Lagrangian Bias in the Local Bias Model

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Based on
Noemi Frusciante & Ravi K. Sheth, JCAP 1211(2012)016

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Motivations

The relationship between the distribution of mass and that of galaxies depends on the complex, nonlinear process of galaxy formation. Bias is an obstacle to the comparison between perturbation theories and observations. (Springel et al., 2005)
Motivations

- Light does not trace the mass → BIAS
- The relationship between the distribution of mass and that of galaxies depends on the complex, nonlinear process of galaxy formation
- Bias is an obstacle to the comparison between perturbation theories and observations

Springel et al. (2005)
The Bias Factor I

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Normalizing density rather than overdensity

Renormalized Bias

Conclusions
Define the dimensionless density perturbation field $\delta(\bar{x}) = \frac{\rho(\bar{x}) - \bar{\rho}}{\bar{\rho}}$
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**Bias types:**

1. **Linear and deterministic (accurate on large scale)**
   
   $\delta_b = b \delta_m$

2. **Non-linear, deterministic**
   
   $\delta_b = b(\delta_m) \delta_m$

3. **Stochastic**
   
   $\delta_b \neq b \langle \delta_b | \delta_m \rangle$

---

REAL BIAS is non-linear, stochastic and quite deterministic see Dekel & Lahav (1999), Sheth & Lemson 1999, Tegmark & Bromley (1999)
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Models:

- **Eulerian Bias**: relation between the final mass density field and the final number density field
- **Lagrangian Bias (halo model, peaks model)**: relation between the initial mass density field and the initial number density field

Matsubara (2011)
The Local Bias Model

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The Local Bias Model

Introduced by Fry & Gaztanaga (1993)

Assumptions: non linear, local and deterministic

\[ \delta_b = f(\delta_m) = \sum_{i > 0} b_i i! (\delta_i - \langle \delta_i \rangle), \]

Note that this model ensures \( \langle \delta_b \rangle = 0 \) by subtracting-off the terms.

It is used to describe the bias with respect to the Eulerian field BUT it is often assumed to describe the bias with respect to the initial Lagrangian field: \( \delta_L \).
The Local Bias Model

- *Introduced by Fry & Gaztählenaga (1993)*

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The Model

On large enough smoothing scales, where the fluctuations are small, this relation can be expanded in a Taylor series

$$\delta_b = f(\delta_m) = \sum_{i>0} \frac{b_i}{i!} (\delta_m^i - \langle \delta_m^i \rangle),$$

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The Local Bias Model

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The Model

On large enough smoothing scales, where the fluctuations are small, this relation can be expanded in a Taylor series

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\]

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- It is used to describe the bias with respect to the Eulerian field BUT it is often assumed to describe the bias with respect to the initial Lagrangian field: \( \delta_L \).
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Problem

PROBLEM: The cross correlation between the biased tracers and the initial field involves higher order terms

\[ \langle \delta_b \delta_L \rangle = \sum_{k>0} b_k k! \langle \delta_{k+1} \rangle \]

We expect (e.g. Peaks and Patches which form halos)

\[ \langle \delta_b \delta_L \rangle = b \langle \delta_L^2 \rangle \]
PROBLEM: The cross correlation between the biased tracers and the initial field involves higher order terms

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Problem

- **PROBLEM:** The cross correlation between the biased tracers and the initial field involves higher order terms

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\langle \delta_b\delta_L \rangle = \sum_{k>0} \frac{b_k}{k!} \langle \delta^{k+1}_L \rangle
\]

- We expect (e.g. Peaks and Patches which form halos)

\[
\langle \delta_b\delta_L \rangle = b \langle \delta^2_L \rangle
\]
Normalizing density rather than overdensity
The correctly normalized bias field is defined by

\[ \delta_B \equiv \frac{1 + \delta_b - \langle 1 + \delta_b \rangle}{\langle 1 + \delta_b \rangle} = \frac{\sum_{k=1}^{\infty} (b_k/k!) (\delta^k_L - \langle \delta^k_L \rangle)}{\sum_{k=0}^{\infty} (b_k/k!) \langle \delta^k_L \rangle} \]
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The cross correlation is

\[ \langle \delta_L^r \delta_B | r \rangle = \xi_{LL'}(r) \frac{\sum_{k=1}^{\infty} (b_k/k!) \langle \delta_L^{k+1} \rangle / \langle \delta_L^2 \rangle}{\sum_{k=0}^{\infty} (b_k/k!) \langle \delta_L^k \rangle} \equiv \xi_{LL'}(r) B_L, \]

Note: this is an exact statement, valid for any \( r, L \) or \( L' \), and for any local deterministic bias function.
The correctly normalized bias field is defined by

\[
\delta_B \equiv \frac{1 + \delta_b - \langle 1 + \delta_b \rangle}{\langle 1 + \delta_b \rangle} = \frac{\sum_k \infty (b_k/k!)(\delta^k_L - \langle \delta^k_L \rangle)}{\sum_k \infty (b_k/k!)\langle \delta^k_L \rangle}
\]

- The cross correlation is

\[
\langle \delta_{L'}\delta_B | r \rangle = \xi_{LL'}(r) \frac{\sum_k \infty (b_k/k!)(\delta^{k+1}_L)/\langle \delta^2_L \rangle}{\sum_k \infty (b_k/k!)\langle \delta^k_L \rangle} \equiv \xi_{LL'}(r) B_L
\]

Note: this is an exact statement, valid for any \( r, L \) or \( L' \), and for any local deterministic bias function.

