New physics in \( e \) ?

Current experimental upper bound on electron EDM:
\[ |d_e| < 8.7 \times 10^{-29} \text{ ecm} \] (ACME 2013, ThO)

Consider new physics at a scale \( \Lambda \) with CP-violating (complex) couplings:
\[ d_e^{np} \sim \frac{\tilde{C}}{\Lambda^2} m_e (\bar{e} \tilde{\nu}_e) (\nu_e) + m^d \]

Assuming \( \tilde{C} \sim 1 \ (\sim 1/16\pi^2) \) with a CP-violating phase of \( 0(\text{d}) \), ACME can probe new physics at scales \( \Lambda \sim 300 \ (30) \text{ TeV} \).

3) Neutrinos

From neutrino oscillation experiments, we know that neutrinos are massive fermions. Cosmological observations yield an upper bound
\[ \sum_{i=\mu,\tau} m_{\nu_i} \leq 0.2 \text{ eV}. \]

Do neutrinos obtain their mass through the Higgs mechanism? And if so, why is it so much smaller than for all other SM fermions?
To address these questions, let us have a closer look at the possible mass terms that respect Lorentz and gauge invariance.

For charged fermions in the SM, the only possible mass term is the so-called **Dirac mass**

\[ L = -m \overline{\psi} \gamma^0 \psi = -m (\overline{\psi}_L \gamma^0 \psi_L + \overline{\psi}_R \gamma^0 \psi_R). \]

A Dirac spinor can be written in terms of two 2-component Weyl spinors \( \chi, \xi \), which transform under the \((1,0)\) representation of the Lorentz group:

\[ \psi = \begin{pmatrix} \chi \\ \varepsilon \chi \xi^* \end{pmatrix}; \ \varepsilon \text{ ensures Lorentz invariance}. \]

If \( \chi, \xi \) transform under the gauge group \( U(1)_em \) as \( \chi \rightarrow U \chi, \xi \rightarrow U^* \xi \), the Dirac mass term has the desired invariance under electromagnetic gauge transformations:

\[ L = -m \overline{\psi} \gamma^0 \psi = -m \overline{\chi} \gamma^0 \chi = -m \left( \xi^T, -\varepsilon^T \chi^T \right) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \chi \\ \varepsilon \chi \xi^* \end{pmatrix} = -m \left( \xi^T \varepsilon^* \xi^* - \varepsilon^T \chi^T \chi \right), \]

where \( \xi^T \varepsilon \chi \rightarrow \xi^T U^* \varepsilon U \chi = \xi^T \varepsilon \chi \), and similarly \( \xi^T \varepsilon^* \xi^* \rightarrow \chi^T U^* \xi^* U \xi^* = \chi^T \xi^* \).
We use the Dirac matrices in the Weyl basis,

\[ \gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}; \quad \sigma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \]

and identity \( \gamma = \gamma_L + \gamma_R \):

\[ \gamma_L = \begin{pmatrix} \chi \\ 0 \end{pmatrix}; \quad \gamma_R = \begin{pmatrix} 0 \\ \chi^* \end{pmatrix}, \]

with \( \gamma_{LR} = \frac{1 + \gamma_5}{2} \gamma \).

It is convenient to introduce the charge conjugation matrix

\[ C = i \gamma^0 \gamma^2 = \begin{pmatrix} -\varepsilon & 0 \\ 0 & \varepsilon \end{pmatrix}. \]

The conjugate of a Dirac spinor is then defined as

\[ \psi^c \equiv C \psi^T = C \gamma^0 \psi^* = \begin{pmatrix} \begin{pmatrix} \chi^* \end{pmatrix} \\ \begin{pmatrix} 0 \end{pmatrix} \end{pmatrix} = \psi = \psi_L + \psi_R. \]

This implies \( (\psi^c)_L = (\psi^c)_R \).

The Dirac mass term can thus also be written as

\[ \mathcal{L} = -m \{ (\psi^c)^T \gamma \psi + h.c. \}. \]

Since neutrinos are not charged under an unbroken gauge symmetry, they might as well be described by a Majorana spinor

\[ \psi_M = \begin{pmatrix} \chi \\ (\varepsilon \chi^*) \end{pmatrix}; \]

i.e., in terms of a single Weyl spinor \( \chi \).
The Majorana spinor is invariant under charge conjugation:

\[ \psi^c = \overline{\psi}^T \mathcal{C} \psi^0 \psi^* = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \psi^t \\ \psi^x \end{pmatrix} = \begin{pmatrix} \psi^t \\ -\psi^x \end{pmatrix} = \psi_m. \]

Majorana particles are thus said to be their own antiparticles.

