University of Heidelberg WS 2010/2011

7. Exercise sheet for the lecture Statistical Physics

Hand in your solutions on Thursday, 02.12.2010 or Friday, 03.12.2010 in the exercise group.

Exercise 7.1: *Reversible isotherm and irreversible free expansion of an ideal gas* **(4 points)**

a) Consider the reversible, isotherm expansion of an ideal gas (see fig. 1).
Calculate
1.) the change in entropy of the gas ΔS_{gas};
2.) the change in entropy of the heat reservoir $\Delta S_{\text{reservoir}}$;
3.) the change in entropy of the total system $\Delta S_{\text{total}} = \Delta S_{\text{gas}} + \Delta S_{\text{reservoir}}$

b) Consider now the free expansion of an ideal gas (see fig. 2).
Calculate as in exercise 7.1 a):
1.)the change in entropy of the gas ΔS_{gas};
2.)the change in entropy of the heat reservoir $\Delta S_{\text{reservoir}}$;
3.)the change in entropy of the total system $\Delta S_{\text{total}} = \Delta S_{\text{gas}} + \Delta S_{\text{reservoir}}$

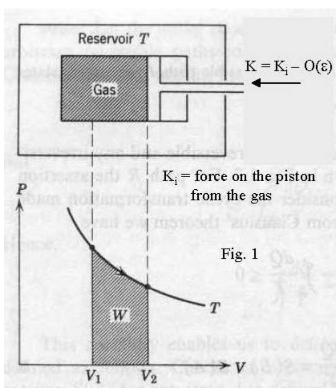


Fig. 1

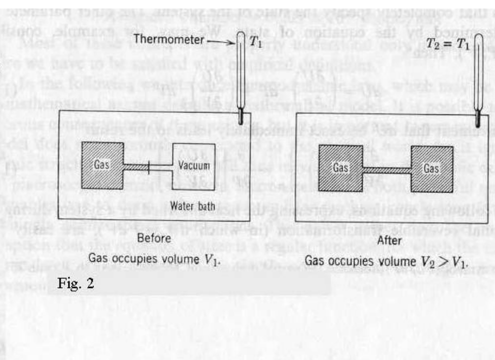


Fig. 2

Exercise 7.2: *Carnot cycle* **(8 points)**

The Carnot cycle of an ideal gas is made up of four reversible processes (see fig. 3): First there is an isothermal expansion of the system ($V_1 \to V_2, p_1 \to p_2, T = \theta_1 = \text{const.}$) with heat Q_1 being absorbed by the system from the thermal reservoir at constant temperature θ_1 (the process is shown by the line from A to B in fig. 3); then there is an adiabatic expansion of the system with the temperature falling from temperature θ_1 to θ_2 ($\theta_1 \to \theta_2, V_2 \to V_3, p_2 \to p_3$, indicated by the line from B to C); next, an isothermal compression of the system at temperature θ_2 with heat Q_2 flowing into the second thermal reservoir (C to D: $V_3 \to V_4, p_3 \to p_4, T = \theta_2 = \text{const.}$); and finally, an adiabatic compression of the system with the temperature rising from θ_2 to θ_1 (D to A: $V_4 \to V_1, p_4 \to p_1, \theta_2 \to \theta_1$).

a) Calculate Q_1 as a function of θ_1, V_2 and V_1 and Q_2 as a function of θ_2, V_4 and V_3.
(2 points)

b) Show that $Q_1/\theta_1 + Q_2/\theta_2 = 0$. (2 points)

c) What is the net work W output during the cycle? The efficiency of an enginge is defined as the work output per cycle divided by the heat input per cycle from the hotter reservoir: $\eta = W/Q_1$. Show that the efficiency of the Carnot cycle is given by $\eta = 1 - \theta_2/\theta_1$.
(2 points)

d) Sketch the Carnot cycle in the $S-T$-diagram. What is the meaning of the enclosed area? What is the meaning of the enclosed area in the $p-V$-diagram? (2 points)

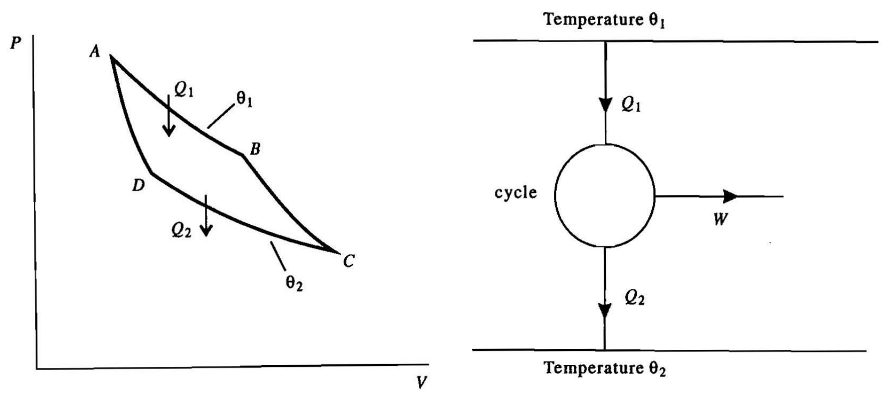
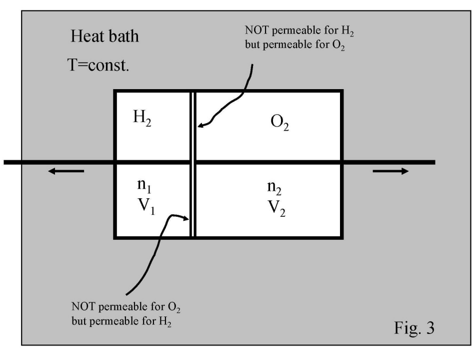


Fig. 3

Exercise 7.3: *Entropy of mixing* **(2 points)**

Consider the situation represented in fig 3, which arises in the reversible conduct of the *Knallgas* reaction. Both the H_2 gas and the O_2 gas can be regarded as ideal gases. The temperature of both gases is T. At constant temperature T, the gases are now reversibly isothermally mixed as follows (see fig 3):The pistons are permeable only for H_2 or for O_2, respectively. The pistons are very slowly moved. In the case of this mixture the system will take up an amount of heat Q from the heat bath of the temperature T and the entropy of the gas will increase by $\Delta S = \frac{Q}{T}$. Calculate the increase in entropy ΔS.



Exercise 7.4: **(6 points)**

Consider ideal gases of bosons as well as of fermions. In this case, the grand canonical partition function for both systems can be summarized by (compare with the expressions derived in the lecture),

$$\ln Z_{gc} = \mp \sum_{\nu} \ln\left(1 \mp e^{-\beta(E_\nu - \mu)}\right), \qquad (*)$$

where for the bose gas the upper signs and for the fermi gas the lower signs apply, respectively.

a) Calculate for both cases

$$\bar{N} = \sum_{\nu} \bar{n}_\nu = \frac{1}{\beta}\frac{\partial}{\partial \mu} \ln Z_{gc}$$

and read off the mean occupation numbers $\bar{n}_\nu$ of the states from the result. **(1 point)**

b) Calculate the entropy

$$S = k\left(1 - \beta\frac{\partial}{\partial \beta}\right) \ln Z_{gc}$$

for both cases. Derive an expression for S which only contains the mean occupation number $\bar{n}_\nu$. What do you get for the entropy S in the classical limit $\bar{n}_\nu \ll 1$? **(5 points)**