I) Lepton flavor physics

The flavor structure of the lepton sector in the SM can be analyzed similarly to how we proceeded for the quark sector.

Recall that the masses of charged leptons are generated by their interaction with the Higgs VEV,

\[ L \rightarrow - Y_E \bar{L}_L E_R H + h.c. \xrightarrow{\langle H \rangle} - \frac{v}{\sqrt{2}} \bar{E}_L Y_E E_R + h.c. \]

The Yukawa matrix $Y_E$ is diagonalized by

\[ E_L \rightarrow V_{eL} e_L, \quad E_R \rightarrow V_{eR} e_R : \quad Y_E = V_{eL}^T Y_E V_{eR} = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \]

In the mass eigenbasis of charged leptons, the $W$ interactions read

\[ L_W = \frac{g}{\sqrt{2}} \bar{N}_L \gamma^\mu V^\dagger_{eL} e_L W_\mu + h.c. \]

For massless neutrinos, the transformation $N_L \rightarrow V_{\nu L} \nu_L$ with $V_{\nu L} = V_{eL}$ renders the charged current flavor-diagonal. For massive neutrinos, the flavor mixing among left-handed leptons is physical,

\[ L_W = \frac{g}{\sqrt{2}} \bar{V}_L \gamma^\mu V^\dagger_{\nu L} V_{\nu L}^\dagger e_L W_\mu + h.c. \]

\[ V_{\text{PMNS}}: \quad \text{Pontecorvo-Maki-Nakagawa-Sakata matrix} \]
we work in a basis where $Y_e$ is diagonal. The neutrino mass and flavor eigenstates are then related by

$$N_e = V_{PMNS}^* \nu_i.$$ Neutrinos are thus mixtures of different flavor eigenstates. (more later)

The PMNS matrix can be parametrized by 3 angles $\theta_{12}, \theta_{23}, \theta_{13}$ and 1 phase $\delta$. From neutrino oscillation experiments, we know that

$$\sin^2 \theta_{12} \sim 0.3, \quad \sin^2 \theta_{23} \sim 0.4, \quad \sin^2 \theta_{13} \sim 0.02.$$ (PDG 2017, $m_1 < m_2 < m_3$)

Flavor mixing in the lepton sector is sizeable! The PMNS matrix does not have a hierarchical structure, unlike the CKM matrix.

1) Rare decays of charged leptons

In the SM, there are no FCNCs with leptons at tree level. At the one-loop level, FCNCs are induced by weak interactions. They can be observed (at least in principle) in flavor-changing processes involving charged leptons, e.g., $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$, $\mu \rightarrow 3e$.

If neutrinos are Majorana particles, the lepton sector has 2 additional phases (→ more later).
$\mu \rightarrow e\nu\bar{\nu}$ decay

This process is induced at the loop level:

\[
\sigma = -\frac{e m_e}{8\pi^2} \bar{e}_L \gamma^\nu \delta_{\mu\nu} f \frac{m_\mu^3}{m_W^2}
\]

The amplitude is given by

\[
M \sim \sum_{i=1}^{3} V_{ei}^* V_{\mu i} f \left(\frac{m_i^2}{M_W^2}\right) \approx f(0) + f'(0) \frac{m_i^2}{M_W^2} + ...
\]

We apply the unitarity condition, $\sum_{i} V_{ei}^* V_{\mu i} = 0$, and obtain

\[
M \sim \sum_{i=2}^{3} V_{ei}^* V_{\mu i} \frac{\Delta m^2_{ei}}{M_W^2}; \quad \Delta m^2_{ei} = m_i^2 - m_1^2.
\]

Due to the small neutrino masses, the GIM mechanism is extremely effective and strongly suppresses the decay rate. Normalized to the dominant decay process $\mu \rightarrow e\nu\bar{\nu}$, we obtain the branching ratio

\[
\text{Br} (\mu \rightarrow e\nu\bar{\nu}) = \frac{T(\mu \rightarrow e\nu\bar{\nu})}{T(\mu \rightarrow e\nu\bar{\nu})} = \frac{3d}{32\pi} \left| \sum_{i=2}^{3} V_{ei}^* V_{\mu i} \frac{\Delta m^2_{ei}}{M_W^2} \right|^2 < 10^{-54}.
\]

