1. Exercise sheet for the lecture Statistical Physics

Hand in your solutions on Friday, 22.10.2010 in the exercise group.

Exercise 1.1 (3 points)
How large is the probability that in a lecture with 200 students at least two have their birthday on the same day? How many persons would have to sit in the lecture so that at least two persons have their birthday on the same day with a probability of $\frac{1}{2}$?

Exercise 1.2 (2 points)
Given M children as well as N similar (distinguishable) candies (without sugar). Each single candy is assigned to child i with probability p_i, where $i = 1, ..., M$. All N candies are distributed independently among the M children, where as many as desired candies fit into each child. How large is the probability that the ith child gets n_i candies?

Exercise 1.3 (3 points)
A deck of 32 Skat cards of the four colors clubs, spades, hearts, diamonds in each case ace, king, queen, jack, ten, nine, eight, seven is randomly distributed among three players A, B, C, each player receiving 10 cards and the remaining to cards go to the so-called Skat. Player A received exactly 2 jacks. How large is the probability p that player B or C holds the other two jacks in his hand?
Note: Regard most simply the relationship of the favorable (= B has exactly two jacks) to the possible hands for players B.

Exercise 1.4 (2 points)
A device uses five silicon chips. Suppose the five chips are chosen at random from a batch of hundred chips out of which five are defective. What is the probability that the device contains no defective chip when it is made up from one batch?

Exercise 1.5 Density matrix (6 points)
Consider a system of a single spin 1/2 particle at rest. The density matrix of the system is parameterized by

$$\rho(\kappa) = \frac{1}{2}\begin{pmatrix} 1 & \kappa \\ \kappa & 1 \end{pmatrix}$$

where κ is supposed to be a real parameter.

a) Determine the eigenvalues of $\rho(\kappa)$ (3 points)

b) Give the general conditions which have to be obeyed by a density matrix. For which values of κ does $\rho(\kappa)$ obey these conditions. (2 points)

c) For which values of κ does $\rho(\kappa)$ represent a pure state. (1 point)

The theorist
Excerpt out of "Physicists continue to laugh" , MIR publishing house Moscow 1968.
If one asks a theoretical physicist, to compute for example the stability of an ordinary table with four legs, then he will shortly have preliminary results, which refer to one-legged tables or to tables with infinitely many legs. He will unsuccessfully spend the remainder of his life by trying to solve the ordinary problem of a table with arbitrary, finite number of legs.

Exercise 1.6 Reduced systems **(4 points)**

Consider a four state system as discussed in the lecture of October 12 (system of two spins):

τ:	$\vert\uparrow\uparrow\rangle$	$\vert\uparrow\downarrow\rangle$	$\vert\downarrow\uparrow\rangle$	$\vert\downarrow\downarrow\rangle$
	$\vert 1\rangle$	$\vert 2\rangle$	$\vert 3\rangle$	$\vert 4\rangle$
	p_1	p_2	p_3	p_4
A_τ	2	0	0	-2
B_τ	1	1	-1	-1

with the probability distribution $\{p_\tau\} = \{p_1, p_2, p_3, p_4\}$. Let A be the observable "total spin" and B be the observable "spin 1" (see lecture). Now consider (see lecture) the case of a reduced system where the probabilities only encode information about the total spin A. The microstates $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ are then indistinguishable. The reduced system is then effectively a three state system:

	$\vert\uparrow\uparrow\rangle$	$\vert\uparrow\downarrow\rangle$, $\vert\downarrow\uparrow\rangle$	$\vert\downarrow\downarrow\rangle$
τ_R:	$\vert 1_R\rangle$	$\vert 2_R\rangle$	$\vert 3_R\rangle$
	$\bar{p}_1 = p_1$	$\bar{p}_2 = p_2 + p_3$	$\bar{p}_3 = p_4$

As you know from the lecture in the reduced state $|2_R\rangle$ the observable B is a probabilistic observable and does not have a definite value. Assume, that in an experiment the system is prepared to be in the reduced state $|2_R\rangle$. After that B is measured. The experiment is repeated very often.

a) What is the expected mean value B_{2R} and the corresponding root-mean-square deviation $(\Delta B)_{2R}$ for the observable B in terms of the probabilities $\{p_1, p_2, p_3, p_4\}$?

b) Express the probabilities $\{p_1, p_2, p_3, p_4\}$ in terms of the probabilities $\{\bar{p}_1, \bar{p}_2, \bar{p}_3\}$ AND B_{2R}. (That is: Knowledge of $\{\bar{p}_1, \bar{p}_2, \bar{p}_3\}$ AND B_{2R} allows one to reconstruct the underlying probability distribution $\{p_1, p_2, p_3, p_4\}$ of the microstates.)