EFFECTIVE FIELD THEORIES in PARTICLE PHYSICS

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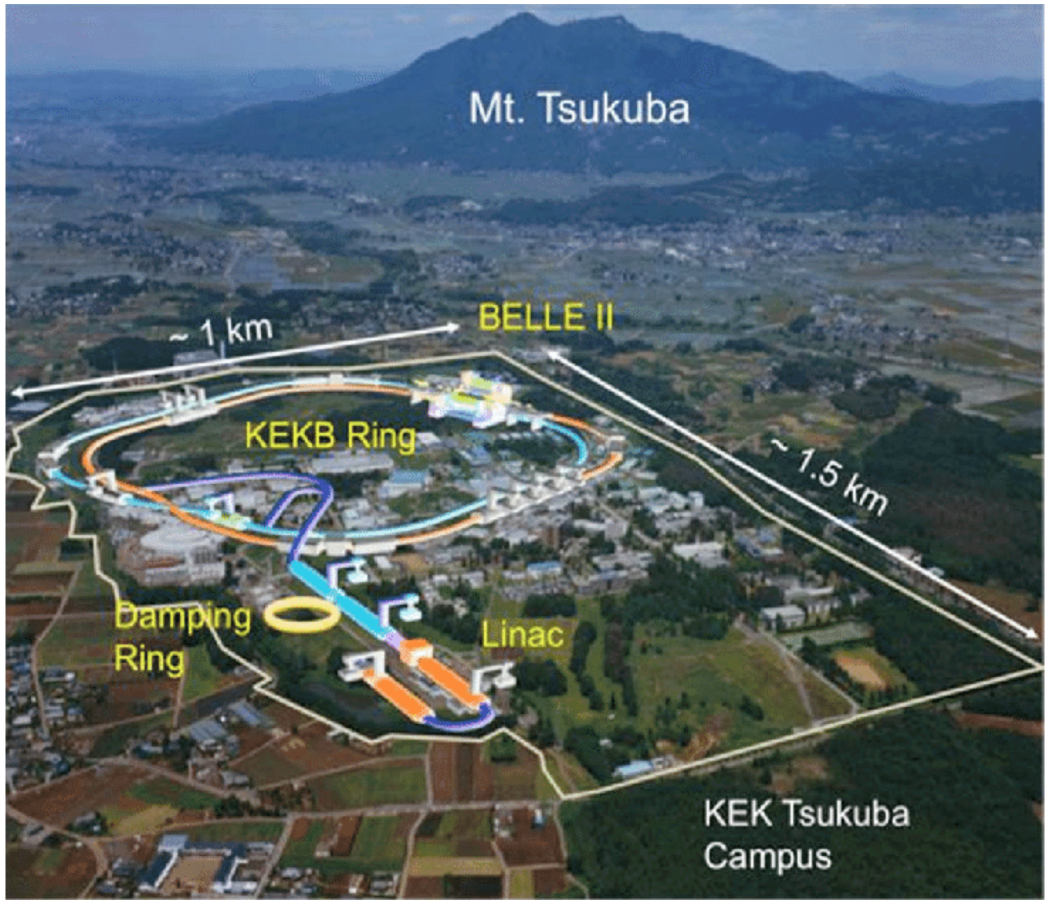
jDPG Theory Workshop • Jan 6-9, 2022 • Grafenschloss Diez

