2. Exercise sheet for the lecture Statistical Physics

Hand in your solutions on October 28 or 29, 2010 in the exercise group.

Exercise 2.1 (2 points)

The probability that a molecule in a gas travels a free path, which lies between x and $x + dx$ be given by an exponential law

$$w(x)dx = \alpha e^{-\alpha x} dx$$

a) How large is the mean free path $\bar{\lambda}$? (1 point)

b) Determine the fraction of the free path, which is larger or smaller than $\bar{\lambda}$, that is determine the probability that $x < \bar{\lambda}$ and the probability that $x > \bar{\lambda}$. (1 point)

Exercise 2.2 (7 points)

A point particle with mass m moves within the range $0 \leq x \leq l$ and gets reflected at the walls at $x = 0$ and $x = l$, respectively.

a) How large is the volume of the classical phase space $\Gamma_0(E)$ with an energy smaller than E? (1 point)

b) Show that $\Gamma_0(E)$ stays constant if the wall at $x = l$ is moved slowly (adiabatic invariance) (2 points)

c) Concerning the corresponding quantum mechanical system: What is the number of states $\Omega_0(E)$ with an energy smaller than E_{max}? Compare to $\Gamma_0(E_{max})$ for large E_{max}. (2 points)

d) Assume equipartition for all states with $E < E_{max}$ as in the lecture. Compute the average energy $\langle E \rangle$ as well as $\langle E^2 \rangle$. (2 points)

Exercise 2.3: *Needed mathematics: Stirling formula* **(6 points)**

Derive the *Stirling formula*

$$\ln(n!) \approx (n + \frac{1}{2}) \ln(n) - n + \frac{1}{2} \cdot \ln(2\pi)$$

a) Calculate first

$$\int_0^\infty dx\, e^{-x} x^n \,,$$

where $n \geq 0$ is an integer. (2 points)

b) Thereafter, expand the natural logarithm (why?) of the integrand around its maximum and consider the result for large n. (2 points)

c) Determine the first correction term. (2 points)

Exercise 2.4: **(5 points)**
The energy levels of an oscillator with frequency ν are given by

$$\epsilon = \frac{1}{2}h\nu,\ \frac{3}{2}h\nu, \ldots, \left(n + \frac{1}{2}\right) h\nu, \ldots$$

A system of N oscillators has the total energy

$$E = \frac{1}{2}Nh\nu + Mh\nu$$

($M \geq 0$ is an integer).

a) Which is the total number of possible states Ω_M for given E?
(2 points)

b) Calculate the entropy

$$S = k \ln \Omega_M$$

by means of the Stirling formula for $N \gg 1, M \gg 1$. (1 point)

c) The temperature T is defined as

$$\frac{1}{T} = \frac{\partial S}{\partial E}.$$

Express the total energy as a function of the temperature and discuss the function $E(T)$. (2 points)