Complementary to searches for dark matter annihilation, we can try to detect dark matter scattering off hadronic nuclei or interacting with the electrons bound in atoms.

Direct detection experiments are typically built to measure the recoil energy of the nucleus, \( E_R \).

\[
\begin{align*}
E_x &= \frac{p_x^2}{2m_x} ; \quad p_x = m_x v \\
E_f &= \frac{(\vec{p}_x - \vec{p}_N)^2}{2m_x} + \frac{\vec{p}_N^2}{2m_N} \\
E_R^{\text{max}} &= \frac{\vec{p}_N^2}{2m_N} = \frac{2m_x m_N}{m_N} ; \quad \mu = \frac{m_x m_N}{m_x + m_N}
\end{align*}
\]

example Xe: \( m_N = 120 \text{ GeV} \) \( \Rightarrow m_x \sim 10 \text{ GeV} \); \( E_R \sim \text{few keV} \)
dark matter velocity \( v \sim 10^{-3} \text{c} \) \( \Rightarrow m_x \sim 1 \text{ TeV} \); \( E_R \sim 100 \text{ keV} \)

\[ E_R \sim 1\,\text{ keV} \]

\[ \langle 1/|\vec{q}| \rangle \sim \text{fm} \] (size of a nucleus)

- What scattering rate can we expect?

\[ \frac{d\sigma}{dE_R} = \frac{A}{m_N m_x} \int_{v_{\text{min}}}^{v_{\text{max}}} d\vec{v} \cdot \vec{P}(\vec{v}) \frac{d\sigma}{dE_R} \]

\[ \langle \vec{P}(\vec{v}) \rangle \sim \text{DM-nucleus scattering cross section} \]

\[ \langle \vec{v} \rangle \sim \text{DM velocity distribution in rest frame} \] of the target
The velocity distribution

\[ f(v) = f(v^2 + \bar{v}_{\text{kin}}^2 + \bar{v}_{\text{rec}}^2) \]

is obtained by boosting \( f(v) \) from the Galactic frame to the lab frame.

\[ v_{\text{max}} = v_\odot \sim 500 \text{ km/s} : \text{Galactic escape velocity} \]

\[ v_{\text{min}} = \sqrt{\frac{m_{\nu} E_r}{2 \mu^2}} : \text{minimum velocity for scattering with recoil energy } E_r \]

**Standard Halo Model:**
Maxwellian distribution

\[ f(v) \sim \exp(-v^2/v_0^2); \quad v_0 \approx 220 \text{ km/s} \]

The scattering rate drops for light WIMPs due to:
- a large minimum velocity and for heavy WIMPs due to
  - a low number density \( n_x = \frac{S_\odot}{m_x} \)
  - a suppressed cross section \( \sigma \) at high recoil energy.

- What is the cross section of DM-nucleus scattering?

Depending on the fundamental interactions, dark matter can either scatter coherently off the entire nucleus or scatter off individual nucleons:

- **Spin-independent**

  \[ \frac{\Delta A^2}{A^2} \]

- **Spin-dependent**

  \[ \frac{\Delta \Sigma}{A^2} \sim \frac{\mathcal{F}(A+1)}{A^2} \]
Spin-independent scattering
differential cross section
\[ \frac{d\sigma}{dE_R} = \frac{2m_n}{\pi v^2} \left| f_p + (A - Z) f_n \right|^2 F^2(E_R) \]

\( Z \) : atomic number
\( A \) : mass number
\( f_p, f_n \) : DM coupling to protons and neutrons
\( F(E_R) \) : nuclear form factor

- For DM interactions that are blind to the weak isospin,
  \( f_p = f_n \implies \frac{d\sigma}{dE_R} = \frac{2m_n}{\pi v^2} \left| \sum_n A^2 F^2(E_R) \right| \)

\( \sigma_n \) : DM-nucleon cross section; \( \kappa = 1 \ (4) \) for Majorana (Dirac) DM
\( \mu_n = m_p m_n / (m_p + m_n) \) : reduced DM-nucleon mass

Fermionic dark matter with
scalar nucleon interaction:
\[ \sigma_n = 4 \mu^2 \frac{\alpha^2}{M^2} \]

Current experimental upper bound:
\[ \sigma_n (m_x = 30 \text{GeV}) \leq 10^{-46} \text{cm}^2. \]

- How can we relate \( f_{n,p} \) to fundamental interactions?

For \( |q| \sim \text{keV} \ll m_x \), we can describe the DM interactions with quarks and gluons in an effective theory.

\[
\begin{align*}
\chi & \rightarrow m_x \rightarrow q \\
\chi & \rightarrow q^2 \rightarrow m_x \\
\chi & \rightarrow \frac{q}{q^2} \rightarrow m_x \\
\chi & \rightarrow \frac{q}{q^2} \rightarrow m_x
\end{align*}
\]

\[ \text{Left} = g_{\chi q} \left( \chi T^a \chi \right) \left( q T^b \frac{q}{q^2} \right) \]

\[ T^a_i = \sum_{i} \left( \chi \lambda_{i} \chi \right) \left( q_{i} \frac{q_{i}}{q^2} \right) \]

\[ g_{\chi q} \sim g_{\chi q} \left( \frac{1}{M^2} \right) \]
Consider the scalar interaction

\[ \mathcal{L}_{\text{eff}} = \mathcal{G}_{\text{eff}}(\bar{q}q)(\bar{q}q). \]

