11. Exercise sheet for the lecture Statistical Physics

Hand in your solutions on Thursday, 20.01.2011 or Friday, 21.01.2011 in the exercise group.

Reminder:
The test will take place on

Monday, February 7, 2011
from 10.00 a.m. till 1 p.m.
in INF 227, HS 1 and HS 2.

Please note: Precondition for the admission to the test is: 60 % of the homework points!

IMPORTANT: If you want to cancel your registration for the course you HAVE to do that in writing **before February 3, 2011**. Hand in your letter to the secretariat of the Institute for Theoretical Physics (addressed to Dr. E. Thommes, Philsophenweg 16, 69120 Heidelberg). If you are registered in the database of the course and if you do not turn up for the test without cancelling your registration in writing, you officially failed the course!

Allowed utilities for the test:
One **handwritten** DIN A4 sheet (both sides) with your personal choice of formulae!
No pocket calculators or pocket computers etc!

Exercise 11.1: *One dimensional Ising model* **(12 Punkte)**
Consider a one dimensional Ising model which consists of three spins (see Fig. 1). Each spin can be in two states: $s_i = 1$ or $s_i = -1$ (up or down). The spins only interact with their direct neighbours with a coupling strength J. The system is in an external magnetic field B with direction along the z axis. Furthermore the system is in thermal contact with an heat bath of temperature T. The Hamiltonian of the system is then given by (in adequate units):

$$H = -J(S_1S_2 + S_2S_3) - B(S_1 + S_2 + S_3)$$

a) Determine the possible micro states of the system. Give the energy and the degeneracy of each state. Sketch the corresponding energy spectrum of the system for the case $2B > J > B > 0$. (3 points)

b) Calculate the canonical partition function $Z(T, B)$ of the system and bring it into a compact form. (3 points)

c) Calculate the mean energy $\bar{E}$ and entropy $\bar{S}$ of the system in the limit $kT \gg B$, J. **(2 points)**

d) Determine the mean magnetisation $\bar{M}(T, B)$. Expand $\bar{M}(T, B)$ for small B to first order in B and consider the limit $kT \gg J$. (4 points)

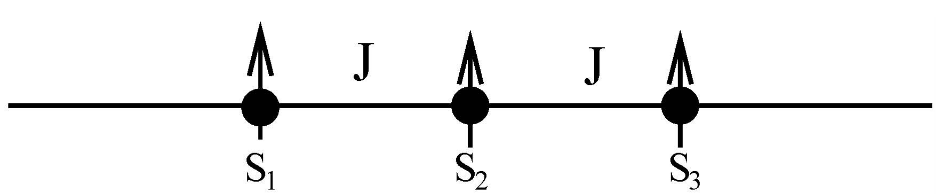

Fig. 1

Exercise 11.2 *Clausius-Clapeyron-equation* **(3 points)**

Determine the vapor pressure as a function of temperature T of a liquid which is in equilibrium with its vapor.

Assume that the heat of vaporisation per particle does not depend on pressure and temperature. Furthermore you could assume that the vapor could be treated as an ideal gas and that the gas has a density which is much less than that of the liquid.

Exercise 11.3 *VAN DER WAALS gas* **(5 points)**

Consider a real gas which can be described by the equation of state of a VAN DER WAALS gas (see lecture):

$$p = \frac{kNT}{V - b_0 N} - b_1 \left(\frac{N}{V}\right)^2$$

a) Show that the values p_*, V_* and T_* in the critical point fulfill

$$\frac{p_* V_*}{NkT_*} = \frac{3}{8}$$

(1 point)

b) Determine the internal energy of a VAN DER WAALS gas. (2 points)

c) Show that for an adiabatic process of a VAN DER WAALS gas one gets the relation

$$(V - Nb_0)T^{3/2} = \text{const}$$

(2 points)