Most powerful tools

Large Hadron Collider

Belle II experiment



Most powerful tools

Large Hadron Collider

$$E(pp) = 13\,\mathrm{TeV}$$

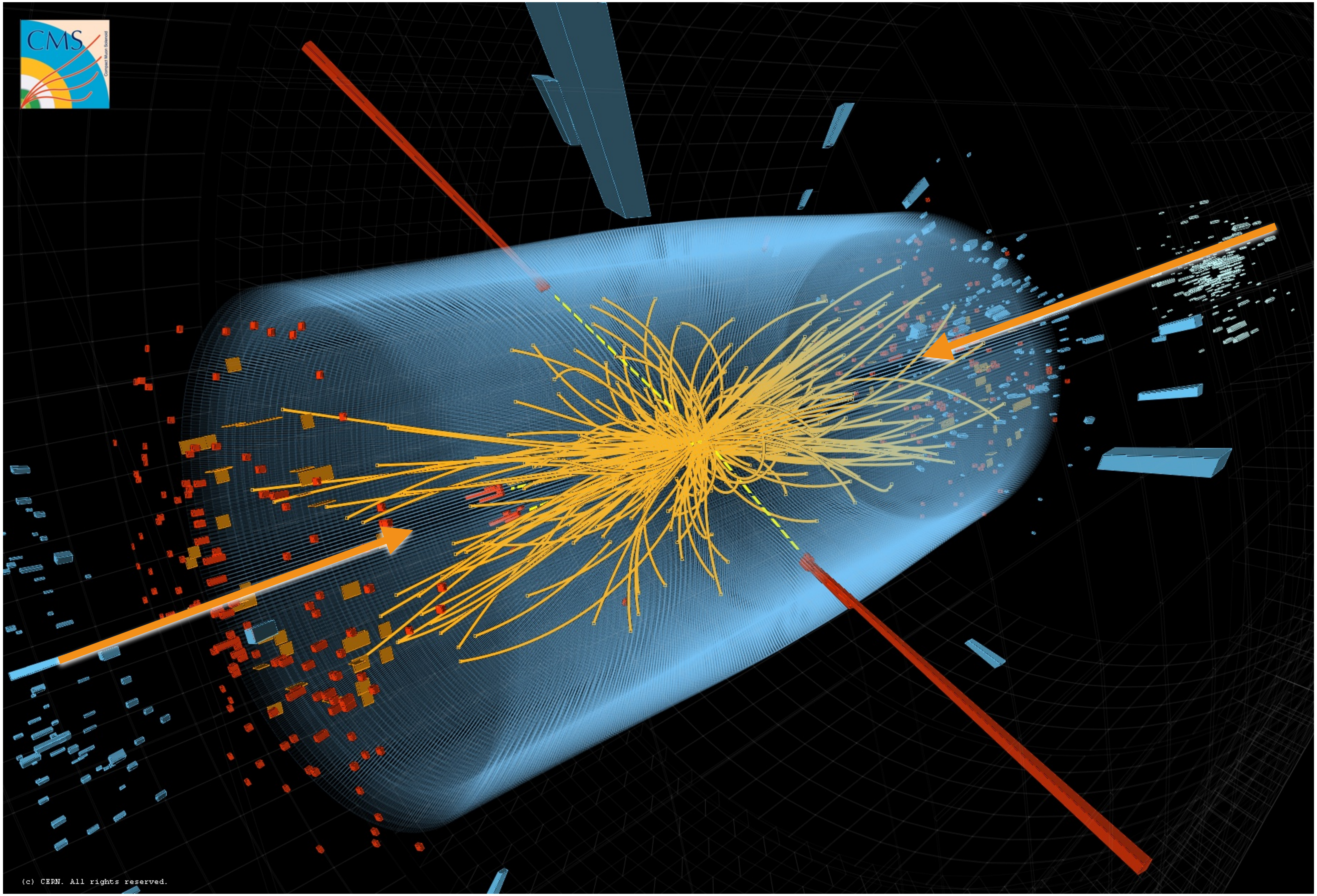
$$\mathcal{L} = 3\,\mathrm{ab}^{-1}$$

Belle II experiment

$$E(e^+e^-) = 10\,\mathrm{GeV}$$

$$\mathcal{L} = 50\,\mathrm{ab}^{-1}$$

Exploring particle collisions



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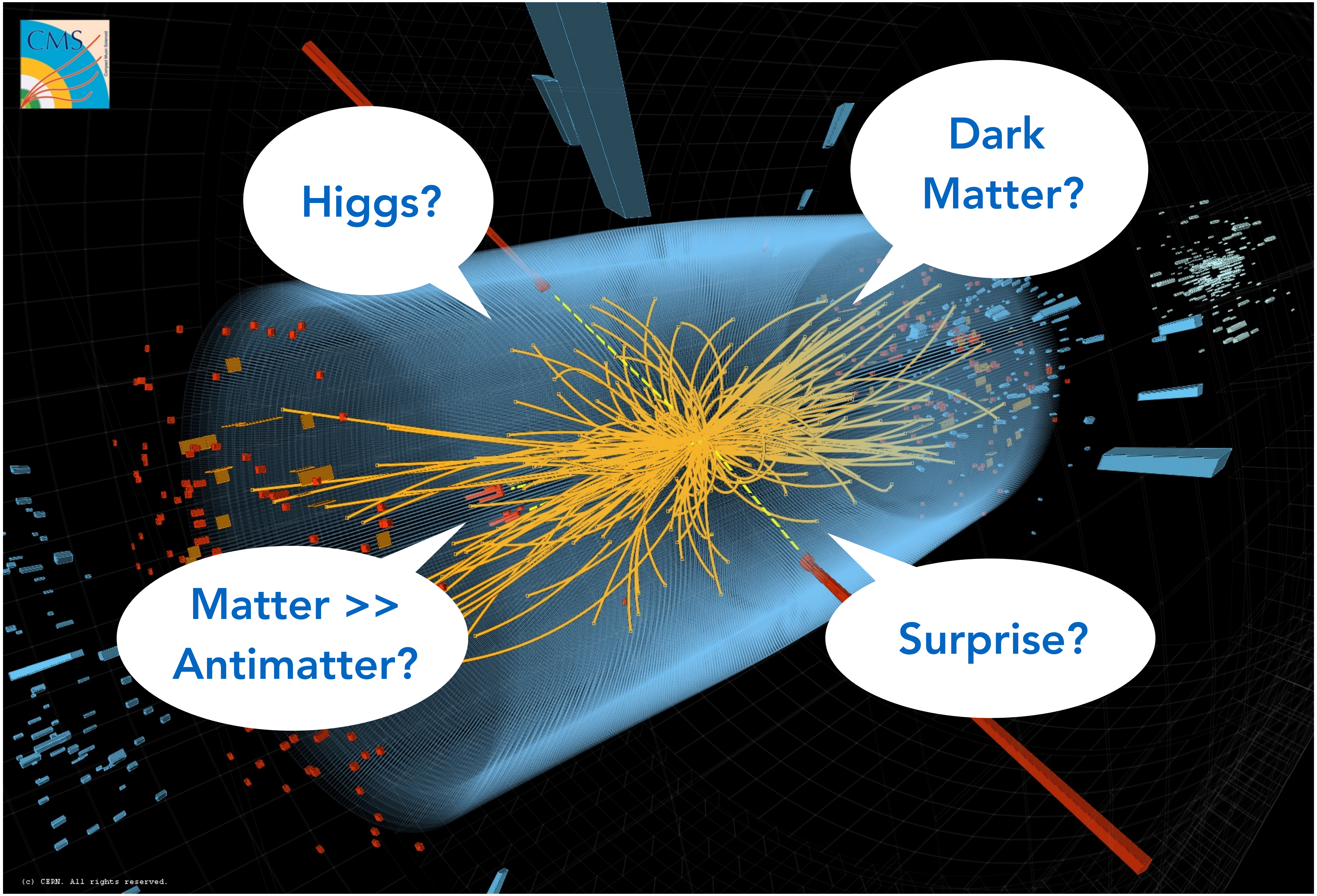
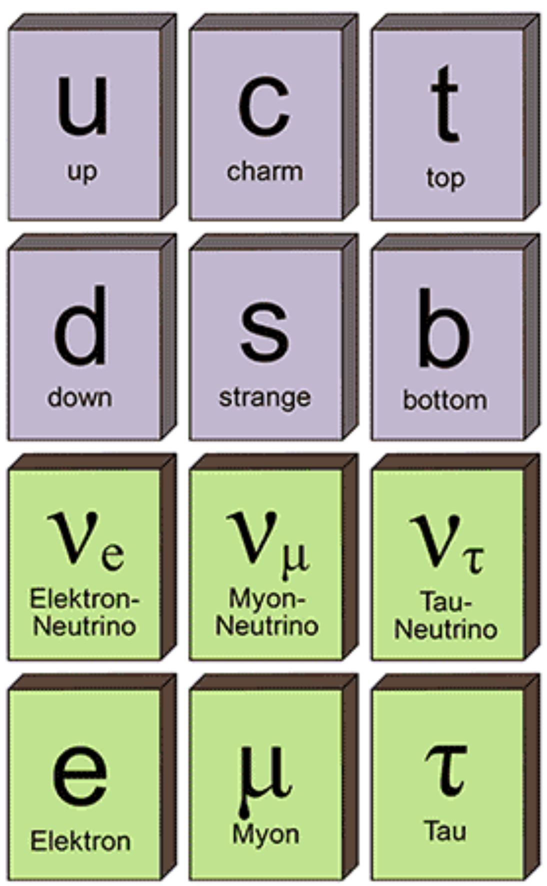