The dark matter interaction with the nucleus \( N \) is then

\[ f_n = \sum_{q = u, d, s} \frac{e^2}{m_q} f_q^n + \frac{2}{27} \left( \sum_{q = d, s} \frac{e^2}{m_q} f_q^n \right), \]

with the nuclear matrix element

\[ m_n f_q^n \bar{u}_n u_n = \langle N | m_q \bar{q} q | N \rangle \quad \text{(and similarly for } f_q^n). \]

The scattering amplitude for DM off protons and neutrons is

\[ M_n = f_p \bar{p} p \bar{N} N + f_n \bar{n} n \bar{N} N. \]

Counting the number of protons and neutrons inside the nucleus \( N \), we obtain the nucleus scattering amp.

\[ M_N = \left( 2 f_p + (A - 2) f_n \right) \bar{X} X \bar{N} N F(q^2), \]

where the form factor \( F(q^2) \) describes the distribution of nucleons inside the nucleus at momentum transfer \( q^2 \).

In the non-relativistic limit, where \( q^2 \approx 2 m_n E_R \ll m_n^2 \), the differential cross section is finally obtained as

\[ \frac{d\sigma}{dE_R} = \frac{2m_N}{\pi q^2} \sum_{\text{spins}} \left| \frac{M_N_{\text{neut}, \text{pol}}}{1} \right|^2, \]

averaged/summed over initial/final-state spins.

Since \( d\sigma/dE_R \propto 1/r^2 \), the event rate decreases exponentially with the recoil energy:

\[ \frac{dR}{dE_R} \sim \int d\omega \frac{1}{2} \mathcal{F}(\omega) \frac{d\sigma}{dE_R} \sim \int d\omega \frac{1}{2} v e^{-\omega^2 \sigma^2} e^{-E_R/E_o}; \]

\[ E_o = \frac{2\mu^2 \sigma^2}{m_N}. \]
Spin-dependent scattering

$\text{Left} \sim (\bar{q} q)(\bar{q} q)$ → spin-independent

$\text{Left} \sim (\bar{q} \gamma_{\mu} q)(\bar{q} \gamma_{\nu} q)$ → nuclear scatter momentum suppressed

$\text{Left} \sim (\bar{q} \gamma_{\mu} \gamma_{5} q)(\bar{q} \gamma_{\nu} \gamma_{5} q)$ → spin-dependent

Cross section for spin-dependent scattering

$$\frac{d\sigma_{\text{SD}}}{dE_{R}} = \frac{16\pi m_{N}^{2}}{h^{2} v^{2}} G_{F}^{2} f(f + 1) (a_{p} <S_{p}^{y}> + a_{n} <S_{n}^{y}>)^{2} F_{\text{SD}}^{2}(E_{R})$$

$a_{p/n}$: effective EM-proton, EM-neutron coupling

$<S_{p/n}>$: average contribution of nucleons to nuclear spin $f$

Dark matter - electron scattering

For light dark matter with $m_{\chi} < \text{few GeV}$, the nuclear recoil energy is too low to be detected. Electron scattering can lead to larger momentum transfer (and different detection methods), offering promising alternative opportunities for direct detection.

* Typical momentum of an electron bound in an atom:

$$k_{e} \sim \frac{1}{k_{\text{max}}} \sim \frac{1}{E_{\text{me}}} \sim \text{keV} ; \quad v_{e} \sim k_{e} \sim \frac{1}{4\pi^{2}} \Rightarrow v_{e}$$

→ ionization energy $E_{\text{ion}} \sim \frac{m_{e}^{2} c^{2}}{2} \sim \text{few eV}$.

* $m_{\chi} \gg m_{e}$: momentum transfer $|\vec{q}| \sim m_{\chi} v_{e} \sim \text{few keV}$
energy deposit similar to ionization energy

\[ E_{\text{deposit}} \approx \frac{1}{2} m_e \left( \frac{p_x}{m_x} \right)^2 \approx \frac{(m_x v_x)^2}{2 m_e} \]

- \( m_x \leq M_e \): momentum transfer limited by
  \[ \frac{1}{2} m_x v_x \leq m_e \leq 10^{-3} \text{ keV} \]
  \[ E_{\text{max}} \approx \frac{(m_x v_x)^2}{2 m_e} \]

For \( m_x \geq \text{MeV} \), \( E_{\text{max}} \approx \frac{m_x v_x^2}{2} \), so that dark matter can transfer almost its entire kinetic energy to the electron.

The cross section to excite an electron from state \( i \) = \{ \text{ke}, l, m_j \} to state \( f \) = \{ \text{ke}', l', m_j' \} is given by

\[ \sigma (\text{exc})_{i\rightarrow f} = \frac{3 \alpha}{16 \pi m_e^2} \int \frac{d \Omega}{4 \pi} \delta (\Delta E_{e} - \Delta E_{x}) | f_{i\rightarrow f}(\mathbf{q}) |^2. \]

- \( \sigma_e \): momentum-averaged scattering cross section
- \( f_{BM}(\mathbf{q}) \): form factor describing possible states of a bound electron
- \( f_{i\rightarrow f}(\mathbf{q}) \): form factor absorbing \( q^2 \) dep. of matrix element

- Typical electron scattering cross section of a thermal WIMP (or BM candidate produced through freeze-in):
  \[ \sigma_e \approx 10^{-39} \text{ cm}^2. \]

New experiments like SENSEI can probe this interesting regime in the range \( \text{MeV} \leq m_x \leq \text{GeV} \).