Using our Majorana spinor, we can construct a new mass term for Majorana particles,

\[ \mathcal{L} = -\frac{M}{2} \overline{\psi}_m \psi_m = -\frac{M}{2} \begin{pmatrix} \psi^t, -\psi^x \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \psi^t \\ \psi^x \end{pmatrix} \]

\[ = -\frac{M}{2} (\psi^t \psi^x - \psi^x \psi^t) \]

\[ = -\frac{M}{2} (-\psi^t \psi^* + \psi^* \psi) \]

\[ = -\frac{M}{2} (\psi^t \psi^* + \psi^* \psi) \]

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\[ = -\frac{M}{2} (\psi^T \mathcal{C} \psi + \text{h.c.}). \]

The Majorana mass term can thus be written in terms of a left-handed Dirac spinor. Furthermore,

\[ \psi_m = \psi_L + \mathcal{C} \overline{\psi}_L^T = \psi_L + (\psi^c)_R. \]

Using \( \overline{\psi}_m \sigma^\mu \psi_m = 2 \overline{\psi}_L \gamma^\mu \psi_L \), one can verify that \( M \) is indeed the mass of \( \psi_m \), i.e.,

\[ \mathcal{L} = \frac{1}{2} \left( i \overline{\psi}_m \gamma^\mu \psi_m - M \overline{\psi}_m \psi_m \right). \]

Here \( i \overline{\psi}_m \gamma^\mu \psi_m \) is the kinetic term of the Majorana particle \( \psi_m \).
So, do neutrinos have Dirac or Majorana masses?

If we postulate a new right-handed neutrino $N_R$ that is sterile, i.e., a singlet under the SM gauge group, we generate the Yukawa interaction

$$L = -Y_N \bar{\nu}_L H N_R + h.c. \xrightarrow{\langle H \rangle = \frac{v^2}{\sqrt{2}}} - \frac{Y_N v}{\sqrt{2}} (\bar{\nu}_L N_R + h.c.)$$

$$= - m_\nu (\bar{\nu}_L N_R + h.c.).$$

$m_\nu$ is a Dirac mass for neutrinos, but

- we have no explanation why neutrinos are so light, i.e., why $m_\nu \ll v$.
- we need to explain why there is no Majorana mass term

$$L = - \frac{M_N}{2} (N_R^+ C N_R + h.c.).$$

This term is allowed by the SM gauge symmetry, but would be forbidden if lepton number was an exact symmetry.

Allowing for the Majorana mass term and assuming that $M_N \gg v$, we obtain an elegant explanation why neutrinos are so light through the so-called seesaw mechanism.
The see-saw mechanism

We supplement the SM by a heavy sterile neutrino $\nu_R$ and write down the most general gauge-invariant Lagrangian,

$$\mathcal{L} = -Y_\nu \bar{\nu} L \tilde{H} N_R - \frac{M}{2} N_R^T C N_R + \text{h.c.}$$

$$\rightarrow \quad -M(\bar{\nu}_L N_R + \bar{N}_R \nu_L) - \frac{M}{2} (N_R^T C N_R - N_R^+ C N_R^*).$$

In terms of Majorana spinors $(m = \frac{\nu_0 \nu}{\sqrt{2}})$

$$\nu_M = \nu_L + (\nu_R^c)_L \quad \text{and} \quad N_M = N_R + (N_R^c)_L,$$

the masses can be written in matrix form,

$$\mathcal{L} = -\frac{1}{2} \begin{pmatrix} \nu_M & N_M \end{pmatrix} \begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix} \begin{pmatrix} \nu_M \\ N_M \end{pmatrix}.$$ (Notice: $-M \bar{\nu}_M \nu_2 = -\frac{M}{2} \left( \bar{\nu}_M \nu_2 + (\bar{\nu}_M)^c \nu_2^c \right)$.)

The mass eigenstates are given by the eigenvalues of this matrix. In the limit $M \gg m$, they approximate as

$$m_1 \approx -\frac{m^2}{M}; \quad m_2 \approx M.$$

The corresponding eigenstates are

$$\nu_1 \approx \nu_M + \varepsilon N_M; \quad \nu_2 \approx N_M - \varepsilon \nu_M,$$

with $\varepsilon \approx \frac{m}{M}$. The physical states are thus two Majorana neutrinos $\nu_1$ and $\nu_2$: one very light neutrino $w/ m_1 \ll v$, and one very heavy neutrino $w/ m_2 \gg v$. 