Particle interactions

matter particles



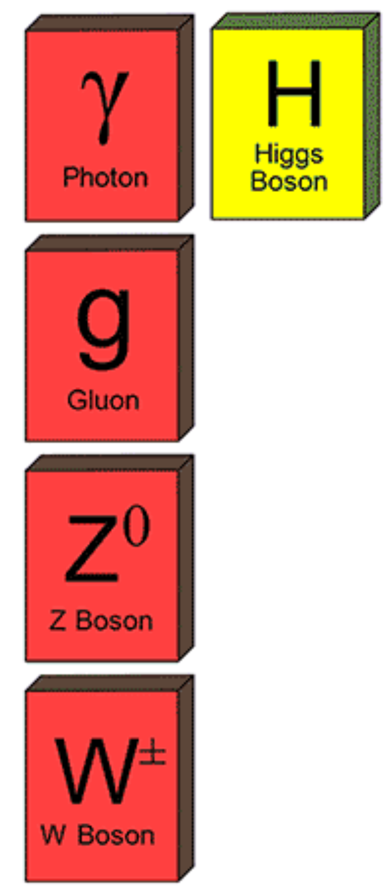
fermions

force carriers

electrodynamics

strong force

weak force

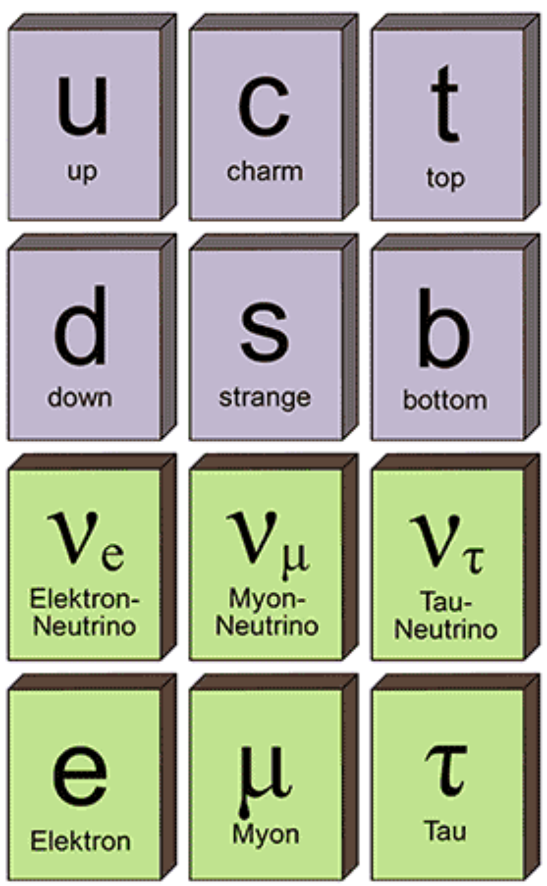


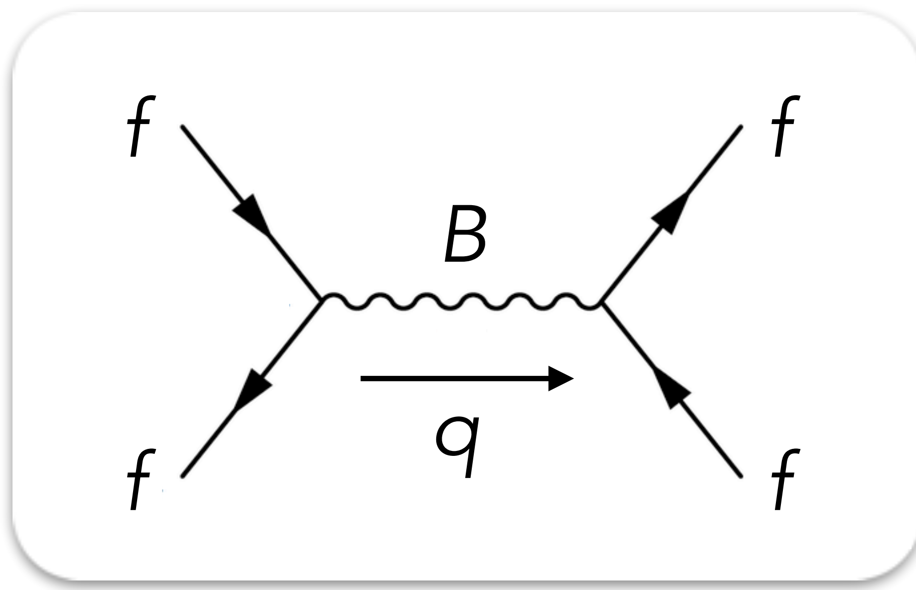
bosons

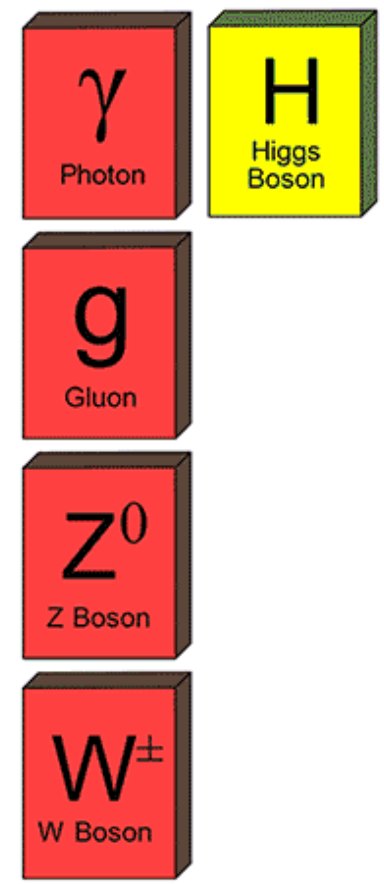
Particle interactions

matter particles

force carriers







