Exercise 1: Flavor violation in the lepton sector

In the standard model, the lepton Yukawa interactions with the Higgs vacuum expectation value are given by

\[ \mathcal{L}_y = -\frac{v}{\sqrt{2}} \bar{E}L Y_E E_R + h.c., \]

where \( Y_E \) is a 3 × 3 matrix in flavor space. The weak doublet of left-handed neutrinos and charged leptons is denoted by \( L_L = (N_L, E_L) \), and \( E_R \) is the weak singlet of right-handed charged leptons.

In the basis of gauge eigenstates, the weak charged interactions are

\[ \mathcal{L}_w = \frac{g}{\sqrt{2}} N_L \gamma^\mu E_L W^+_{\mu} + \frac{g}{\sqrt{2}} \bar{E}_L \gamma^\mu N_L W^-_{\mu}. \]

a) Diagonalize the matrix \( Y_E \) by performing unitary transformations on the lepton fields, \( E_L \rightarrow V_{el} e_L, \quad E_R \rightarrow V_{en} e_R. \)

Is there flavor mixing among leptons in weak charged interactions?

b) Now we introduce right-handed neutrinos \( N_R = (1, 0) \). Can you write down a mass term for neutrinos? Does this affect the flavor structure of weak charged interactions?

Exercise 2: CKM elements at work

a) Consider the two decays of the neutral D meson,

\[ D^0(c\bar{u}) \rightarrow K^-(s\bar{u})\pi^+(u\bar{d}) \quad \text{and} \quad D^0(c\bar{u}) \rightarrow K^+(u\bar{s})\pi^-(d\bar{u}). \]

Express the ratio of the branching fractions for the two decays in terms of the CKM matrix elements, i.e.,

\[ \frac{Br(D^0 \rightarrow K^-\pi^+)}{Br(D^0 \rightarrow K^+\pi^-)} = ???. \]

b) The Particle Data Group (PDG) has collected experimental information about meson decays:


Look up the branching fractions \( (\Gamma_i/\Gamma) \) for \( D^0 \rightarrow K^-\pi^+ \) and \( D^0 \rightarrow K^+\pi^- \) and compare the ratio with your result from a). Compare the branching fraction for \( D^0 \rightarrow K^-\pi^+ \) with the semi-leptonic decay \( D^0 \rightarrow K^- e^+\nu_e. \)