Meson decays

Mesons can decay through weak interactions.

1) leptonic decay, e.g., $\pi^- \rightarrow e^- \bar{\nu}_e$

2) semi-leptonic decay, e.g., $B^+ \rightarrow D^0 l^+ \nu_e$

3) hadronic decay, e.g., $\bar{B}^0 \rightarrow \pi^+ \pi^-$

Separate QCD and electroweak physics: $M_W > 192 \Rightarrow \Lambda_{QCD}$

$\rightarrow$ Chapter IV.

Pion decay

The pion decay amplitude is given by

$$ M = -i \frac{\alpha}{4\pi} G_F V_{ud} f_\pi P_L \bar{u}(p_\mu) \gamma_\mu P_L \nu (p_\nu), $$

with the pion decay constant $f_\pi$. 
$f_\pi$ is the value of a pion-to-vacuum transition, described by a hadronic matrix element

$$<0 | \overline{u} \gamma^\mu \gamma_5 d | \pi^0 (p_\pi) > = -i f_\pi p_\pi^\mu.$$  

Since the pion is a P-odd pseudo-scalar meson, only the axial-vector part of the weak current contributes to its decay.

It can be shown (see Manohar, Wise: „Heavy Quark Physics“) that the pion decay constant is equal to the constant $f$ in the chiral limit of QCD,

$$f = f_\pi = f_K 1.$$  

Experimentally, one finds from

$$\pi^- \rightarrow \mu^- \nu_\mu : f_\pi \approx 131 \text{ MeV}$$  
$$K^- \rightarrow \mu^- \nu_\mu : f_K \approx 164 \text{ MeV}.$$  

The difference between $f_\pi$ and $f_K$ demonstrates once more that $SU(3)_L$ is broken by the strange-quark mass.

**Helicity suppression in pion decay**

Since $m_\pi^+ (140 \text{ MeV}) \gg m_\mu (106 \text{ MeV}) \gg m_e (0.5 \text{ MeV})$, one could expect that the decay $\pi^+ \rightarrow e^+ \nu_e$ is more likely than $\pi^+ \rightarrow \mu^+ \nu_\mu$, the latter being phase-space suppressed. Experimentally, however, one finds

$$\frac{P(\pi^+ \rightarrow e^+ \nu_e)}{P(\pi^+ \rightarrow \mu^+ \nu_\mu)} = (1.230 \pm 0.004) \times 10^{-4}.$$
This result can be understood by considering angular momentum conservation:

\[ h = \frac{1}{2} \]

\[ \nu_L \xrightarrow{\bullet} \pi^+ \rightarrow e^+ \]

\[ J = 0 \]

(reminder: helicity \( h = \frac{\vec{s} \cdot \vec{p}}{|\vec{p}|} \); \( h = \pm \frac{1}{2} \) for massless \{right-handed fermion\}

In the limit \( m_e = 0 \), the anti-lepton \( e^+ \) has helicity \( h = +\frac{1}{2} \), in contradiction with \( J = 0 \).

→ The pion decay is helicity-suppressed,

\[ \frac{T'(\pi^+ \rightarrow e^+\nu_e)}{T'(\pi^+ \rightarrow \mu^+\nu_\mu)} = \frac{m_e^2}{m_\mu^2} \cdot \left(1 - \frac{m_e^2}{m_\mu^2}\right)^2 \left(1 - \frac{m_e^2}{m_\mu^2}\right)^2 = 1.275 \times 10^{-4}. \]

Semi-leptonic meson decays

**Example:** \( B^0 \rightarrow D^+e^-\bar{\nu}_e \)

\[ \bar{B}^0 \xrightarrow{\gamma} B^0 \quad \xrightarrow{\bar{\nu}_e\rightarrow \bar{
u}_e} D^+ \rightarrow \bar{p}' \]

\( q = \bar{p} - p' \) is the momentum transfer to the \( e^-\bar{\nu}_e \) pair.

\[ f_+(q^2) \text{ and } f_-(q^2) \]

The matrix element for the \( B^0 \rightarrow D^+ \) transition is conveniently expressed in terms of Lorentz-invariant formfactors.
\[ \langle D(p') | \bar{c} | \mu | b | \bar{B}^0(p) \rangle = f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu, \]

where \( q^2 = (p-p')^2 = m_b^2 + m_\mu^2 - 2p \cdot p'. \)

Under parity, the pseudo-scalar mesons transform as
\[ P | B(p) \rangle = - | B(-p) \rangle; \quad P | D(p') \rangle = - | D(-p') \rangle. \]

Since strong interactions conserve parity, one has
\[ \langle D(p') | \bar{c} | \mu | b | \bar{B}^0(p) \rangle = - \langle D(-p') | \bar{c} | \mu | b | \bar{B}^0(-p) \rangle, \]
\[ \langle D(p') | \bar{c} | \mu | b | \bar{B}^0(p) \rangle = - \langle D(-p') | \bar{c} | \mu | b | \bar{B}^0(-p) \rangle; \]
\[ i = 1, 2, 3. \]

This is consistent with our ansatz \( \otimes. \)

The axial-vector current does not contribute to \( B \to D \)
decays, since \( p_\mu \) and \( p'_\mu \) are the only Lorentz vectors in
the process. (In \( B \to D^* \), use the polarization vector \( \epsilon_\mu(B^*) \)
to construct an axial-vector matrix element.)

The complete amplitude for \( \bar{B}^0 \to D^+ e^- \bar{\nu}_e \) reads
\[ M = \sqrt{2} \, G_F \, V_{cb} \, f_+(q^2)(p+p')_\mu \, \bar{u}(p_\mu) \gamma^\mu \gamma_5 P_L v(p_\nu). \]

\( f_-(q^2) \) does not contribute for \( m_\mu = 0. \)

The differential decay rate is then given by
\[ \frac{d\Gamma}{dq^2}(\bar{B}^0 \to D^+ e^- \bar{\nu}_e) = \left| G_F \, V_{cb} \, f_+(q^2) \right|^2 \frac{4\pi^3}{92 \, \pi^2 \, m_b^2} \left[ q^2 - m_b^2 - m_D^2 \right]^2 \right]^{3/2}, \]

where \( 0 \leq q^2 \leq (m_b - m_D)^2. \)

The form factor \( f_+(q^2) \) can be calculated using lattice
gauge theory.