- The auto-correlation function of the biased tracers will reduce to a series of the form [Similar result by Szalay (1988)]

\[
\langle \delta_B'\delta_B | r \rangle = B^2_L \xi_{LL}(r) + \frac{C_L}{2} [\xi_{LL}(r)]^2 + \ldots
\]
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McDonald (2006) suggests redefining the mean density, and hence all bias factors order by order.

\[
\delta_j B = 1 + \delta_b - \langle 1 + \delta_b \rangle_j \langle 1 + \delta_b \rangle_j \sum_{j,k=1} \left( \frac{b_k}{k!} \left( \delta_L^k - \langle \delta_L^k \rangle \right) \right) \sum_{j,k=0} \left( \frac{b_k}{k!} \langle \delta_L^k \rangle \right).
\]

The cross-correlation between the mass and biased fields is

\[
\langle \delta_L' \delta_j^B \rangle_r = \sum_{j,k=1} \left( \frac{b_k}{k!} \langle \delta_L^k \delta_L^k \rangle_r \right) \sum_{j,k=0} \left( \frac{b_k}{k!} \langle \delta_L^k \rangle \right).
\]

To 4th order in \( \delta_L \), this is

\[
\langle \delta_L' \delta_j^{B(4)} \rangle_r = \left[ b_1 + \left( b_3^2 - b_1 b_2^2 \right) \langle \delta_L^2 \rangle \right] \xi_{LL'}(r) \equiv b_4 \times \xi_{LL'}(r).
\]
What do you do when this cannot be done analytically?
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Mc Donald (2006) suggests redefining the mean density, and hence all bias factors order by order

$$\delta_{B}^{(j)} = \frac{1 + \delta_{b} - \langle 1 + \delta_{b} \rangle_{j}}{\langle 1 + \delta_{b} \rangle_{j}} = \frac{\sum_{k=1}^{j} (b_{k} / k!) (\delta_{L}^{k} - \langle \delta_{L}^{k} \rangle)}{\sum_{k=0}^{j} (b_{k} / k!) \langle \delta_{L}^{k} \rangle}.$$
Renormalized Bias I

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Mc Donald (2006) suggests redefining the mean density, and hence all bias factors order by order

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The cross-correlation between the mass and biased fields is

\[ \langle \delta'_L, \delta^{(j)}_B | r \rangle = \frac{\sum_{k=1}^j (b_k/k!\langle \delta^k_L \delta_L | r \rangle)}{\sum_{k=0}^j (b_k/k!\langle \delta^k_L \rangle)} . \]
Renormalized Bias I

**What do you do when this cannot be done analytically?**

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- **The cross-correlation between the mass and biased fields is**

\[
\langle \delta^j_L, \delta_B \rangle_r = \frac{\sum_{k=1}^{j} (b_k/k!)(\delta_k^L \delta_L | r)}{\sum_{k=0}^{j} (b_k/k!)(\delta_k^L)}.
\]

- **To 4th order in \( \delta_L \), this is**

\[
\langle \delta^4_L, \delta_B \rangle_r = \left[ b_1 + \left( \frac{b_3}{2} - \frac{b_1 b_2}{2} \right) \langle \delta_L^2 \rangle \right] \xi_{LL^\prime}(r) \equiv b_{\times}(4) \xi_{LL^\prime}(r)
\]
Renormalized Bias II

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The Bias coefficient is defined as

$$b(4) \equiv b_1 + (b_3^2 - b_1b_2^2) \langle \delta^2 \rangle$$

Note: The Cross-correlation does not include higher order terms!

Examples:
Lognormal field:

$$1 + \delta_b = \exp(b \delta_L) \exp(-b_2^2 \langle \delta^2_L \rangle / 2)$$

$$b_3 = b_1b_2 \rightarrow b(4) \equiv b_1 = B_L$$

Peaks:

$$1 + \delta_p = \exp(b \delta_L - c \langle \delta^2_L \rangle ) \sqrt{1 + c \langle \delta^2_L \rangle } \exp(-b_2^2 \langle \delta^2_L \rangle / 2)$$

$$b_3 \neq b_1b_2 \rightarrow b(4) = b(1 - c \langle \delta^2_L \rangle )$$

$$B_L = b_1 + c \langle \delta^2_L \rangle$$
The Bias coefficient is defined

\[ b^{(4)}_\times \equiv b_1 + \left( \frac{b_3}{2} - \frac{b_1 b_2}{2} \right) \langle \delta_L^2 \rangle \]

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Renormalized Bias II

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- **Peaks**: \( 1 + \delta_p = \exp(b\delta_L - c\delta^2_L/2) \sqrt{1 + c\langle \delta^2_L \rangle} \exp \left( \frac{-b^2\langle \delta^2_L \rangle/2}{1 + c\langle \delta^2_L \rangle} \right) \)

  \[ b_3 \neq b_1 b_2 \quad \rightarrow \quad b^{(4)}_\times = b(1 - c\langle \delta^2_L \rangle) \quad \rightarrow \quad B_L = \frac{b}{1 + c\langle \delta^2_L \rangle} \]
Assuming that the Lagrangian bias is local and deterministic with respect to the initial Gaussian field, we showed that with a CORRECT NORMALIZATION
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- The two point cross-correlation between the tracer and mass is always only linearly proportional to the auto-correlation signal of the DM.
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- The auto correlation function of locally biased tracers can be written as a Taylor series in the auto-correlation function of the mass.
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- These relations allow for simple tests of whether or not halo bias is indeed local in Lagrangian space.
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- These relations allow for simple tests of whether or not halo bias is indeed local in Lagrangian space.

**Note:** The multiplicative normalization holds also for Eulerian mass field although providing an explicit expression is more complicated.