the dipole moment $d_e$ and an anomalous magnetic moment $a_e$ with

$$\mu_e = g_e \frac{e}{2m_e} \; ; \; (g_e - 2) = 2a_e.$$  

In quantum field theory, $a_e$ and $d_e$ are induced by quantum corrections to the interaction of lepton $l$ with a static background field. These corrections can be calculated from loop diagrams at zero momentum transfer.

**Muon $g-2$**

a) electromagnetic contributions (dominant)

$$a_e^{\text{QED}} = \frac{e}{2\pi} \quad \text{(Schwinger, 1948)}$$

b) vacuum polarization

leptonic (flavor-specific)

hadronic

not calculable in perturbative QCD

optical theorem: $\delta(e^+e^- \rightarrow \gamma^{*}\rightarrow \text{hadrons}) = \frac{4\pi \alpha^2}{5} \text{Im} \left(\frac{\text{had.}}{s}\right)$

$\Rightarrow$ measure hadronic contributions $a_{\mu}$.
c) weak contributions

\[ \frac{M_{\mu}}{M_{W}} \sim \frac{M_{W}}{M_{W}} \sim \frac{M_{W}^{4}}{M_{W}^{4}} \]

\[ a_{\mu}^{E W} = \frac{\sqrt{2} G_{F} M_{W}^{2}}{4 \pi^{2}} \left( 5 + (-1+4 \sin^{2} \theta_{W})^{2} \right) + O \left( \frac{M_{\mu}^{4}}{M_{W}^{4}} \right) \]

\[ \approx (194.82 \pm 0.02) \times 10^{-11} \quad \text{(quoted from Fegerleiner)} \]

**in summary:**

\[ a_{\mu}^{S M} = a_{\mu}^{Q E D} + a_{\mu}^{Q C D} + a_{\mu}^{E W} = 1.65941786(66) \times 10^{-3} \]

\[ a_{\mu}^{e x p} = 1.6592080(63) \times 10^{-3} \quad \text{(mostly BNL exp.)} \]

**observed discrepancy**

\[ \Delta a_{\mu} = a_{\mu}^{e x p} - a_{\mu}^{S M} = (294 \pm 89) \times 10^{-11} \quad (3.38) \]

**New physics in \( a_{\mu} ? \)**

For example, consider a new vector boson \( Z' \) with couplings to muons:

\[ L \supset \bar{\mu} \left( g_{V}^{\mu} \gamma^{\mu} + g_{A}^{\mu} \gamma^{\mu} \gamma_{5} \right) \mu Z' \]
\[ \Delta \alpha'_\mu = \frac{(g^\mu)^2 - 5 (g^\mu)^2}{12 \pi^2} \frac{M_{\mu}^2}{M_{Z^0}^2} + O(M_{\mu}/M_{Z^0}) \]

\[ \approx \Delta \alpha_\mu \text{ for } M_{Z^0} \approx 16 \text{ GeV}, \ q_{V}^{\mu} \approx 10^{-2}. \]

The new muon g-2 experiment at Fermilab will measure \( \alpha_\mu \) with a precision improved by a factor of 4. This will greatly help us knowing if there is a new contribution to \( \alpha_\mu \).

**Electric dipole moments**

EDMs are CP-violating quantities. In the SM, the only source of CP violation is the phase \( \delta \) in the CKM matrix.

Let us first consider the EDM of a down-quark.

1-loop:

\[ \lambda_{1d}^{(1)} \sim \frac{e}{16 \pi^2} G_F M_d \text{ Im} (V_{td} V_{td}^*) = 0 \]

* assuming zero neutrino masses and not considering the \( \bar{q} \)-term in QCD.
2-loop:

\[ d_d^{(2)} \sim \frac{e}{(16\pi^2)^2} \frac{G_F^2 M_d M_c}{16 \pi^2} \]

\[ \times \text{Im} (V_{td} V_{tb} V_{cd} V_{cd}^*) \neq 0 \]

\[ \text{Feynman invariant, } \mathcal{F} \sim 10^5 \]

But: \( \leq d_d^{(2)} = 0 \) ! (Shabalin, 1984; due to antisymmetry of \( \mathcal{F} \))

The down-quark EDM is first induced at 3-loop level:

\[ d_d^{(3)} \sim \frac{e}{(16\pi^2)^2} \frac{G_F^2}{16 \pi^2} \frac{2}{6} G_F^2 M_d M_c \]

\[ \times \text{Im} (V_{td} V_{tb} V_{cd} V_{cd}^*) \neq 0 \]

\[ \rightarrow |d_d| = 10^{-34} \text{ e cm} \]

Lepton EDM first induced at 4-loop level:

\[ d_e \sim \frac{e}{(16\pi^2)^2} \frac{G_F^2}{16 \pi^2} \frac{2}{6} G_F^2 m_e m_c m_s \]

\[ \rightarrow |d_e| = 10^{-44} \text{ e cm} \ll |d_d| \]

(Pospelov, Ritz, 2013)
New physics in $d_e$?

Current experimental upper bound on electron EDM:

$|d_e| < 8.7 \times 10^{-29}\text{cm}$ \text{(ACME 2013, ThO)}

Consider new physics at a scale $\Lambda$ with CP-violating (complex) couplings:

$$d_e^{\text{NP}} \sim \frac{\tilde{C}}{\Lambda^2} m_e (\bar{e} \tilde{\delta}_{\mu\nu} \gamma_5 e)^{\mu\nu}$$

Assuming $\tilde{C} \sim 1$ ($\sim 1/16\pi^2$) with a CP-violating phase of $0(\Lambda)$, ACME can probe new physics at scales $\Lambda \sim 300 \ (30) \text{TeV}$. 