Currently strongest limit from MEG experiment (1605.05081):

\[
\text{Br} (\mu \rightarrow e\nu\bar{\nu}) < 4.2 \times 10^{-13} \quad (90\% \text{ CL}).
\]

An observation of $\mu \rightarrow e\nu$ would be a clear sign of new physics.
\( \mu \to 3e \) decay

\[
\begin{align*}
0 & \sim W \nu \bar{e} \ell \bar{\nu} \mu F_{\mu e}^\nu \mu e; \\
0 & \sim (\bar{\nu}_e \bar{\nu}_\mu \mu_e)(\bar{e} \gamma^\nu \mu e).
\end{align*}
\]

Currently strongest limit from SINDRUM experiment:

\[
Br(\mu \to 3e) < 1.0 \times 10^{-12}.
\]

(future: \( Br(\mu \to 3e) \leq 10^{-10} \) expected from mu3e experiment)

\( \mu \to e \) conversion

\[
\begin{align*}
0 & \sim W \nu \bar{e} \ell \nu \bar{\mu} \mu F_{\mu e}^\nu \mu e; \\
0 & \sim (\bar{\nu}_e \bar{\nu}_\mu \mu_e)(\bar{e} \gamma^\nu \mu e).
\end{align*}
\]

Current limit: \( Br(\mu + A \mu \to e + A \mu) < 7 \times 10^{-13} \).

(future: Mu2e / COMET: \( Br(\mu + A \mu \to e + A \mu) < \text{few} \times 10^{-14} \))

In the search for new physics, the complementarity of \( \mu \to e\gamma \), \( \mu \to 3e \) and \( \mu \to e \) conversion is interesting. Depending on the new effective interaction, the sensitivity of the respective observables can change significantly.
2) Electric and magnetic moments

In non-relativistic electrodynamics, the interaction of a particle (with spin \(\vec{S}\)) with an electric and magnetic field is described by the Hamiltonian

\[
\mathcal{H} = -\mu \frac{\vec{S} \cdot \vec{B}}{|\vec{S}|} - d \frac{\vec{S} \cdot \vec{E}}{|\vec{S}|};
\]

\(\mu\): magnetic dipole moment (MDM),
\(d\): electric dipole moment (EDM).

Transformation properties under parity and time reversal:

- **P**: \(\vec{B} \rightarrow +\vec{B}, \vec{E} \rightarrow -\vec{E}, \vec{S} \rightarrow +\vec{S}\);
- **T**: \(\vec{B} \rightarrow -\vec{B}, \vec{E} \rightarrow +\vec{E}, \vec{S} \rightarrow -\vec{S}\).

\(\rightarrow\) \(\mu\) is \(P\)-even and \(T\)-even \(\xrightarrow{\text{CP}}\) CP-conserving.
\(d\) is \(P\)-odd and \(T\)-odd \(\xrightarrow{\text{CP}}\) CP-violating.

In relativistic quantum electrodynamics, MDMs and EDMs are induced by the following effective operators,

\[
-\mu_e \frac{\vec{S}}{|\vec{S}|} \cdot \vec{B} \quad \leftrightarrow \quad e (\bar{e} \gamma^{\mu} e) A_\mu + a_e \frac{e}{4m_e} (\bar{e} \gamma_\mu \gamma_5 e) \gamma_5; \\
-d_e \frac{\vec{S}}{|\vec{S}|} \cdot \vec{E} \quad \leftrightarrow \quad d_e \frac{i}{2} (\bar{e} \gamma_\mu \gamma_5 e) \gamma_5.
\]

The renormalizable coupling \(e (\bar{e} \gamma^{\mu} e) A_\mu\) induces a magnetic moment with the gyromagnetic factor \(g_e = 2\).

The dipole operators induce a (CP-violating) elec-