3. Exercise sheet for the lecture Statistical Physics

Hand in your solutions on Thursday, 4.11.2010 or Friday, 5.11.2010 in the exercise group.

Exercise 3.1: *Conditional probability, BAYES theorem* **(3 points)**
In 80% of the working days a physicist takes the train to drive home. He arrives at home on time for supper on two-thirds of these days. On average he comes home on time for supper on 3 from 5 working days.
One evening he comes home on time. What is the probability that he took the train that day?

Use the BAYES theorem which says (if $P(A) \neq 0$ and $P(B) \neq 0$):

$$P(A|B) = \frac{P(A)}{P(B)} P(B|A)$$

with the conditional probabilities $P(B|A) :=$ probability of B *assuming* A has occured. If you don't know BAYES theorem or if you can't remember it, take any book about probability theory and read the sections "conditional probability" and "BAYES theorem".

Exercise 3.2: *Random Walk: A drunken fellow and the lamp pole* **(6 points)**
A lamp pole stands in the center between the two ends of a straight road (one dimensional problem !) before a tavern. Closing time.

a) A drunkard starts his one-dimensional movement from the lamp pole along the road. He makes steps of equal length with the same probability either in one or the other direction. How large is the probability that the man after N steps gets back to the lamp pole, (i) if N is even or (ii) is odd? (3 points)

b) Two drunkards start together from the lamp pole along the same road (one dimensional). They both make steps of equal length with the same probabilities as in (a). Determine the probability that they meet after N steps again, assuming that they make their steps at the same time. (3 points)

Exercise 3.3: **(10 points)**
A container with volumes V contains N gas molecules. The number of molecules in a partial volume v of the container is n. The probability to find a certain molecule in v is equal to $p = \frac{v}{V}$ (if the system is in thermal equilibrium which we assume here).

(i) Find the probability distribution $f(n)$ of the number n, and (2 points)

(ii) calculate the mean values $\bar{n}$ and $\overline{(n - \bar{n})^2}$. (2 points)

(iii) Show that if N and n are both large, $f(n)$ is approximately Gaussian, that is

$$f(n) \sim \exp\left[-\frac{1}{2\Delta^2}(n - \bar{n})^2\right] \quad .$$

Give Δ^2 in terms of N and p. (3 points)

(iv) Show that in the limit $\frac{v}{V} \to 0$ and $V \to \infty$ with constant $\frac{N}{V}$, $f(n)$ approaches the Poisson distribution $f(n) = e^{-\bar{n}}(\bar{n})^n/n!$. (3 points)

In einem Flugzeug wird ein ansonsten untadeliger Fluggast dabei ertappt, eine Maschinenpistole bei sich zu führen.

Zur Rede gestellt erklärt er: Die Wahrscheinlichkeit,
dass ein bewaffneter Terrorist im Flugzeug sitzt, ist gering,
dass aber gleich zwei Bewaffnete im Flugzeug sitzen, ist praktisch 0.
Um dieser Beruhigung willen habe ich die Maschinenpistole bei mir.

Aus: Christoph Bandelow, Einführung in die Wahrscheinlichkeitstheorie (BI)