Lecture 5: The kind of dark matter

Based on observations, we have collected evidence that particle dark matter should be
- electrically neutral
- stable compared to the lifetime of our universe
- "cold", i.e., non-relativistic at the time of decoupling.

Big open questions remain:
- What are its mass, spin, couplings?
- Does it interact with itself?
- When was it formed?
- Is there an entire dark sector?

Standard-model neutrinos are neutral and the lightest neutrino is stable.
- Could neutrinos be dark matter?

For relativistic fermions in thermal equilibrium, the yield is given by

\[ Y_\nu = \frac{n_\nu(T)}{s(T)} = \frac{45}{2\pi^4} \, \xi(3) \, \frac{5/4 \, \frac{g}{g_{\ast s}(T)}}{g_{\ast s}(T)} \approx 0.21 \, \frac{g}{g_{\ast s}(T)}. \]

Notice that it does not explicitly depend on \( T \) (unlike for non-relativistic particles).

On Problem Set 2, you will show that the energy density of the lightest neutrino is given by

\[ \sum_\nu \xi \frac{m_\nu}{94 \, eV} \Rightarrow \sum_\nu \xi \frac{m_\nu}{94 \, eV} \]
Today's strongest limit on the sum of neutrino masses has recently been set by the KATRIN experiment in tritium β-decay,

\[ \sum m_{\nu_i} < 1.1 \text{eV} \text{ @ 90\% CL.} \]

This implies \( \mathcal{N}_\nu h^2 < 0.02 \). The relic abundance of neutrinos is thus way too small to account for all of the dark matter.

How "warm" can dark matter be?

**Def.**
- Cold dark matter is non-relativistic before radiation-matter equality, i.e., at \( T > T_{eq} \approx 5 \text{eV} \).
- Hot dark matter is relativistic at \( T > T_{eq} \).

Structures grow in the universe during matter domination, i.e., at \( T < T_{eq} \).

The characteristic scale for the smallest possible structures is the free-streaming length of a collisionless particle between regions of over- and under-density,

\[ \lambda_{FS}(t) = \int_0^t \int_0^t \frac{k}{a(t')} \, dt'. \]

The free-streaming length today is then given by

\[ \lambda_{FS}(t_0) \approx 1 \text{Mpc} \left( \frac{k \text{eV}}{m_{\nu}} \right) \left( \frac{T_0}{T_f} \right), \]

assuming that the species is relativistic during freeze-out.
Particles lighter than \( m_\chi \approx 1 \text{keV} \) wash out structures smaller than 1 Mpc, which is about the smallest observed scale.

- \( m_\chi > 1 \text{keV} \): hot dark matter - no small-scale structures
- \( m_\chi = 1 \text{keV} \): warm dark matter - wash-out
- \( m_\chi > 1 \text{keV} \): cold dark matter - in principle arbitrary small structures

For neutrinos: \( \tau_\nu / \tau_\chi \approx 0.7 \); \( \Sigma_i m_{\nu_i} < 1.1 \text{eV} \)

\( \rightarrow \lambda_{\nu\chi} \approx 600 \text{ Mpc} \).

Neutrinos are too warm to account for small-scale structures.

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When was the relic abundance formed?

Strong bound on the presence of extra light degrees of freedom is obtained from the observed abundance of light elements.

**Big Bang nucleosynthesis:**

\( T \approx 1 \text{MeV} \):

\[ n + \nu_e \leftrightarrow p + e^- \quad \text{in thermal equilibrium} \]

\[ n + e^+ \leftrightarrow p + \nu_e \]

\[ n + p \leftrightarrow ^2\text{H} + \gamma \]

\[ ^2\text{H} + p \leftrightarrow ^3\text{He} + \gamma \]

\[ ^2\text{H} + ^3\text{He} \leftrightarrow ^4\text{He} + p \]
neutron abundance at time of nucleosynthesis: \( R_n(t_{\text{nucl}}) \approx \frac{1}{6} e^{-t_{\text{nucl}}/\tau_n} \approx \frac{1}{6} \)

\( \tau_n \): neutron lifetime; \( \tau_n = 886.7 \pm 0.88 \) s

\( t_{\text{nucl}} = 330 \) s

Since two neutrons go into one helium, the mass fraction of helium is

\[
\frac{4n_{\text{He}}}{N_H} = \frac{2n_n}{N_p} \approx \frac{2 R_n(t_{\text{nucl}})}{1 - R_n(t_{\text{nucl}})} \approx \frac{1}{4}
\]

New light degree of freedom increase the number of effective d.o.f. during neutrino freeze-out:

\[
H \sim \sqrt{g_*} \frac{T_v^2}{M_{\text{pl}}} \sim T_v \sim \frac{g_*^2}{T_v} T_v^5 \rightarrow T_v \sim g_*^{1/6}
\]

A larger \( g_* \) would increase the neutrino freeze-out temperature, thus the \( n/p \) ratio and thus the helium abundance.

\( \rightarrow \) Dark matter should leave thermal equilibrium before big bang nucleosynthesis.