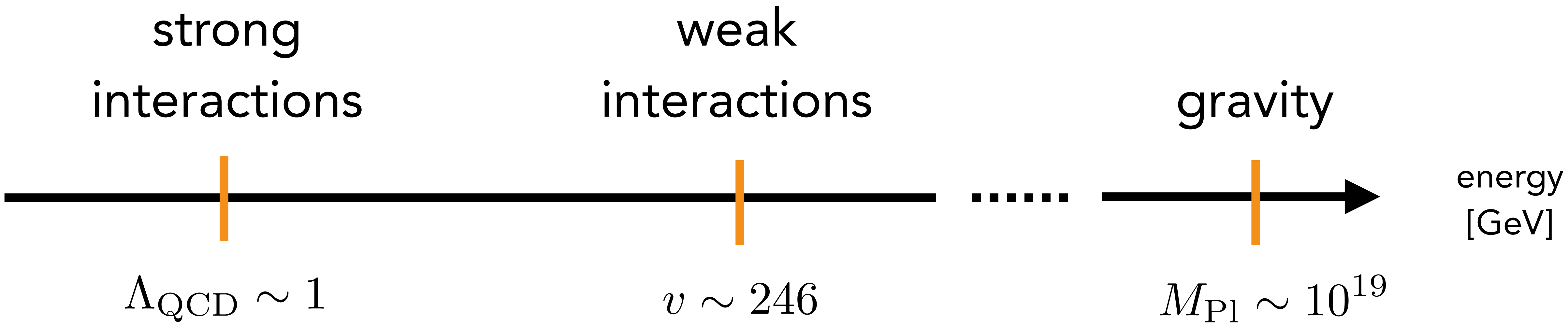
fermions

gauge symmetry

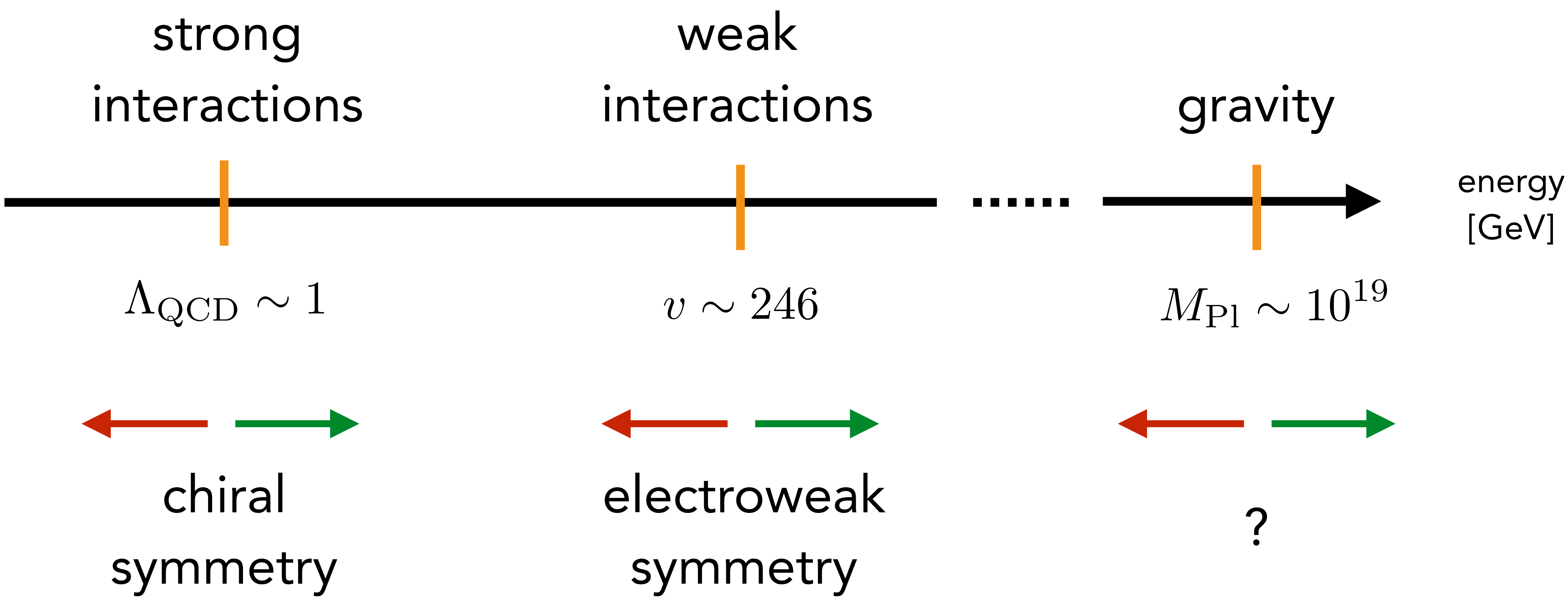
bosons

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

Interactions at different scales



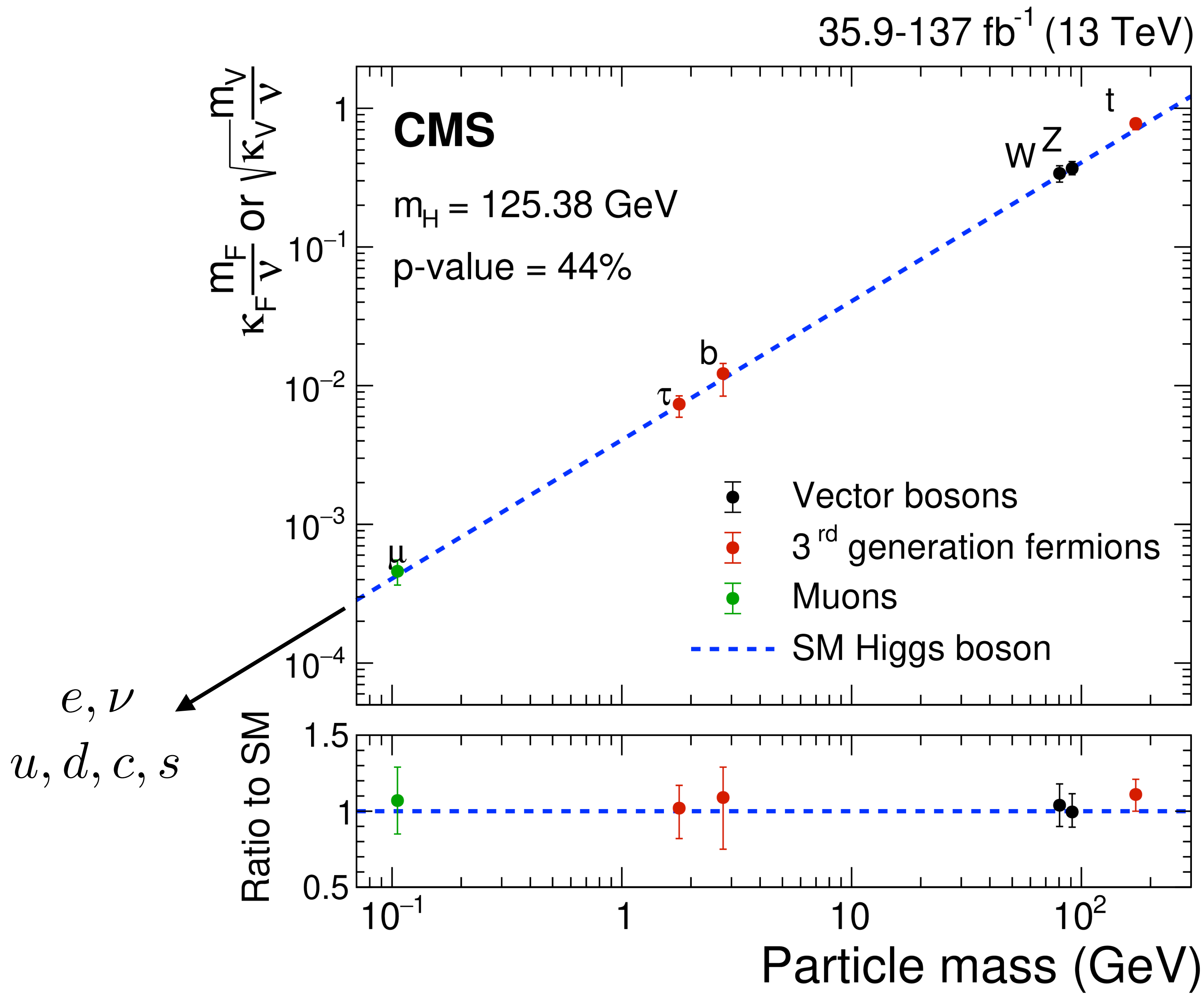
Interactions at different scales



symmetry breaking affects physics at low energies

