Exercise 1: $K^0 - \bar{K}^0$ oscillation probability
Assuming CP invariance, the observed mass eigenstates $K_S$ and $K_L$ are given by the following linear combinations of the flavor states $K^0$ and $\bar{K}^0$,

$$|K_S\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle), \quad |K_L\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle).$$

The time evolution of these physical states is given by $|K_{S,L}(t)\rangle = e^{-i m_{S,L} t} e^{-\Gamma_{S,L} t/2} |K_{S,L}\rangle$, where $m_{S,L}$ and $\Gamma_{S,L}$ are the mass and the total decay width of the respective state.

Assume that at $t = 0$ the system is in a pure flavor state, $K(t = 0) = K^0$. The time-dependent probability to find it in a state $K'$ at time $t$ is defined as

$$P(K(t = 0) \rightarrow K')(t) = |\langle K' | K(t) \rangle|^2.$$

What is the probability to find the system at time $t = t'$ in a pure $K^0$ state? What is the probability to find it in a $\bar{K}^0$ state?

Exercise 2: $K_S - K_L$ interference as confirmation of CP violation
In the presence of CP violation, the physical states $K_S$ and $K_L$ decaying to CP eigenstates can interfere. For a neutral kaon which is produced at $t = 0$ as a $K^0 (\bar{K}^0)$ and propagates freely in vacuum, the time-dependent decay rate to $\pi^+ \pi^-$ is given by

$$\Gamma[K^0 (\bar{K}^0) (t = 0)](t) \propto e^{-\Gamma S t} + |\eta_{\pi\pi}|^2 e^{-\Gamma L t} \pm 2 |\eta_{\pi\pi}| e^{-(\Gamma_S + \Gamma_L) t/2} \cos (\Delta m t - \phi_{\pi\pi}),$$

where the $+$ ($-$) sign applies for the $K^0 (\bar{K}^0)$. The complex number $\eta_{\pi\pi} = |\eta_{\pi\pi}| e^{i \phi_{\pi\pi}}$ describes the CP-violating amplitude ratio

$$\eta_{\pi\pi} = \frac{A(K_L \rightarrow \pi\pi)}{A(K_S \rightarrow \pi\pi)}.$$

a) Motivate the formula above for the time-dependent decay rate.

   - Explain the selection of the $K^0 \rightarrow \pi\pi$ events.
   - How is the proper-time distribution in Figure 4 obtained?
   - How do the authors finally obtain $|\eta_{\pi\pi}|$ and the phase $\phi_{\pi\pi}$?