5. Exercise sheet for the lecture Statistical Physics

Hand in your solutions on Thursday, 18.11.2010 or Friday, 19.11.2010 in the exercise group.

Please note:

According to the recommendation of the faculty there will be only ONE test, that is there will be NO test in the middle of the term!
The originally annouced test for November 27, 2010 will be omitted! Also the originally planed test for January 29, 2011 will be omitted. Instead there will be only ONE test in the last week of the lecture period:

The single test will take place on

Monday, February 7, 2011
from 10.00 a.m. till 1 p.m.
in INF 227, HS 1 and HS 2.

Please note: Precondition for the admission to the test is: 60 % of the homework points!

Exercise 5.1 *Entropy of an ideal gas* **(8 points)**

Consider n moles of Argon in a volume V at temperature T. Argon can be treated as an ideal gas. The entropy $S(E,V,N)$ of an ideal gas is given by (see lecture):

$$S(E,V,N) = k_B(3N)\left\{\frac{1}{3}\ln\left(\frac{V}{N}\right) + \frac{1}{2}\ln\left(\frac{2}{3}\frac{E}{N}\right) + \frac{1}{2}\ln\left(\frac{M}{2\pi\hbar^2}\right) + \frac{5}{6}\right\}$$

a) Use

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{N,V}$$

to find an expression for the internal energy of the gas $E = E(T,V,n)$ as a function of T,V and n with $n := \frac{N}{N_A}$ and the Avogadro constant $N_A = 6,022 \cdot 10^{23} \frac{1}{\mathrm{mol}}$.
What is the specific heat per mol for Argon ? (1 point)

b) Assume that the volume and the temperature of the gas are changed to V' and T'. Calculate the change in entropy. (2 points)

Now consider an heat isolated box with volume V_g, which is devided into two volumes V_1 and V_2 by an isolated wall. In each volume V_1 and V_2 is 1 mol of Argon. The temperature of the gas in volume V_1 is T_1 and in the volume V_2 is T_2 with $T_1 \neq T_2$.

c) To which temperature and which pressure will the system adjust when the wall is removed ? (2 points)

d) What is the change ΔS in entropy ? Show that $\Delta S \geq 0$. What will you get in the special case of $T_1 = T_2$ and $V_1 = V_2$? (3 points)
Hint: First consider the gases in the two volumes separately and calculate the change for each gas in entropy for bringing it to the volume $V_g/2$ and temperature T_g.

Exercise 5.2: *Non interacting spins on a lattice, continuation of exercise 4.2.* **(10 points)**
Consider again the system from exercise 4.2, that is N particles placed on lattice sites with each particle having a spin $\frac{1}{2}$ with an associated magnetic moment μ_B, placed in a homogenous, constant external magnetic field $\vec{B} = B\,\vec{e}_z$. We now want to use the canonical ensemble to calculate thermodynamic quantities.

a) Calculate the canonical partition function $Z(T, B) = \sum_{n_1=0}^{N} g_{n_1} e^{-\beta E_{n_1}}$. with g_{n1}= degree of degeneracy of the energy values E_{n_1} and n_1 = number of particles with spin up (see exercise 4.2). (2 points)

b) Calculate the average internal energy of the system $\bar{E} = -\frac{\partial}{\partial\beta} \ln Z$ from the partition function $Z(T, B)$. Compare the result with the result of exercise 4.2 d). (1 point)

c) Determine the average total magnetic moment $\bar{M} = \langle 2\mu_B \sum_{i=1}^{N} \hat{S}_i^z \rangle$.
Consider first how one gets $\bar{M}$ from the partition function Z. Compare the result with the result of exercise 4.2 e). (2 points)

d) Calculate the entropy of the system $S(T, B) = k_B\,(1 - \beta\frac{\partial}{\partial\beta}) \ln Z(T, B)$
Compare with the result of exercise 4.2. b). What emerges for $S(T, B)$ in the case $B = 0$?
. (4 points)

e) Check whether the 3. law of thermodynamics is fulfilled,
i.e. whether $S(T, B) \longrightarrow 0$ for $T \to 0$. (1 point)

Exercise 5.3: **(4 points)**
A zipper has N links, where each link has a closed state with energy 0 and an open one with energy ϵ. In addition, we require that the zipper can only be opened from the left end and that a link only opens, if all links to its left are already opened. Assume $N \gg 1$.

a) Calculate the canonical partition function. (2 points)

b) Determine the average number of open links in the limit
$\beta\epsilon \gg 1$ for a given temperature T (2 points)

This system is a simple model for the separation of a DNS-molecule into its two strands.