8. Exercise sheet for the lecture Statistical Physics

Hand in your solutions on Thursday, 09.12.2010 or Friday, 10.12.2010 in the exercise group.

Exercise 8.1:*Diluted ideal gas, classical limit* **(7 points)**

a) Calculate $\ln(Z_{gc})$ (cf. (*) from exercise 7.4) for free particles in a cube with edge length L in the thermodynamic limit for $\mu < 0$ and $e^{-\beta(E_\nu - \mu)} \ll 1$. In doing so, substitute the sum over the one-particle states by an integral and expand the logarithm.
What do you get for fermions and bosons, respectively ? (5 points)

b) Use the result from a) to calculate the average number of particles $\bar{N}$ as well as the average energy $\bar{E}$. Express $\bar{E}$ in terms of $\bar{N}$ and T. (2 points)

Exercise 8.2:*Photon gas and thermodynamic potentials* **(6 points)**

The canonical partition function for a photon gas was derived in the lecture. It was shown that

$$\ln Z_{can} = \frac{\pi^2 V}{45(\hbar c)^3}\frac{1}{\beta^3} = \frac{1}{3}\frac{\sigma}{k^4}V\frac{1}{\beta^3}$$

with the *Stefan-Boltzmann constant*

$$\sigma = \frac{\pi^2 k^4}{15(\hbar c)^3}\,.$$

a) Calculate (3 points)
   - the free energy $F(T,V) = -\frac{1}{\beta}\ln Z_{can}(\beta,V)$,
   - the mean entropy $\bar{S}(T,V) = k(1-\beta\frac{\partial}{\partial\beta})\ln Z_{can}(\beta,V)$
   - as well as the mean energy $\bar{E}(T,V) = -\frac{\partial}{\partial\beta}\ln Z_{can}(\beta,V)$.

b) Determine the thermodynamic potential $E(S,V)$. (1 point)

c) Calculate the pressure p of the photon gas as a function of the energy density $\epsilon = \frac{E}{V}$. (1 point)

d) What do you get for the so-called enthalpy $G = F + pV$ of the photon gas ? (1 point)

Exercise 8.3: *Stefan-Boltzmann law* **(4 points)**

a) Calculate the temperature of the surface of the earth under the assumption that the sun and the earth can approximately be treated as black bodies. (The surface temperature of the sun is 5500 K, its radius is $r = 7 \cdot 10^8$ m and the distance between the sun and the earth is $d_E = 1{,}5 \cdot 10^{11}$ m.) (2 points)

b) How should the surface temperature of the other planets depend on their distance d_P from the sun under the same assumption as above? Compare with the data

	Merkur	Venus	Mars	Jupiter	Saturn	Uranus	Neptun
d_P/d_E	0,4	0,7	1,5	5,2	9,5	19,3	30,2
Surface/atmosphere-T $[K]$	~ 500	731	214	163	134	70	70

(2 points)

Exercise 8.4: *Cosmic Microwave Background* **(4 points)**

Imagine the Universe to be a spherical cavity with radius 10^{26}m. If the temperature in the cavity is 3 K, estimate the total number of thermally excited photons in the Universe, and the energy content of these photons.

Hint:

$$J_m := \int_0^\infty \frac{x^m}{e^x - 1} dx = \int_0^\infty x^m (e^{-x} + e^{-2x} + e^{-3x} + ...) dx = \Gamma(m+1) \sum_{n=1}^{\infty} \frac{1}{n^{m+1}}$$

[We consider] the distribution of the energy U among N oscillators of frequency ν. If U is viewed as divisible without limit, then an infinite number of distributions are possible. We consider however - and this is the essential point of the whole calculation - U as made up of an entirely determined number of finite equal parts, and we make use of the natural constant $h = 6.55 \times 10^{-27}$ erg sec. This constant when multiplied by the common frequency of the oscillators gives the element of energy in ergs ...

Max Planck

Chancen

Nach einem Kolloquim, in dem Hans KOPFERMANN von Experimenten zur Bestimmung der Planckschen Konstante berichtet hatte, stellte NERNST einige Fragen. Man wußte, dass er selbst 1932, als die Quantentheorie längst allgemein akzeptiert wurde, noch Zweifel hatte. Nachdem KOPFERMANN seine Fragen hinreichend beantwortet hatte, sagte NERNST laut zu dem neben ihm sitzenden MAX PLANCK: *Na, Herr Kollege, dann haben Sie ja noch Chancen!*

Aus *Anita Ehlers: Liebes Hertz! Physiker und Mathematiker in Anekdoten*