6. Exercise sheet for the lecture Statistical Physics

Hand in your solutions on Thursday, 25.11.2010 or Friday, 26.11.2010 in the exercise group.

Exercise 6.1 *Heating the air in a room* **(5 points)**
Consider heating up the air in a room from 0°C to 20° C. During the heating process the airpressure in the room stays constant (at the value of the airpressure outside). Assume the room has a volume of 100 m^3. Compute the change of internal energy and the change of entropy of the air in the room.
Hint: Consider the air in the room as an ideal gas. The entropy / mol of air at 0° C and 1hPa pressure is S =196 J/(mol K).

Exercise 6.2 *Quantummechanical harmonic oscillator in an heat bath* **(10 points)**
Consider a one dimensional quantummechanical harmonic oscillator with the energy levels $E_n = \hbar\omega(n + \frac{1}{2})$ in an heat bath of temperature T.

a) Calculate the canonical partition function Z. (2 points)

b) Calculate the free energy F and the entropy S of the system. (2 points)

c) Calculate the heat capacity $C_V = T\left(\frac{\partial S}{\partial T}\right)_V$. Find an approximation for low temperatures. (2 points)

Now consider a three dimensional isotropic quantummechanical harmonic oscillator with energy levels

$$E_{n_1,n_2,n_3} = \hbar\omega(n_1 + n_2 + n_3 + \frac{3}{2})$$

where each of n_1, n_2 and n_3 can be $0, 1, 2, 3, ...$

d) Find the degeneracies of the levels of energy $\frac{7}{2}\hbar\omega$ and $\frac{9}{2}\hbar\omega$. (2 points)

e) Given that the system is in thermal equilibrium with a heat bath at a temperature T, show that the $9\hbar\omega/2$ level is more populated than the $7\hbar\omega/2$ level if k_BT is larger than $\hbar\omega/\ln(5/3)$. (2 points)

Exercise 6.3 *Rigid rotator - model for a molecule* **(7 points)**
Consider a molecule, such as carbon monoxide, made up of two different atoms, one carbon and one oxygen separated by a distance d. Such a molecule can exist in quantum states of different orbital angular momentum with each state having energy

$$\epsilon_l = \frac{\hbar^2 l(l+1)}{2I}$$

Here $I = \mu d^2$ is the moment of inertia of the molecule about an axis through its centre of mass and μ the reduced mass given by $1/\mu = 1/m_1 + 1/m_2$. $l = 0, 1, 2, 3, ...$ is the quantum number associated with the orbital angular momentum. Each energy level of the rotating molecule has a degeneracy $g_l = (2l + 1)$.

a) Find the general expression for the canonical partition function. (1 points)

b) Show that at high temperatures it can be approximated by an integral. (2 points)

c) Evaluate the high-temperature mean energy U and heat capacity C_V. (2 point)

d) Find the low-temperature approximations to the canonical partition function , the mean energy U and C_V. (2 points)

Exercise 6.4: *Absorption of atoms on a surface* **(3 points)**

Suppose there is a flat surface of a solid in contact with a vapour which acts as the thermal and particle reservoir. The atoms stick to sites on the solid surface with at most one atom per site. Each site on the surfache of the solid has an energy ϵ if an atom is stuck to it, and an energy of zero if no atom is stuck to it. Assume that there are M sites.

a) Give the energy and the degeneracy of the system, if there are $N < M$ atoms trapped on the M sites. (1 point)

b) Calculate the grand partition function Z_{gc} of the system. (2 points)

Schlimmstenfalls kann ich mir noch vorstellen,
dass Gott eine Welt hätte schaffen können,
in der es keine natürlichen Gesetze - also kurz gesagt:
ein Chaos - gibt. Aber dass es statistische Gesetze
mit endgültigen Lösungen geben soll,
d.h. Gesetze, die Gott in jedem einzelnen Fall
zwingen zu würfeln,
das finde ich im höchsten Maße unangenehm.

A. Einstein zu James Franck, zitiert von
C. P. Snow in French, Einstein, S. 67

aus "Einstein sagt",S. 152, Herausgegeben von Alice Calaprice, Piper Serie