electrodynamics: unbroken symmetry $\longrightarrow$ no scale

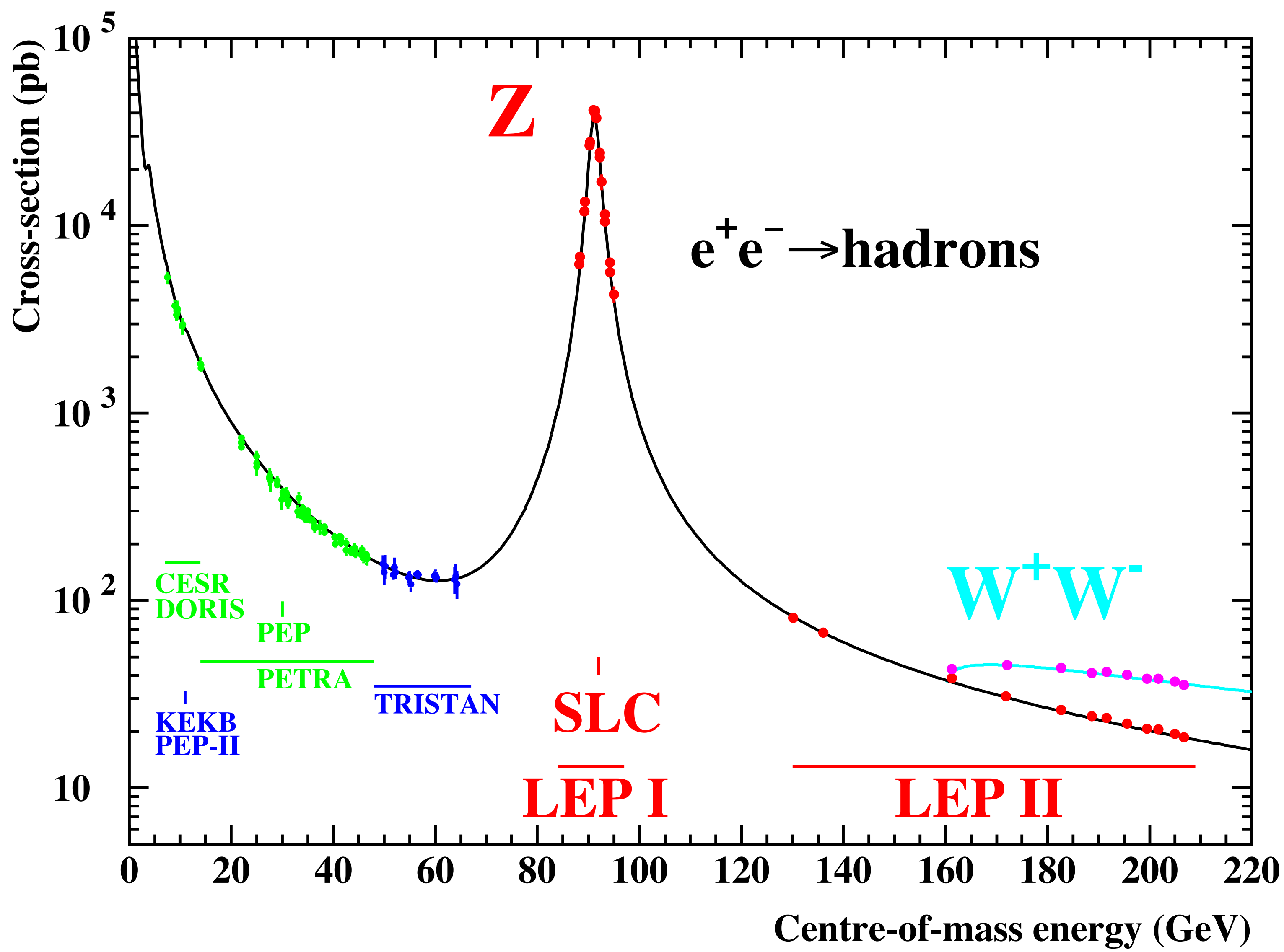
Particles at different scales



$$m_f = y_f \frac{v}{\sqrt{2}}$$

multiple mass scales - origin unknown

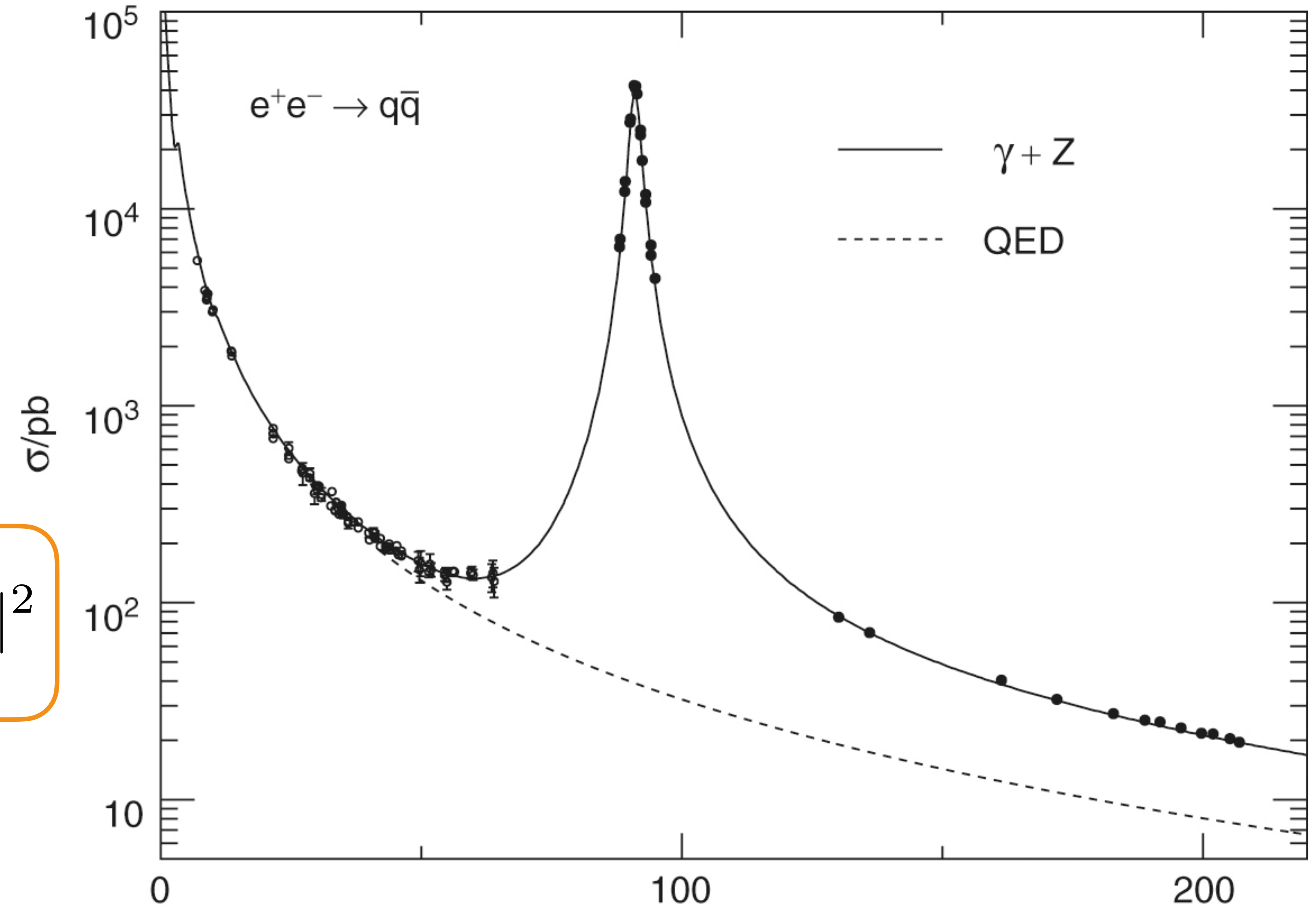
Resonant or virtual?



LEP collaboration 2005

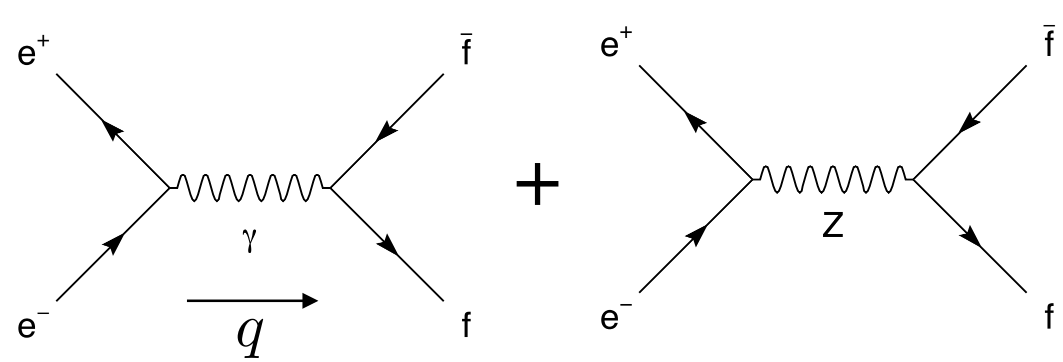
Resonant or virtual?

