Exercise 1: Flavor violation in the lepton sector
In the standard model, the lepton Yukawa interactions with the Higgs vacuum expectation value are given by
\[ \mathcal{L}_y = -\frac{v}{\sqrt{2}} E_L Y_E E_R + h.c., \]
where \( Y_E \) is a 3 \( \times \) 3 matrix in flavor space. The weak doublet of left-handed neutrinos and charged leptons is denoted by \( L_L = (N_L, E_L) \top \), and \( E_R \) is the weak singlet of right-handed charged leptons.
In the basis of gauge eigenstates, the weak charged interactions are
\[ \mathcal{L}_w = \frac{g}{\sqrt{2}} N_L \gamma^\mu E_L W^\mu_L + \frac{g}{\sqrt{2}} E_R \gamma^\mu N_L W^\mu_L. \]

a) Diagonalize the matrix \( Y_E \) by performing unitary transformations on the lepton fields,
\[ E_L \rightarrow V_{eL} e_L, \quad E_R \rightarrow V_{eR} e_R. \]
Is there flavor mixing among leptons in weak charged interactions?

b) Now we introduce right-handed neutrinos \( N_R = (1, 1)_0 \). Can you write down a mass term for neutrinos? Does this affect the flavor structure of weak charged interactions?

Exercise 2: CKM elements at work

a) Consider the two decays of the neutral \( D \) meson,
\[ D^0(c \bar{u}) \rightarrow K^-(s \bar{u})\pi^+(u \bar{d}) \quad \text{and} \quad D^0(c \bar{u}) \rightarrow K^+(u \bar{s})\pi^-(d \bar{u}). \]
Express the ratio of the branching fractions for the two decays in terms of the CKM matrix elements, i.e.,
\[ \frac{\text{Br}(D^0 \rightarrow K^-\pi^+)}{\text{Br}(D^0 \rightarrow K^+\pi^-)} = ???. \]

b) The Particle Data Group (PDG) has collected experimental information about meson decays:
Look up the branching fractions \( (\Gamma_i/\Gamma) \) for \( D^0 \rightarrow K^-\pi^+ \) and \( D^0 \rightarrow K^+\pi^- \) and compare the ratio with your result from a). Compare the branching fraction for \( D^0 \rightarrow K^-\pi^+ \) with the semi-leptonic decay \( D^0 \rightarrow K^- e^+\nu_e \).