Z_{gc}$

Table 2:

thermod. potential	independent variable	conjugate variable	thermodynamic relations
energy E	S, V, N $dE =$ $TdS - pdV + \mu dN$	$T = \left(\frac{\partial E}{\partial S}\right)_{V,N}$ $p = -\left(\frac{\partial E}{\partial V}\right)_{S,N}$ $\mu = \left(\frac{\partial E}{\partial N}\right)_{S,V}$	$\left(\frac{\partial^2 E}{\partial V \partial S}\right)_N = \left(\frac{\partial T}{\partial V}\right)_{S,N} = -\left(\frac{\partial p}{\partial S}\right)_{V,N}$ . . .
enthalpie $H = E + pV$	$S, \quad p, \quad N$ $dH =$ $TdS + Vdp + \mu dN$	$T = \left(\frac{\partial H}{\partial S}\right)_{p,N}$ $V = \left(\frac{\partial H}{\partial p}\right)_{S,N}$ $\mu = \left(\frac{\partial H}{\partial N}\right)_{S,p}$	$\left(\frac{\partial^2 H}{\partial p \partial S}\right)_N = \left(\frac{\partial T}{\partial p}\right)_{S,N} = \left(\frac{\partial V}{\partial S}\right)_{p,N}$ . . .
free energy $F = E - TS$	$T, \quad V, \quad N$ $dF =$ $-SdT - pdV + \mu dN$	$S = -\left(\frac{\partial F}{\partial T}\right)_{V,N}$ $p = -\left(\frac{\partial F}{\partial V}\right)_{T,N}$ $\mu = \left(\frac{\partial F}{\partial N}\right)_{T,V}$	$\left(\frac{\partial^2 F}{\partial V \partial T}\right)_N = -\left(\frac{\partial S}{\partial V}\right)_{T,N} = -\left(\frac{\partial p}{\partial T}\right)_{V,N}$ . . .
free enthalpy $G =$ $E - TS + pV$	$T, \quad p, \quad N$ $dG =$ $-SdT + Vdp + \mu dN$	$S = -\left(\frac{\partial G}{\partial T}\right)_{p,N}$ $V = \left(\frac{\partial G}{\partial p}\right)_{T,N}$ $\mu = \left(\frac{\partial G}{\partial N}\right)_{T,p}$	$\left(\frac{\partial^2 G}{\partial p \partial T}\right)_N = -\left(\frac{\partial S}{\partial p}\right)_{T,N} = \left(\frac{\partial V}{\partial T}\right)_{p,N}$ . . .
grand can. pot. $J =$ $E - TS - \mu N$	$T, \quad V, \quad \mu$ $dJ =$ $-SdT - pdV - Nd\mu$	$S = -\left(\frac{\partial J}{\partial T}\right)_{V,\mu}$ $p = -\left(\frac{\partial J}{\partial V}\right)_{T,\mu}$ $N = -\left(\frac{\partial J}{\partial \mu}\right)_{T,V}$	. . . . . .