$$\sigma \sim \int d\Phi \, |\mathcal{M}|^2$$

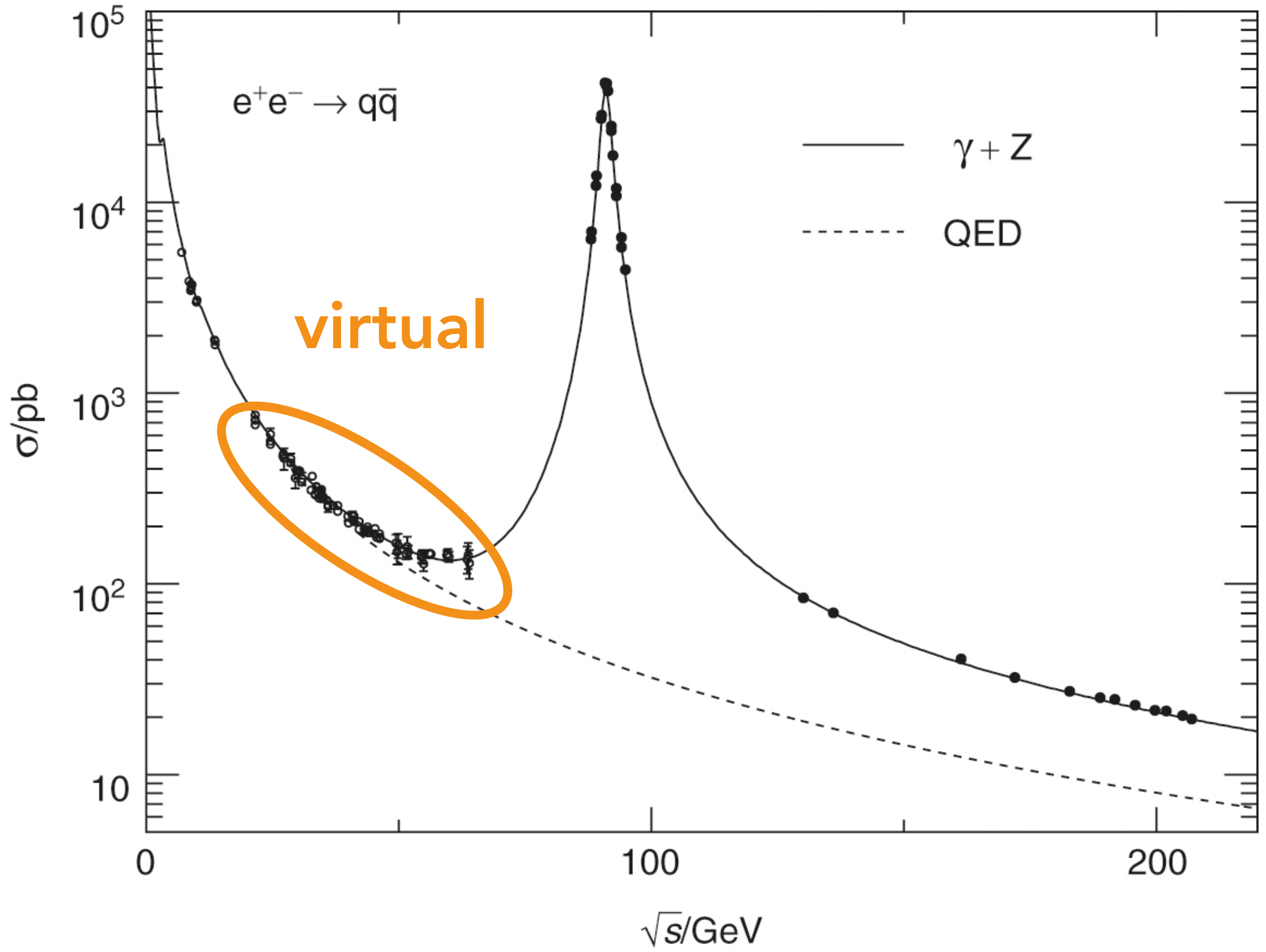


$$\sqrt{s} = \sqrt{q^2}$$

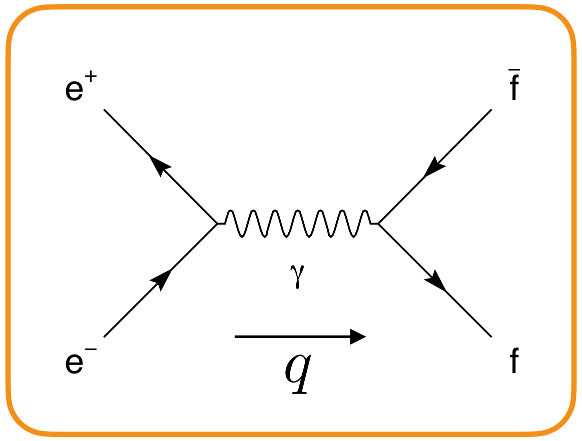
$$\mathcal{M} \sim$$



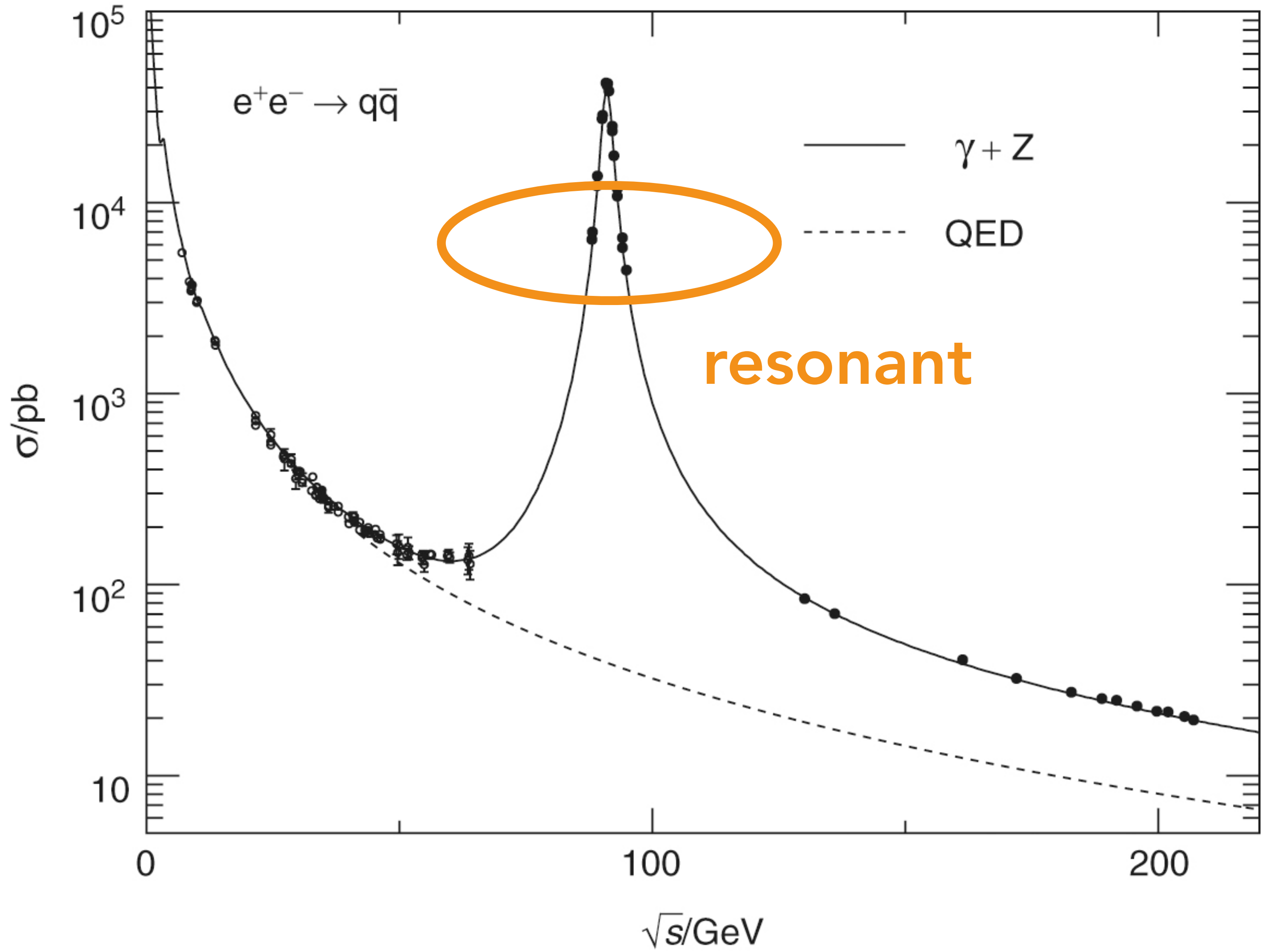
Electrodynamics



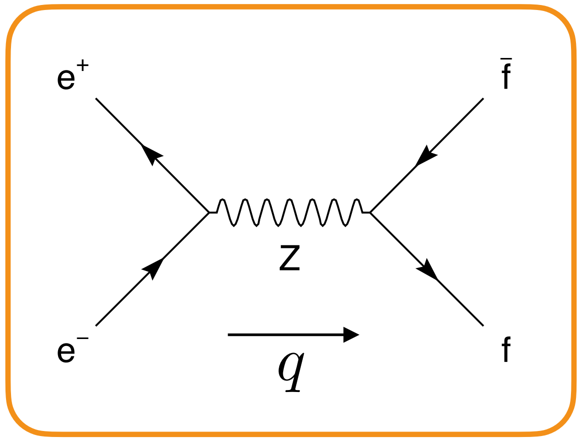
QED: $\mathcal{M} \sim \frac{1}{q^2}$



Weak interactions



EW: $\mathcal{M} \sim \frac{1}{q^2 - m_Z^2 + i m_Z \Gamma_Z}$



$\xrightarrow{q^2 \ll m_Z^2} -\frac{1}{m_Z^2}$

Effective Field Theories in Particle Physics

- describe particle interactions involving multiple scales
- virtual effects of heavy particles at low energies

Part I: Basics

Part II: Weak Effective Theory

Part II: Standard Model Effective Field Theory

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Part II: Weak Effective Theory

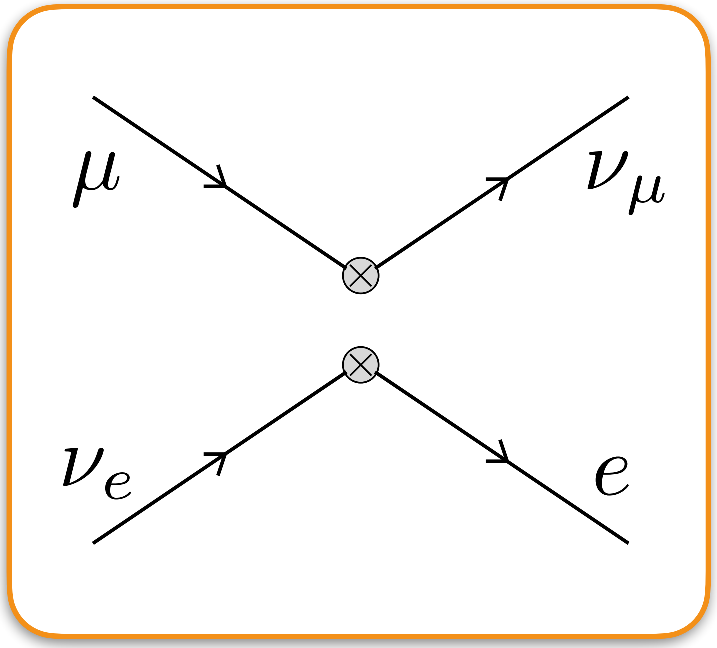
Part II: Standard Model Effective Field Theory

Other EFTs in particle physics:

- Heavy Quark Effective Theory (HQET)
- Soft-Collinear Effective Theory (SCET)
- Chiral Perturbation Theory (ChiPT)

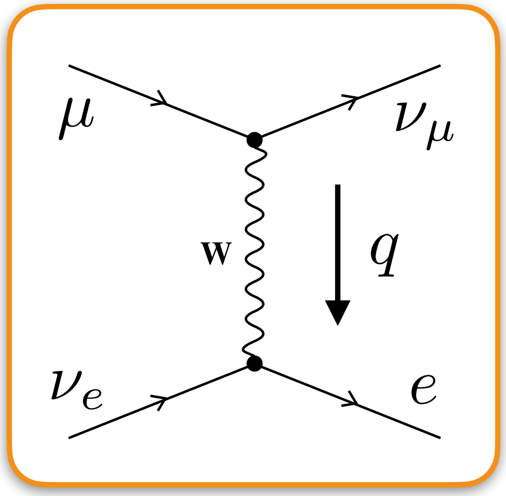
Part I

Basics of Particle EFT



Fermi's theory of weak interactions

Muon decay: $\mu^- \to \nu_\mu e^- \bar{\nu}_e$



$$\mathcal{M} = i\,\frac{g^2}{2}\,(\bar{e}_L \gamma^\rho \nu_L)\,\frac{g_{\rho\sigma}}{q^2 - m_W^2}\,(\bar{\nu}_L \gamma^\sigma \mu_L)$$

$$q^2 \ll m_W^2 : \quad \frac{1}{q^2 - m_W^2} \approx -\frac{1}{m_W^2}\left[1 + \frac{q^2}{m_W^2} + \ldots\right]$$

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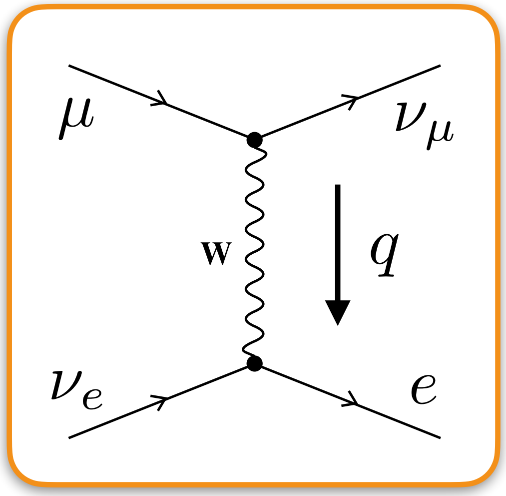

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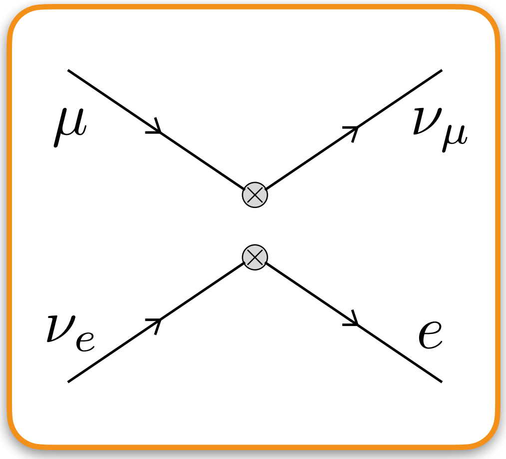


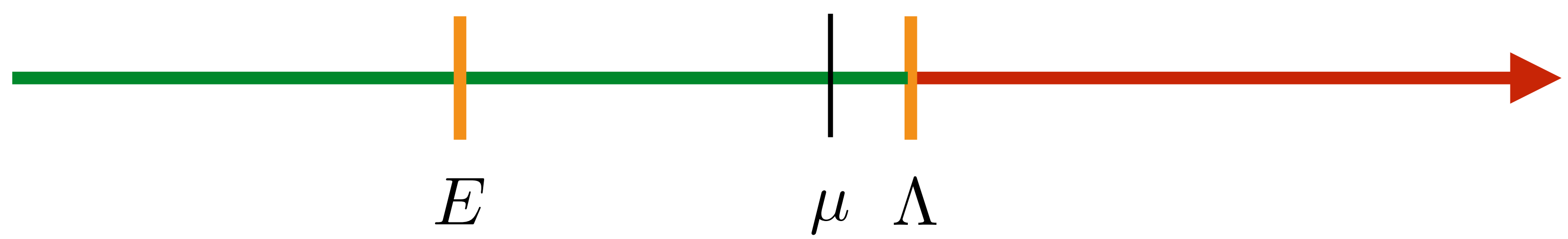
$$\mathcal{M}_{\text{eff}} = -i\frac{4G_F}{\sqrt{2}}\left(\bar{e}_L\gamma^\rho\nu_L\right)\left(\bar{\nu}_L\gamma_\rho\mu_L\right)$$

Fermi constant $$G_F = \frac{\sqrt{2}g^2}{8m_W^2} = 1.166 \times 10^{-5}\,\text{GeV}^{-2}$$

Effective Field Theory

approximation of full theory at experimentally relevant scales

- valid up to **cutoff scale** $\mu < \Lambda$



action in full theory: $S(\phi) = S(\phi_{E<\mu}, \phi_{E>\mu})$

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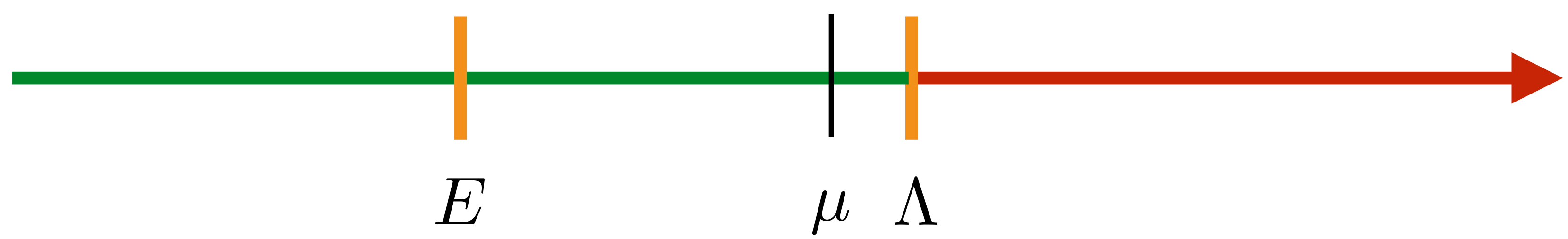

action in full theory: $$S(\phi) = S(\phi_{E<\mu}, \phi_{E>\mu})$$

remove high ‚frequencies‘: $$S_\mu(\phi_{E<\mu}) = \int d^4x \, \mathcal{L}_{\text{eff}}(x)$$

- **effective Lagrangian** $$\mathcal{L}_{\text{eff}}(x) = \sum_i \frac{C_i}{\Lambda^{\gamma_i}} O_i(\phi_{E<\mu}(x))$$

Operator expansion

$$\mathcal{L}_{\text{eff}}(x) = \sum_i \frac{C_i}{\Lambda^{\gamma_i}} O_i(\phi_{E<\mu}(x))$$

O_i : **local operators** with mass dimension $\gamma_i + 4$

C_i : **Wilson coefficients**

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- $\gamma_i > 0 : \; [O_i] > 4 \quad \longrightarrow \quad \mathcal{L}_{\text{eff}} = \sum_i \frac{C_i}{\Lambda} O_i^{(5)} + \sum_j \frac{C_j}{\Lambda^2} O_j^{(6)} + \ldots$

power counting:

$$[\mathcal{L}] \stackrel{!}{=} 4 \quad \longrightarrow \quad [F] = \frac{3}{2}, \; [B] = 1, \; [V_{\mu\nu}] = 2, \; [\partial_\mu] = 1$$

examples: $[(\bar{q}\gamma_\mu q)(\bar{\ell}\gamma^\mu \ell)] = 6 \qquad [(\overline{Q} H \sigma_{\mu\nu} q) F^{\mu\nu}] = 6$

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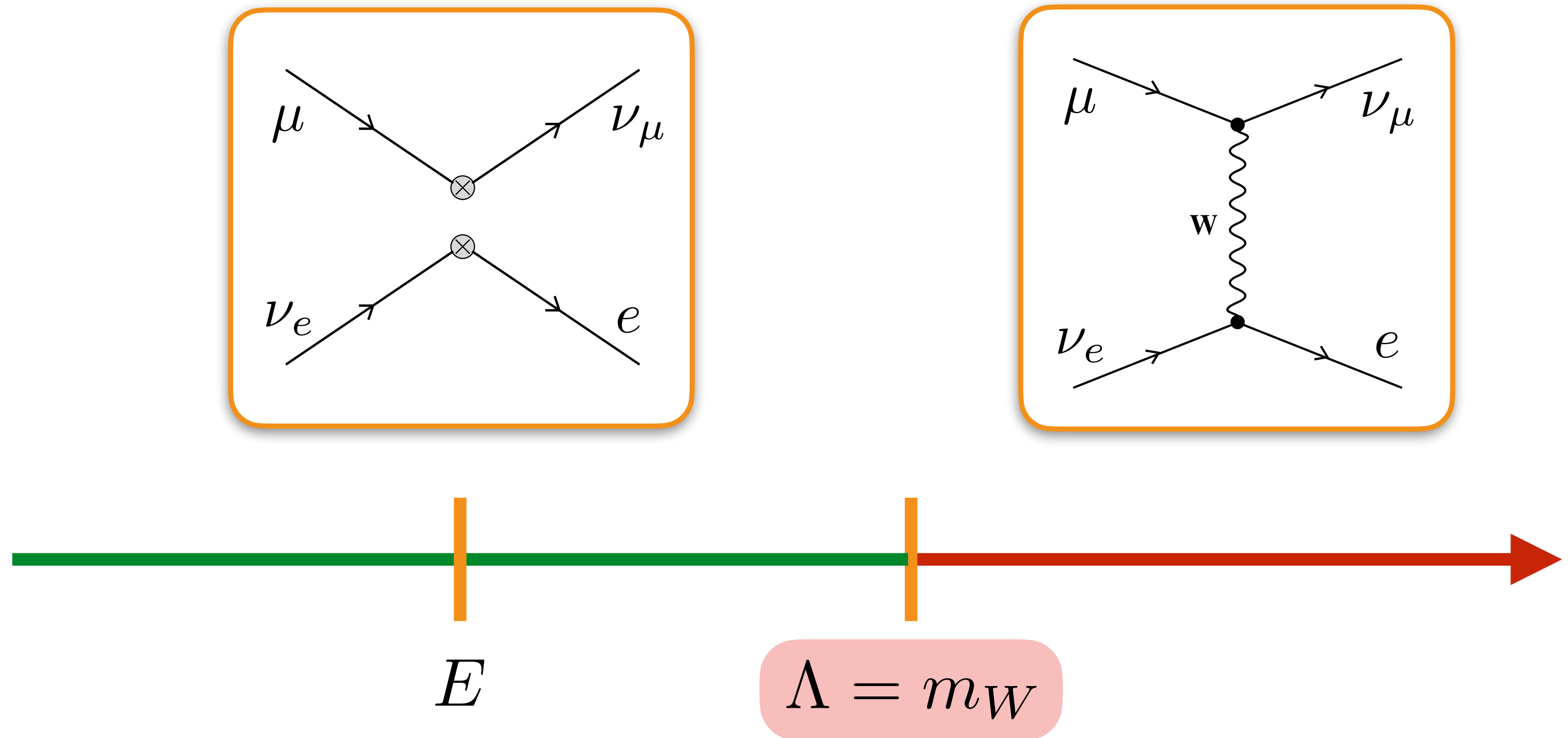
- in observables: $\sigma \sim C_i \left(\frac{E}{\Lambda}\right)^{\gamma_i} + \mathcal{O}(C_i^2)$

Operator effects are suppressed for $E \ll \Lambda$.

Interpretation: Fermi's theory



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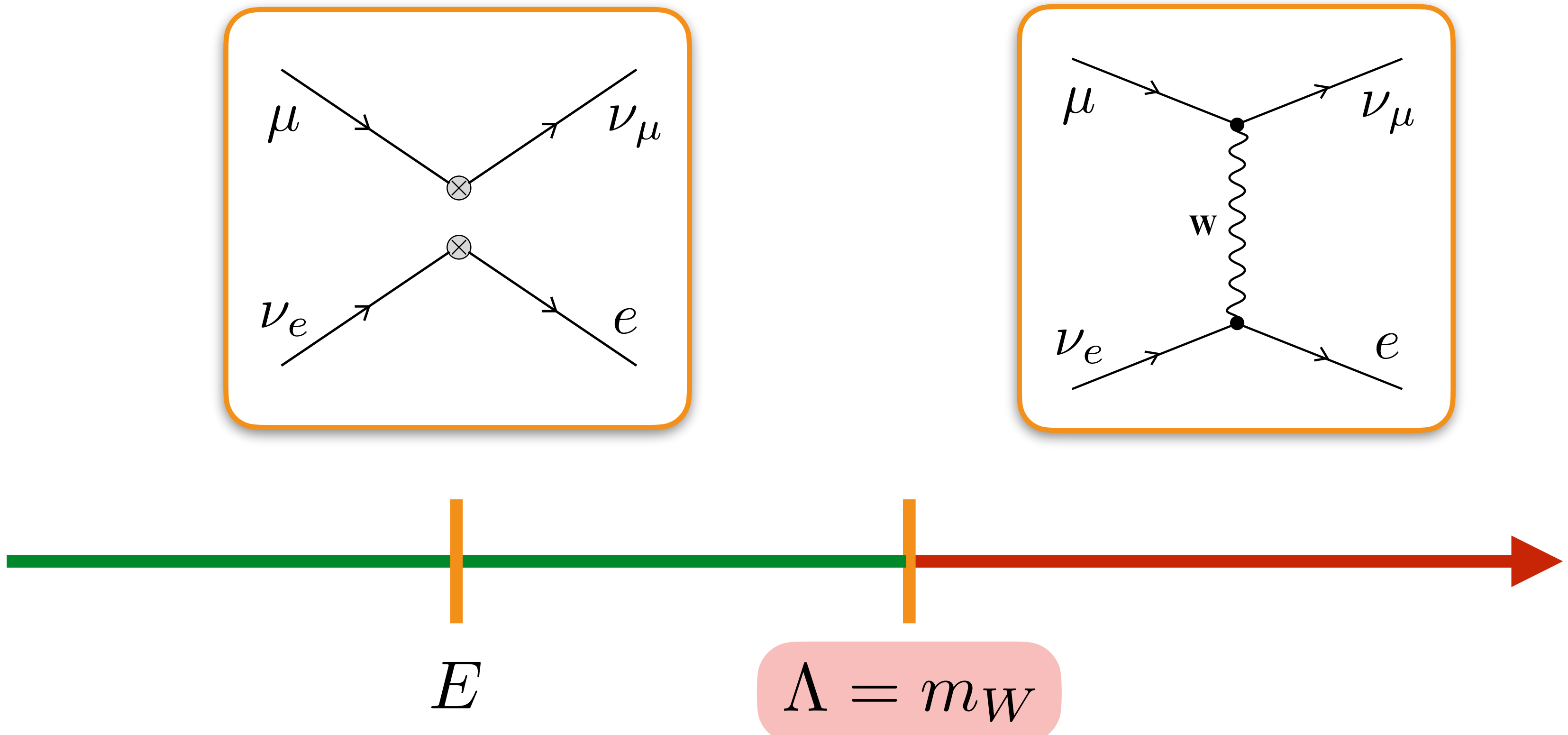

- effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \sum_i \frac{C_i}{\Lambda^2} O_i^{(6)} + \ldots \quad = -\frac{g^2}{2m_W^2} \left(\bar{e}_L \gamma^\rho \nu_L\right)\left(\bar{\nu}_L \gamma_\rho \mu_L\right) + \ldots$$

- weak *effective* interactions at low energies $E = m_\mu$:

$$\Gamma(\mu \to \nu_\mu e \bar{\nu}_e) \sim \left(\frac{m_\mu^2}{m_W^2}\right)^2 \longrightarrow \quad G_F \cdot \text{GeV}^2 \ll 1$$

Summary Part I

Main features of an effective theory:

- good **approximation** of full theory at energies $E \ll \Lambda$
- **effective Lagrangian** describes particle interactions

$$\mathcal{L}_{\text{eff}} = \sum_i \frac{C_i}{\Lambda^{\gamma_i}} O_i$$

- series of **local operators** O_i with expansion parameter E/Λ
- operators respect **symmetries** of the full theory.

Any effective theory is only valid up to cut-off.

Your turn

- What is the effective theory for $e^+e^- \to Z^* \to q\bar{q}$?
- At what energies is it valid?